CHANGING THE SNELL PARAMETER OF SLANT STACKS

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The motivation for changing the Snell parameter of slant stacks is that a finite offset spread makes it impossible to coherently stack the full time range of the data for a given Snell parameter. Early reflections stack well for large Snell parameter values while late reflections stack better at smaller Snell parameter values. Therefore, it is advantageous to be able to take slant stack sections which have stacked well over a certain time range and transform their Snell parameter into another for which this time range may be absent. A transformation operator to accomplish this is derived from the double square root equation in a manner similar to the derivation of the deviation operator of Yilmaz and Claerbout (SEP-16).

The conversion operator comes from simultaneously applying the operators which migrate and diffract a slant stack section to and from a zero-offset section. However, the migration is done with the Snell parameter with which the slant stack section was created and is then diffracted using the new Snell parameter. The combined effect of both operators is a ***partial*** migration. The method of Yilmaz and Claerbout which manipulates frequency domain exponential operators by summing their exponential arguments will be used here. The slant stack migration operator from Ottolini (SEP-15) is

$$\frac{vk_z}{\omega} = -\{[1 - (Y + \hat{H}_1)^2]^{\frac{1}{2}} + [1 - (Y - \hat{H}_1)^2]^{\frac{1}{2}}\} \qquad (1)$$

where Y is the midpoint normalized wavenumber and $\hat{H}_i$ is the slant stack coordinate transformation of the offset normalized wavenumber for a given Snell parameter p:

$$\hat{H}_i = vp_i = \sin \theta_i \tag{2}$$

The diffraction operator is simply the complement of equation (1). Therefore, we take the conversion operation from Snell parameter p_1 to p_2 to be

$$\frac{vk_z}{\omega} = -\{[1 - (Y - \hat{H}_1)^2]^{\frac{1}{2}}\}$$

$$+ \{[1 - (Y + \hat{H}_2)^2]^{\frac{1}{2}} + [1 - (Y - \hat{H}_2)^2]^{\frac{1}{2}}\} \tag{3}$$

The operator of equation (3) is relatively expensive to implement in the frequency domain. Since the partial migration effect of this operator moves the data less than a full migration, a finite difference approximation of equation (3) is adequate. The spatial derivative operator contained in Y is removed from the square roots by a binomial expansion about small Y, i.e. small midpoint dip. Each square root of equation (3) becomes

$$[1 - (Y \pm \hat{H}_i)^2]^{\frac{1}{2}} \simeq [1 - \hat{H}_i^2]^{\frac{1}{2}} \left[1 \pm \frac{\hat{H}_i Y}{(1 - \hat{H}_i^2)} - \frac{Y^2}{(1 - \hat{H}_i^2)^2}\right] \tag{4a}$$

It is convenient at this point to convert $\hat{H}$ to sines and cosines according to equation (2):

$$[1 - (Y \pm \hat{H}_i)^2]^{\frac{1}{2}} \simeq \cos \theta_i \left[1 \pm \frac{\sin \theta_i Y}{\cos^2 \theta_i} - \frac{Y^2}{\cos^4 \theta_i}\right] \tag{4b}$$

Expanding all four square roots in equation (3) gives

$$\frac{vk_z}{\omega} \simeq 2\,[\cos\theta_2 - \cos\theta_1] + Y^2[\cos^{-3}\theta_1 - \cos^{-3}\theta_2] \qquad (5)$$

The wavenumbers k_z and $k_y = 2Y\omega/v$ are then Fourier-transformed resulting in the differential equation

$$P_z = 2\,\frac{\omega}{v}\,[\cos\theta_2 - \cos\theta_1]P + \frac{v}{4\omega}\,[\cos^{-3}\theta_1 - \cos^{-3}\theta_2]\,P_{yy} \qquad (6)$$

Equation (6) is a 15-degree equation similar to equation 10-4-7 in ***Fundamentals of Geophysical Data Processing***. This equation may be implemented by splitting the first (retardation) and second (diffraction) terms in the manner of Kjartansson (SEP-15). Another implementation scheme is analagous to Yilmaz's (SEP-18), which uses a post-NMO deviation operator. A moveout correction is added to equation (1), while at the same time a compensating depth-to-migrated-time coordinate transformation is made in equation (6). There is an advantage in having the results in migrated time, but the coordinate transformation is only valid for flat events. The derivation of this second implementation is not particularly instructive and is not given here.