ONE MORE TIME

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In order to apply f-k migration in a variable velocity medium, we all know that a stretching at the time axis

$$t \rightarrow t_n = \sqrt{\frac{2}{\bar{v}^2} \int_0^t dt\ t\ v_{rms}^2} \qquad (1)$$

is required. [In equation (1) v_{rms} is the time variable rms velocity, and $\bar{v}$ is an arbitrary constant velocity usually chosen so that t_n and t are about the same size.]

Apparently we don't all know why the particular form (1) is required (why not just $t_n = v_{rms}^2 t/\bar{v}^2$, for example?) or what it really does and doesn't do. Here, then, is another attempt to make everything perfectly clear.

The object of stretching the time axis is to make all diffraction hyperbolas, no matter where they are on the section, have the same moveout velocity. This is manifestly impossible: in a variable velocity world diffraction curves aren't hyperbolas, and any attempt to convert one into a hyperbola will only louse up its neighbors. However, in a layered medium the shallow parts of diffraction curves do look like hyperbolas, and it is possible to stretch the time axis so that these ***shallow*** parts all move out with the same velocity.

Look at the diffraction from a scatterer at (y,t) in a layered medium. At the nearby basement y + Δy, the diffraction from this point will begin at t + Δt, where

$$\Delta t \simeq \frac{2\Delta y^2}{v_{rms}^2 t} \tag{2}$$

Since v_{rms} is time-dependent, Δt is a complicated function of time. We would like to define a new time coordinate t_n such that moveout Δt_n is inversely proportional to time:

$$\Delta t_n = \frac{2\Delta y^2}{\bar{v}^2 t_n} \tag{3}$$

This requires that

$$\frac{\Delta t_n}{\Delta t} = \frac{v_{rms}^2 t}{\bar{v}^2 t_n} \tag{4}$$

If this relation is to be exact in the limit as Δt → 0, then

$$\frac{dt_n}{dt} = \frac{v_{rms}^2 t}{\bar{v}^2 t_n} \quad , \text{ or}$$

$$2\, t_n\, dt_n = \frac{2}{\bar{v}^2} v_{rms}^2\, t\, dt \tag{5}$$

which, when integrated, yields equation (1).

It should be clear from the above analysis that time-stretching is strictly valid only near the apex of a diffraction curve. Events will be migrated using the rms velocities at their apparent rather than their actual locations. Thus, if velocities are varying sufficiently rapidly, some mislocation of steeply dipping events can be expected. In practice, of course, an improvement over time-stretching requires (1) a comparatively expensive algorithm and (2) a detailed, accurate knowledge of velocity structure.