SIGN CONVENTION IN FOURIER TRANSFORM

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Choice of the sign of i in Fourier transformation is popularly regarded as a purely arbitrary matter of convention. Actually, a particular choice of sign at the outset of some analysis can reduce the number of subsequent minus signs. Human confusion by double negatives and aversion to minus signs will explain the fact that the disciplines of physics and electrical engineering (EE) have settled on opposite sign conventions. Physics is so enmeshed with its sign convention that the Schrodinger equation of quantum physics (Schiff, 1955) does not by itself represent a law of nature. It is necessary to add that the i that appears in the equation must be replaced by $-i$ if the sign convention of Fourier time transformation is reversed! (Finite-difference-oriented geophysicists will immediately suspect that the Schrodinger equation is not a law of nature at all but merely an approximation valid when the potential energy is nearly time invariant.)

Electrical engineers describe time variation of voltages and currents at circuit modes and loops. For them it is convenient to choose a sign convention such that time derivative ∂_t may be represented in Fourier transform space by $+i\omega$ rather than $-i\omega$. For physicists the situation is complicated by the fact that they often wish to take Fourier transforms over three-dimensional space coordinates as well as time. Naturally they would like the gradient operator in Cartesian coordinates $\nabla = (\partial_x, \partial_y, \partial_z)$ to have a Fourier representation given with plus signs, say $i(k_x, k_y, k_z)$. They would also, however, like waves to ordinarily be moving in the positive direction along the spatial axis, say in the direction of the $\vec{k}$ vector. Here there is a problem since a wave moving positively along the spatial axis is described by either $\cos(\omega t - kx)$ or $\cos(kx - \omega t)$ but not $\cos(\omega t + kx)$ as would be necessary for time and space derivatives to both have positive Fourier duals. In Cartesian space it might be realistic to demand that waves propagate along the negative space axis, but this would be wholly unrealistic in spherical coordinates where atoms radiate energy positively out the radial r-axis. Likewise, cylindrical coordinates are often used in exploration geophysics where there is a point source in a layered medium. In nature, it is highly improbable to observe waves coming from infinity collapsing along the radial axis. So whatever sign convention we pick for Fourier transformation of time must be the opposite of the choice for space. Electrical engineers infrequently deal with the space axes, so naturally they chose to simplify temporal frequency expressions. Physicists are often involved with three-dimensional space axes where they may use the complicated special functions of mathematics which arise from the separation of partial differential equations in curvilinear coordinates. Naturally, they chose to simplify the space calculations. In seismic exploration we are usually spared the need for curvilinear coordinates. The most complicated space axes we need to consider may be midpoint $y = (g + s)/2$ and offset $h = (g - s)/2$. So even conventional seismic data analysis involves the four space coordinates (s,g,y,h). What sign convention should we choose?

Some reasons for the EE choice are: (1) The Fourier transform subroutine in our array processor library uses the EE sign convention for time transforms (which is the physics convention for space transforms). (2) The solid earth is time invariant but not space invariant, so in principle we should always do our space calculations by finite differences rather than Fourier transforms.

Some reasons for the physics choice are: (1) Time is one-dimensional and uniform, but space is multidimensional and multifaceted. (2) In theoretical work we nearly always use Fourier transforms on space axes and we often do so in computations too. (3) *Fundamentals of Geophysical Data Processing* uses physics notation consistently throughout, as do all the tutorial papers in the SEP reports.

REFERENCE

SCHIFF, Leonard I. (1955), *Quantum Mechanics* (McGraw-Hill), pp. 20-21, equations (6.13) and (6.16).