SUMMARY AND CONCLUSION

We reviewed the fundamentals of extrapolation of wavefields which are acquired via seismic experiment with many shots and receivers. Mathematically, the wavefield extrapolation is described by the double square root equation. We determined that conventional processing is equivalent to a separable approximation to the double square root operator. Underlying this separation are two assumptions- ***zero-dip*** NMO and ***zero-offset*** migration. Through the separable approximation, downward extrapolation is achieved in two independent steps: seismic data is moveout corrected and stacked in offset space, and then migrated in midpoint space.

To the conventional processing scheme, we added ***partial migration before stack***. The theory for this process is also based on the double square root equation. The algorithm emerges from the deviation operator, which is formally defined as the difference between the double square root operator and its separable form. Pre-stack partial migration is applied on common offset sections, individually. It removes the effect of wide offsets and corrects for dipping events, thus producing a more coherent stack.

In the case of lateral velocity variation, a new term emerged from our analysis of the separable approximation. This term involves pure lateral shift due to non-zero offset and the lateral velocity gradient. A rough estimate (500 ft/sec difference over a distance of 1000 ft) on a field dataset indicates that a lateral velocity estimation scheme may be formulated based on the theory developed in Chapter 3. In particular, the offset angle and lateral velocity gradient are both significant near surface. Therefore, the theory has important implications for the statics problem. Offset-dependent statics estimation is perhaps a proper title for future work in this area.

Finally, the theory for the double square root equation and related operators was extended to 3-D recording geometry. This is another area of great potential for future research in exploration seismology. The continual increase in 3-D seismic exploration requires the research geophysicist to develop accurate and adequate processing procedures for 3-D data. Moreover, it is essential to develop a 3-D data acquisition technique that would not only be practical but would also make the related theory as uncluttered as possible.

References

Claerbout, J. F., 1976, Fundamentals of geophysical data processing: New York, McGraw-Hill.

Claerbout, J. F., 1978, Class notes: Dept. of Geophysics, Stanford University.

Conte, S. D., 1965, Elementary numerical analysis: McGraw-Hill.

Doherty, S. M., 1975, Structure independent velocity estimation, Ph.D. dissertation: Dept. of Geophysics, Stanford University.

Doherty, S. M. and Claerbout, J. F., 1976, Structure independent velocity estimation: Geophysics, v. 41, p. 850-881.

Dohr, G. P. and Stiller, P. K., 1975, Migration velocity determination: part II. applications: Geophysics, v. 40, p. 6-16.

Judson, D. R., Schultz, P. S. and Sherwood, J. W. C., 1978, Equalizing the stacking velocities of dipping events via Devilish: paper presented at the 48th Annual International SEG Meeting in San Francisco, October 1978.

Gardner, G. F., French, W. S. and Matzuk, T., 1974, Elements of migration and velocity analysis: Geophysics, v. 39, p. 811-825.

Sattlegger, J. W., 1975, Migration velocity determination: part I. philosophy: Geophysics, v. 40, p. 1-5.

Sherwood, J. W. C., Schultz, P. S., and Judson, D. R., 1976, Some recent developments in migration before stack, a short note released by Digicon, Inc.

Stolt, R. H., 1978, Migration by Fourier transform: Geophysics, v. 43, p. 23-48.

APPENDIX A

SECOND ORDER SQUARE ROOT EXPANSIONS

A-1

Consider the simplest case

$$(1 - A)^{1/2} = 1 - \frac{A}{2} - \frac{A^2}{8} \qquad \text{(A-1)}$$

where $1 \gg A$.

A-2

To second order in A, i.e. $1 > a \gg A$,

$$\left[1 - (a + A)^2 \right]^{1/2} = \left[1 - (a^2 + 2aA + A^2) \right]^{1/2}$$

"a" could be close to 1; therefore, factoring $(1 - a^2)$ from the square root prior to expansion we get

$$\left[1 - (a + A)^2 \right]^{1/2} = (1 - a^2)^{1/2} \left[1 - \frac{2aA + A^2}{1 - a^2} \right]^{1/2}$$

Applying (A-1) to the square root on the right,

$$\left[1 - (a + A)^2 \right]^{1/2} = (1 - a^2)^{1/2} \left[1 - \frac{2aA + A^2}{2(1 - a^2)} - \frac{(2aA + A^2)^2}{8(1 - a^2)^2} \right]$$

Expanding the right-hand side and ignoring terms of higher than second order in A, the final expression is

$$\left[1 - (a + A)^2 \right]^{1/2} = (1 - a^2)^{1/2} \left[1 - \frac{aA}{1 - a^2} - \frac{A^2}{2(1 - a^2)^2} \right] \tag{A-2}$$

A-3

To second order in A, i.e. $b > a \gg A$,

$$\left[b^2 - (a + A)^2 \right]^{1/2} = \left[(b^2 - a^2) - (2aA + A^2) \right]^{1/2}$$

Similarly to the development of (A-2),

$$= (b^2 - a^2)^{1/2} \left[1 - \frac{2aA + A^2}{b^2 - a^2} \right]^{1/2}$$

$$= (b^2 - a^2)^{1/2} \left[1 - \frac{2aA + A^2}{2(b^2 - a^2)} - \frac{(2aA + A^2)^2}{8(b^2 - a^2)^2} \right]$$

Finally,

$$\left[b^2 - (a + A)^2 \right]^{1/2} = (b^2 - a^2)^{1/2} \left[1 - \frac{aA}{b^2 - a^2} - \frac{b^2 A^2}{2(b^2 - a^2)^2} \right] \tag{A-3}$$

Setting b = 1 , (A-3) reduces to (A-2), as should be the case.

A-4

Scaling (A-2) with α yields

$$\left[1 - \left(\frac{a + A}{\alpha} \right)^2 \right]^{1/2} = \left(1 - \frac{a^2}{\alpha^2} \right)^{1/2} \left[1 - \frac{aA}{\alpha^2 \left(1 - \frac{a^2}{\alpha^2} \right)} - \frac{A^2}{2\alpha^2 \left(1 - \frac{a^2}{\alpha^2} \right)^2} \right]$$

$$= \left(1 - \frac{a^2}{\alpha^2} \right)^{1/2} \left[1 - \frac{aA}{\alpha^2 - a^2} - \frac{\alpha^2 A^2}{2(\alpha^2 - a^2)^2} \right]$$

Finally,

$$\left[1 - \left(\frac{a + A}{\alpha} \right)^2 \right]^{1/2} = \frac{1}{\alpha} (\alpha^2 - a^2)^{1/2} \left[1 - \frac{aA}{\alpha^2 - a^2} - \frac{\alpha^2 A^2}{2(\alpha^2 - a^2)^2} \right] \qquad \text{(A-4)}$$

Setting α = 1 , (A-4) reduces to (A-2) as we would expect.

A-5

To second order in A and B, i.e. $1 > (a^2 + b^2)^{\frac{1}{2}} >> (A^2 + B^2)^{\frac{1}{2}}$.

$$\left[1 - (a + A)^2 - (b + B)^2 \right]^{1/2} = \left[1 - (a^2 + 2aA + A^2 + b^2 + 2bB + B^2) \right]^{1/2}$$

$$= (1 - a^2 - b^2)^{1/2} \left[1 - \frac{2aA + A^2 + 2bB + B^2}{2(1 - a^2 - b^2)} - \frac{(2aA + A^2 + 2bB + B^2)^2}{8(1 - a^2 - b^2)^2} \right]$$

Expanding and ignoring terms of higher than second order in A, B and AB, we have the final form

$$\left[1 - (a + A)^2 - (b + B)^2 \right]^{1/2} = (1 - a^2 - b^2)^{1/2} \qquad \text{(A-5)}$$

$$\cdot \left[1 - \frac{aA + bB}{1 - a^2 - b^2} - \frac{A^2 + B^2}{2(1 - a^2 - b^2)} - \frac{(aA + bB)^2}{2(1 - a^2 - b^2)^2} \right]$$

Setting $b = B = 0$, (A-5) reduces to (A-2).

APPENDIX B

STATIONARY PHASE APPROXIMATIONS

B-1

Consider the double square root operator

$$DSR(Y,H) = \left[1 - \left(Y + H\right)^2\right]^{1/2} + \left[1 - \left(Y - H\right)^2\right]^{1/2} \qquad \text{(B-1)}$$

where

$$\begin{bmatrix} Y \\ H \end{bmatrix} = \frac{v}{2\omega} \begin{bmatrix} k_y \\ k_h \end{bmatrix}$$

We want to operate on the transformed wavefield $P(k_y,k_h,0,\omega)$ with (B-1). Subsequent inverse Fourier transformation will yield the wavefield $P(y,h,z,t)$:

$$P(y,h,z,t) = \iiint P(k_y,k_h,0,\omega)\ e^{i\Phi z}\ dk_y\ dk_h\ d\omega \qquad \text{(B-2)}$$

where the total phase, normalized with respect to z, is

$$\Phi = -\frac{\omega}{v} DSR(Y,H) - k_y \frac{y}{z} - k_h \frac{h}{z} + \omega \frac{t}{z} \qquad \text{(B-3)}$$

The main contribution to integration in (B-2) occurs when the phase stays nearly constant. We therefore set the variation of the phase with respect to variables k_h , k_y and ω to zero . Using (B-3) we have,

$$\frac{\partial\Phi}{\partial k_h} = - \frac{\omega}{v} \frac{\partial DSR(Y,H)}{\partial H} \frac{\partial H}{\partial k_h} - \frac{h}{z} = 0$$

$$\frac{\partial\Phi}{\partial k_y} = - \frac{\omega}{v} \frac{\partial DSR(Y,H)}{\partial Y} \frac{\partial Y}{\partial k_y} - \frac{y}{z} = 0$$

$$\frac{\partial\Phi}{\partial\omega} = - \frac{1}{v} DSR(Y,H) - \frac{\omega}{v} \left[\frac{\partial DSR(Y,H)}{\partial H} \frac{\partial H}{\partial\omega} + \frac{\partial DSR(Y,H)}{\partial Y} \frac{\partial Y}{\partial\omega} \right] + \frac{t}{z} = 0$$

Substituting (B-1) and carrying out the differentiation we obtain

$$\frac{1}{2} \frac{G}{\left[1 - G^2 \right]^{1/2}} - \frac{1}{2} \frac{S}{\left[1 - S^2 \right]^{1/2}} = \frac{h}{z} \qquad \text{(B-4a)}$$

$$\frac{1}{2} \frac{G}{\left[1 - G^2 \right]^{1/2}} + \frac{1}{2} \frac{S}{\left[1 - S^2 \right]^{1/2}} = \frac{y}{z} \qquad \text{(B-4b)}$$

$$\frac{1}{\left[1 - G^2 \right]^{1/2}} + \frac{1}{\left[1 - S^2 \right]^{1/2}} = \frac{v\ t}{z} \qquad \text{(B-4c)}$$

where

$$G = Y + H \quad , \quad S = Y - H$$

Eliminating G and S among (B-4a,b,c),

$$\left[(y + h)^2 + z^2 \right]^{1/2} + \left[(y - h)^2 + z^2 \right]^{1/2} = v\ t \qquad \text{(B-5)}$$

which is an ellipse in (y,z)-space at constant t.

Specializing to the zero-offset case (h=0), (B-5) takes the form

$$\left(y^2 + z^2 \right)^{1/2} = \frac{v\ t}{2} \qquad \text{(B-6)}$$

which is a circle in (y,z)-space at constant t and a hyperbola in (y,t)-space at constant z.

B-2

Consider the stacking operator

$$St(H) = 2 \left(1 - H^2 \right)^{1/2} - 2 \qquad \text{(B-7)}$$

The total phase is

$$\Phi = - \frac{\omega}{v} St(H) - k_h \frac{h}{z} + \omega \frac{t}{z} \qquad \text{(B-8)}$$

Differentiating (B-8) with respect to k_h and ω and setting the result to zero, we obtain

$$\frac{H}{\left(1 - H^2 \right)^{1/2}} = \frac{h}{z} \qquad \text{(B-9a)}$$

and

$$\frac{1}{\left(1 - H^2\right)^{1/2}} = \frac{v\ t}{2\ z} + 1 \qquad \text{(B-9b)}$$

Eliminating H between (B-9a) and (B-9b),

$$\left(h^2 + z^2\right)^{1/2} = \frac{v\ t}{2} + z \qquad \text{(B-10a)}$$

For non-retarded St(H), (B-10a) takes the form

$$\left(h^2 + z^2\right)^{1/2} = \frac{v\ t}{2} \qquad \text{(B-10b)}$$

which is the normal moveout equation. Let us define the zero-offset two-way time as

$$t' = \frac{2\ z}{v}$$

Substituting into (B-10a)

$$\left[h^2 + \left(\frac{v\ t'}{2}\right)^2\right]^{1/2} = \frac{v\ t}{2} + \frac{v\ t'}{2}$$

Rearranging, we get the classical NMO shift

$$t = t' \left\{ \left[1 + \left(\frac{2\ h}{v\ t'} \right)^2 \right]^{1/2} - 1 \right\} \qquad \text{(B-11)}$$

which is the equation for moveout correction. In the main text, the NMO shift is defined as Δt (Equation 1-33), which is the same as t in (B-11).

B-3

Consider the non-retarded separable operator

$$\mathrm{Sep}(Y,H) = 2 \left(1 - Y^2 \right)^{1/2} + 2 \left(1 - H^2 \right)^{1/2} \qquad \text{(B-12)}$$

The total phase is

$$\Phi = - \frac{\omega}{v} \mathrm{Sep}(Y,H) - k_y \frac{y}{z} - k_h \frac{h}{z} + \omega \frac{t}{z} \qquad \text{(B-13)}$$

Differentiating with respect to k_h , k_y , and ω ,

$$\frac{H}{\left(1 - H^2 \right)^{1/2}} = \frac{h}{z} \qquad \text{(B-14a)}$$

$$\frac{Y}{\left(1 - Y^2 \right)^{1/2}} = \frac{y}{z} \qquad \text{(B-14b)}$$

$$\frac{2}{v}\frac{1}{\left(1 - Y^2\right)^{1/2}} + \frac{2}{v}\frac{1}{\left(1 - H^2\right)^{1/2}} = \frac{t}{z} \qquad \text{(B-14c)}$$

Eliminating Y and H, we have the final expression

$$\left(h^2 + z^2\right)^{1/2} + \left(y^2 + z^2\right)^{1/2} = \frac{v\ t}{2} \qquad \text{(B-15)}$$

B-4

Consider the approximate form of the deviation operator

$$\text{Dev}(Y,\hat{H}) = C(\hat{H})\ Y^2 \qquad \text{(B-16)}$$

where $C(\hat{H})$ is

$$C(\hat{H}) = \left[1 - \left(1 - \hat{H}^2\right)^{-3/2}\right] \qquad \text{(B-17a)}$$

or

$$C(\hat{H}) = -\frac{3}{2}\hat{H}^2 \qquad \text{(B-17b)}$$

and

$$\hat{H} = \frac{2\ h}{v\ t} \qquad \text{(B-18a)}$$

which, for constant velocity medium, can be rewritten as

$$\hat{H} = \frac{h / z}{\left[1 + h^2 / z^2 \right]^{1/2}} \tag{B-18b}$$

The total phase is

$$\Phi = - \frac{\omega}{v} \, Dev(Y,\hat{H}) - k_y \frac{y}{z} + \omega \frac{t}{z} \tag{B-19}$$

Differentiating with respect to k_y and ω ,

$$C(\hat{H}) \; Y = - \frac{y}{z} \tag{B-20a}$$

$$3 \; C(\hat{H}) \; Y^2 = \frac{v \; t}{z} \tag{B-20b}$$

Eliminating Y between (B-20a) and (B-20b) we obtain

$$y^2 = \frac{v \; t \; z}{3 \; C(\hat{H})} \tag{B-21}$$

Since $C(\hat{H})$ is a function of z and h by way of (B-18b), (B-21) describes a parabola in (y,t)-space.