THE DOUBLE SQUARE ROOT EQUATION IN MIDPOINT-OFFSET COORDINATES

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The double square root (DSR) equation in shot geophone coordinates has the relatively simple form

$$P_z = -i \frac{\omega}{v} \left\{ \left[1 - G^2 \right]^{\frac{1}{2}} + \left[1 - S^2 \right]^{\frac{1}{2}} \right\} P(k_g, k_s, \omega, z) \qquad (1)$$

where S and G are the normalized wavenumbers

$$S = \frac{vk_s}{\omega} \qquad \text{and} \qquad G = \frac{vk_g}{\omega}$$

The first square root governs the wavefield from the source to the reflector and S may be interpreted as the sin of the takeoff angle γ_s. The path from the reflector to the receiver is governed by the second square root, and G is the sin of the incident angle γ_g.

To convert the DSR equation into midpoint-offset coordinates as shown in figure 1 the transformation is

$$y = \frac{s + g}{2} \qquad \text{and} \qquad h = \frac{g - s}{2}$$

Using the chain rule we have

$$\frac{\partial}{\partial g} = \frac{\partial y}{\partial g} \frac{\partial}{\partial y} + \frac{\partial h}{\partial g} \frac{\partial}{\partial h} = \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial h}$$

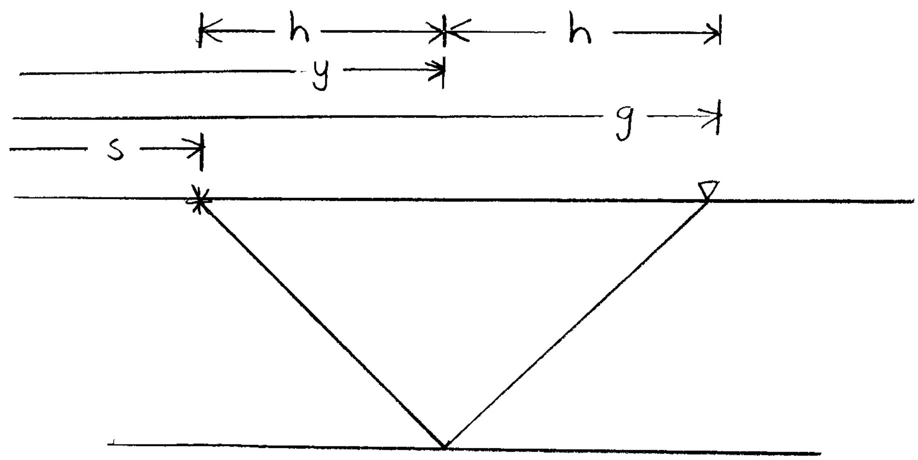

FIG. 1.

This implies, in the Fourier domain,

$$ik_g = \frac{1}{2} ik_y + \frac{1}{2} ik_h$$

or

$$\frac{vk_g}{\omega} = \frac{vk_y}{2\omega} + \frac{vk_h}{2\omega}$$

$$G = Y + H$$

Similarly, $S = Y - H$.

Substituting into equation (1), the DSR equation in midpoint-offset coordinates becomes

$$P_z = -i\frac{\omega}{v}\left\{\left[1 - (Y+H)^2\right]^{\frac{1}{2}} + \left[1 - (Y-H)^2\right]^{\frac{1}{2}}\right\} P(k_y, k_h, \omega, z) \qquad (2)$$

Interpreting Y and H

To understand the role of Y and H in equation (2), it is necessary to find their relationship to dip angle and offset. Consider the dipping bed shown in figure 2.

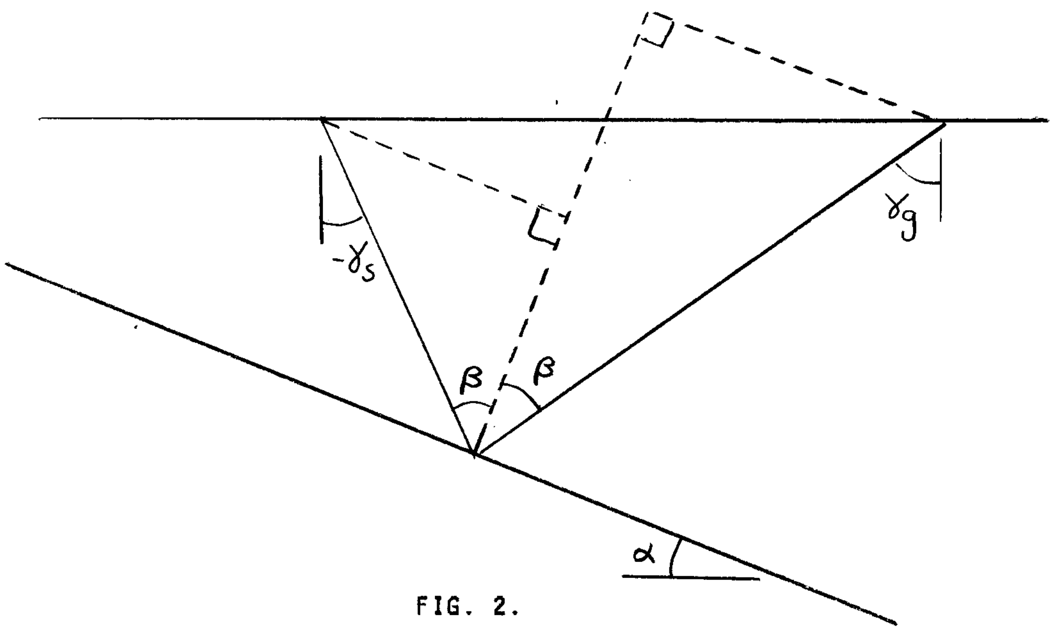

FIG. 2.

The dip angle of the reflector is α and the offset is expressed as the offset angle β. Also shown are the source takeoff angle γ_s and the incident receiver angle γ_g. The object of the following trigonometry is to relate Y and H to α and β. First we start by relating γ_s and γ_g to α and β. Adding up the angles of the small constructed triangle we have

$$\left(\frac{\pi}{2} - \gamma_s - \alpha\right) + \beta + \frac{\pi}{2} = \pi$$

or

$$\gamma_s = \beta - \alpha$$

Adding up the angles around the larger triangle we have

$$\gamma_g = \beta + \alpha$$

Previously we had noted that $S = \sin(\gamma_s)$, and that $S = Y - H$. Hence

$$Y - H = S = \sin\gamma_s = \sin(\beta - \alpha)$$

and similarly

$$Y + H = G = \sin\gamma_g = \sin(\beta + \alpha)$$

Solving for Y and H in this pair of equations we have

$$Y = \frac{1}{2}\sin(\beta + \alpha) + \frac{1}{2}\sin(\beta - \alpha)$$

$$H = \frac{1}{2}\sin(\beta + \alpha) - \frac{1}{2}\sin(\beta - \alpha)$$

With a more little trigonometry these relations can be reduced to

$$Y = \sin\alpha\ \cos\beta \qquad (3)$$

$$H = \sin\beta\ \cos\alpha \qquad (4)$$

Conventional Processing in Terms of the DSR Equation

With relations (3) and (4) we can examine some special cases of the DSR equation. First, if there is zero offset between the source and geophone then $\beta = 0$. This means $H = 0$ and $Y = \sin\alpha$. Equation (2) then reduces to the familiar zero offset migration equation

$$P_z = -2i\frac{\omega}{v}(1 - Y^2)^{\frac{1}{2}}\ P \qquad (5)$$

Second, if there is no dip (α=0), then equation (2) becomes

$$P_z = \left[-2i\frac{\omega}{v}(1 - H^2)^{\frac{1}{2}} + 2i\frac{\omega}{v}\right] P - 2i\frac{\omega}{v} P \qquad (6)$$

The operator in square brackets is the common midpoint stacking operator, while the last term in the equation is the time-to-depth conversion. Standard processing of reflection data usually starts with a common midpoint stack [the square bracketed part of equation (6)], and follows with a zero-offset migration [equation (5)]. The combined operation becomes

$$P_z = \left[-2i\ \frac{\omega}{v}\ (1 - H^2)^{\frac{1}{2}} + 2i\frac{\omega}{v}\right] P\ \ -2i\frac{\omega}{v}(1 - Y^2)^{\frac{1}{2}}\ P \qquad (7)$$

Equation (7) represents the approximation made to equation (2), when stacking and migration are applied as separate processes. Its validity is limited to small dip angles and small offsets. These limitations are the motivation for adding additional terms to equation (7), to make it valid for a wider range of angles and offsets.