SIMPLICITY: 1) PURIFICATION OF BINARY MIXTURES

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This short note presents an example of the application of the homogeneity measure, introduced by Claerbout in his paper ***The Purification of Binary Mixtures*** (this report, page 221). It is instructive to analyze the single parameter behavior of this function; therefore the least squares solution is compared to the simplicity solution.

The problem is to get a pure near trace N and a pure far trace F given records which are contaminated by some crossfeed phenomena:

$$x = N + \epsilon_0 F \tag{0a}$$

$$y = F + \epsilon_1 N \tag{0b}$$

An answer is found by combining the observations x and y and solving for a weight which optimizes a given measure:

$$z = x + \epsilon y \tag{1}$$

Least squares uses the L_2 norm as a measure, and the solution is found by minimizing the energy of the new trace z,

$$z^t z = (x + \epsilon y)^t (x + \epsilon y) \tag{2}$$

Using this criteria we obtain

$$\epsilon = -\frac{x^t y}{y^t y} \qquad (3)$$

Figure 1 shows the solution using this approach. The answer has 6.3% error and is above visual resolution.

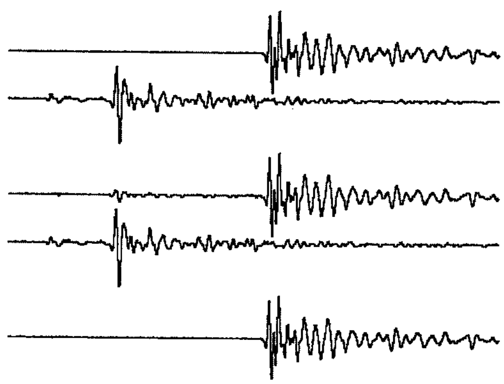


FIG. 1. Purification using the least squares approach. The top two traces are the unknown near trace N and far trace F; the traces labeled x and y constitute the input to the algorithm; weights $\epsilon_0 = 0.15$ and $\epsilon_1 = 0.$ were used. The last trace is the result of purifying the contaminated trace to get a pure far trace. The least squares solution is $\epsilon = 0.1405$. The error is above visual resolution as can be seen in the near trace residual in the purified trace.

The next approach is to use the ***simplicity*** measure defined as

$$S = \ln \frac{1}{N} \sum_{j=1}^{N} p_j - \frac{1}{N} \sum_{j=1}^{N} \ln p_j \qquad (4)$$

Claerbout in the paper previously referenced discusses two possibilities for defining the positive function p_j. The first one consists in using the envelope of the seismic trace,

$$p_t = (\bar{x}_t + \epsilon \bar{y}_t)(x_t + \epsilon y_t) \tag{5}$$

One possibility for finding the maximum is to expand S in a Taylor series

$$S = S_0 + \epsilon \frac{\partial S}{\partial \epsilon} + \frac{\epsilon^2}{2} \frac{\partial^2 S}{\partial \epsilon^2} \tag{6}$$

to get

$$\epsilon = - \frac{\frac{dS}{d\epsilon}}{\frac{d^2 S}{d\epsilon^2}} \tag{7}$$

Figure 2 shows the behavior of the simplicity measure using the complex envelope as a positive function of the data. Unfortunately the possibility of using second derivatives for the optimum value of the parameter ϵ will not work with this class of curve. The maximum value was found using an optimum linear search. The error in the estimate for ϵ is only 0.6% and well below visual resolution.

An alternative way of defining a positive function is to do a local average over neighboring points. Figure 4 shows the behavior of the simplicity measure as a function of different lengths for the window average. In general, the measure is insensitive to the length of the average, and the estimate of ϵ has a maximum error of 1.1%.

In general, the simplicity measure worked remarkably well in comparision to the least squares result for this single parameter problem. The next step is to introduce more parameters into the problem and hope the function will preserve its good condition so an optimization algorithm can be used to estimate the unknown parameters.

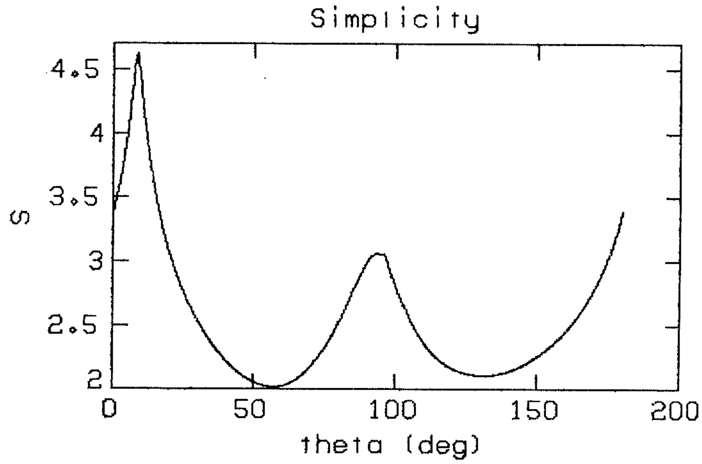


FIG. 2. Plot showing the simplicity curve as a function of angle where $z = x\cos(\theta) + y\sin(\theta)$. Two maximum values are found which correspond to the simplest near trace and the simplest far trace solutions. Note that the behavior of the function makes it not feasible to use second derivatives to estimate the optimum θ.

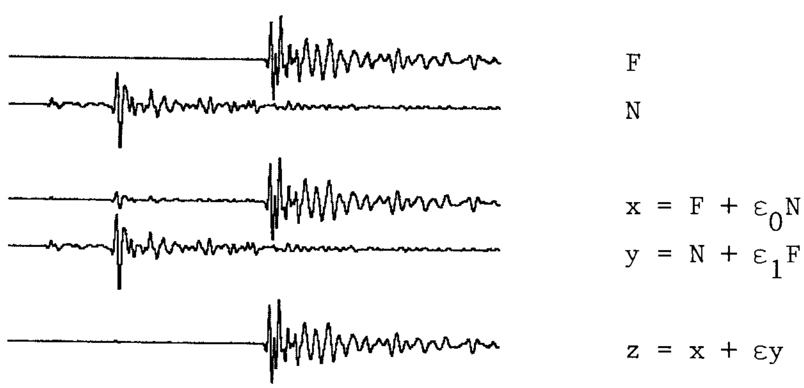


FIG. 3. Solution using the complex envelope approach. The first two traces are the unknown solution. The weights are the same as in Figure 1. The solution yields an $\varepsilon = 0.1509$, which is an order of magnitude better than the least squares answer.

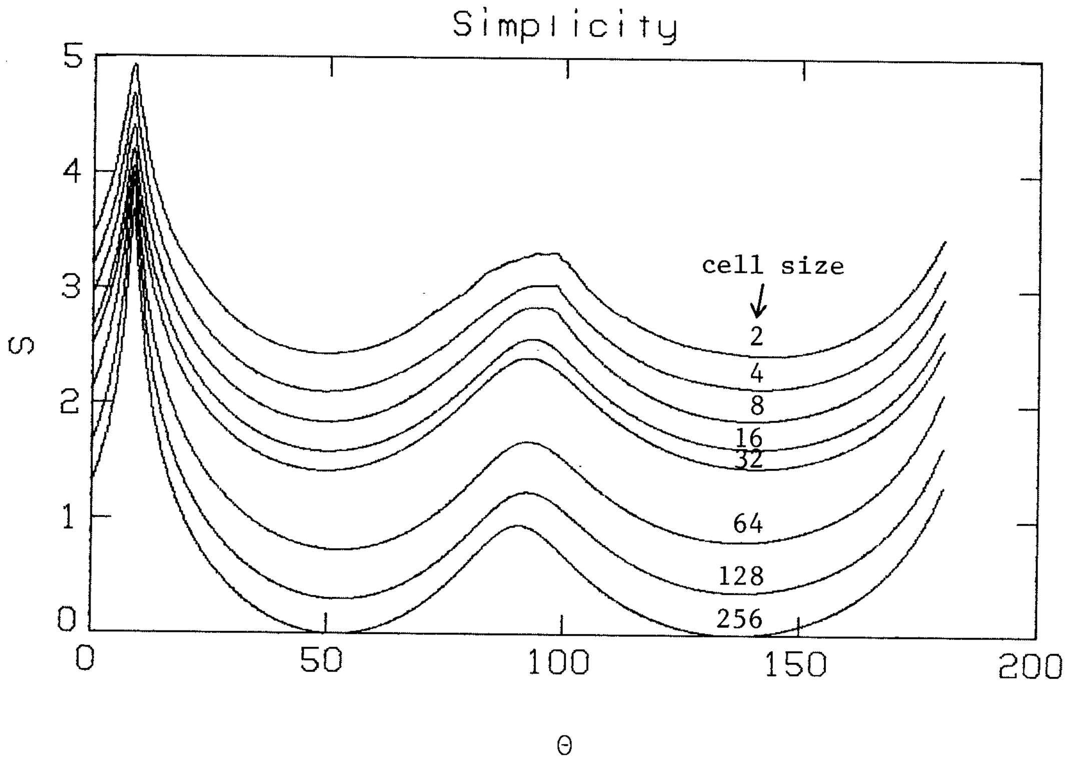
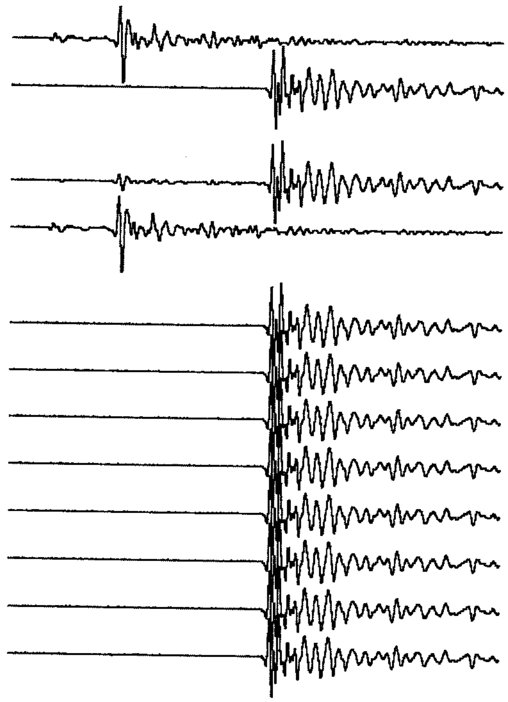


FIG. 4. Plot showing simplicity curves as function of angle for different sizes of window average. The solution is found optimizing the simplicity measure for $z = x \cos(\theta) + y \sin(\theta)$. The behavior of the measure is almost independent of cell size.



N

F

$$x = F + \varepsilon_0 N$$
$$y = N + \varepsilon_1 F$$

cell	ε
2	0.1505
4	0.1511
8	0.1512
16	0.1502
32	0.1506
64	0.1511
128	0.1517
256	0.1516

FIG. 5. These traces show the answer for the purification problem using the neighboring average approach for defining a positive function of the data. The answer is an order of magnitude better than the least squares solution.