# DOWNWARD- AND UPWARD-CONTINUING HEAD WAVES

*Bert Jacobs and Jon F. Claerbout*

*Abstract*

An algorithm is designed that can model head waves in a z-variable velocity medium and a variety of recording geometries. The only innovation is a new source function, to replace the spike at zero travel-time, for scatterers which lie on top of refracting interfaces.

*Introduction*

Head waves are a common feature of seismic sections and gathers that are not included in wave equation modeling or imaging procedures. In a zero-dip environment, refractions spend part of their time moving along horizontal ray paths that our one-way algorithms do not model. If we consider an upward-continuation of the wavefield when the shots and geophones are near a refracting interface, we can localize the source of our difficulty. Below the refractor the head wave is traveling in the wrong direction and attenuating rapidly, so considering it to be zero is not a bad approximation. Above, the head wave can be continued like any other wave. At the refractor we normally sum in a spike wherever there is a scatterer, since this is thought to be the seismogram when the shots and geophones are at the z-level of the scatterer. At a refracting surface this is not true because the seismogram recorded there is much more complicated. In other words, if head waves are to be accounted for, then our algorithms are not adding in the right source functions at the scatterer locations.

### *Exploding ships at the seafloor*

The key to the theory in this paper will be the source function of Figure 1. To understand this diagram, consider the following situation. A seismic exploration ship is sent into dangerous waters, capsizes, and sinks to the bottom of the sea, dragging its geophone array with it. Its owner sends a second ship after it. When this second ship is directly above the first, the sunken ship suddenly explodes. Our oil company finds itself in possession of two sets of seismic records - one recorded at the surface and the other obtained by an array on the seafloor. The peculiar recording geometry and their two seismograms (with direct arrivals removed by dip filtering) are all in Figure 2.

The seismogram from the sunken array consists of two types of events. First, it contains a big spike at the zero offset geophone from the explosion of the ship. Secondly, it contains an arm connected to the spike. Geophysicists employed by the firm claim that this branch is due to the passage of a seafloor refraction. They also claim that they can upward-continue this seismogram and get the same record as that obtained by the surface vessel. This is easy, since the migration can be done by superimposing two known migrations, an operation similar to that done in Figure 1.

What can we learn from this misfortune? If we consider the exploding ship as a big reflection coefficient, then we can get the surface seismogram by upward-continuing the seafloor seismogram - with the refraction. Usually a spike is used as a source function, and this is equivalent to leaving out the refractions. The seismogram the sunken ship has recorded is a prototype source function for scatterers on refracting interfaces, provided that we are recording common-shot gathers.

### *Extension to CMP gathers and zero-offset sections*

We now know how to model a head wave recorded with common-shot geometry. Consideration of the ray path diagram of Figure 3 will make it possible to model refractions recorded with common-midpoint and zero-offset recording geometry.

In Figure 3, from top to bottom, are ray path diagrams corresponding to explosive-reflector, common-midpoint, and zero-offset recording

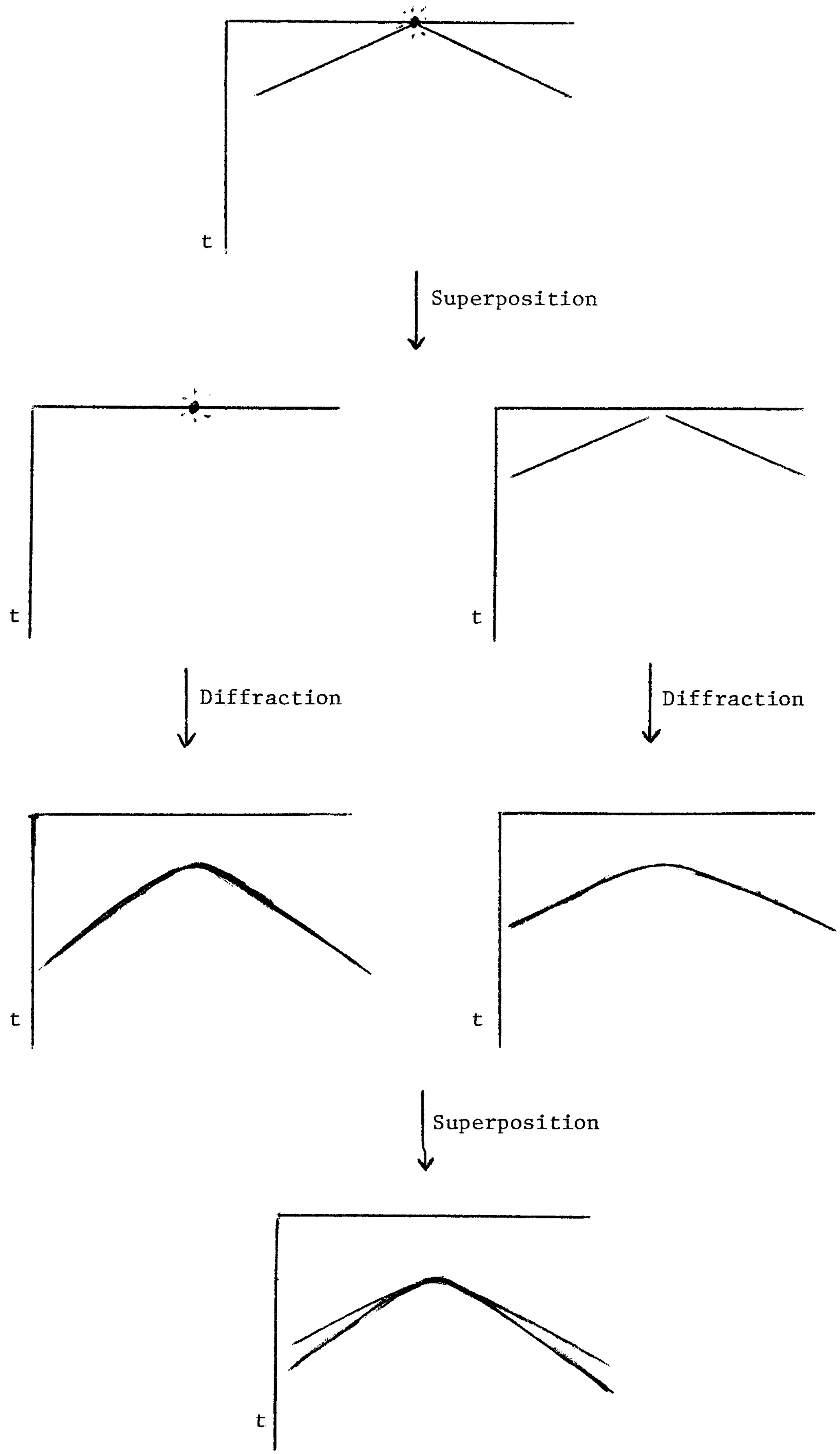


FIGURE 1.--The diagram at the top is the source function for a point scatterer on a refracting interface. It is the seismogram when both shots and geophones are at the scatterer's z-level. Its diffraction pattern at the surface can be built up from known diffraction patterns by using superposition.

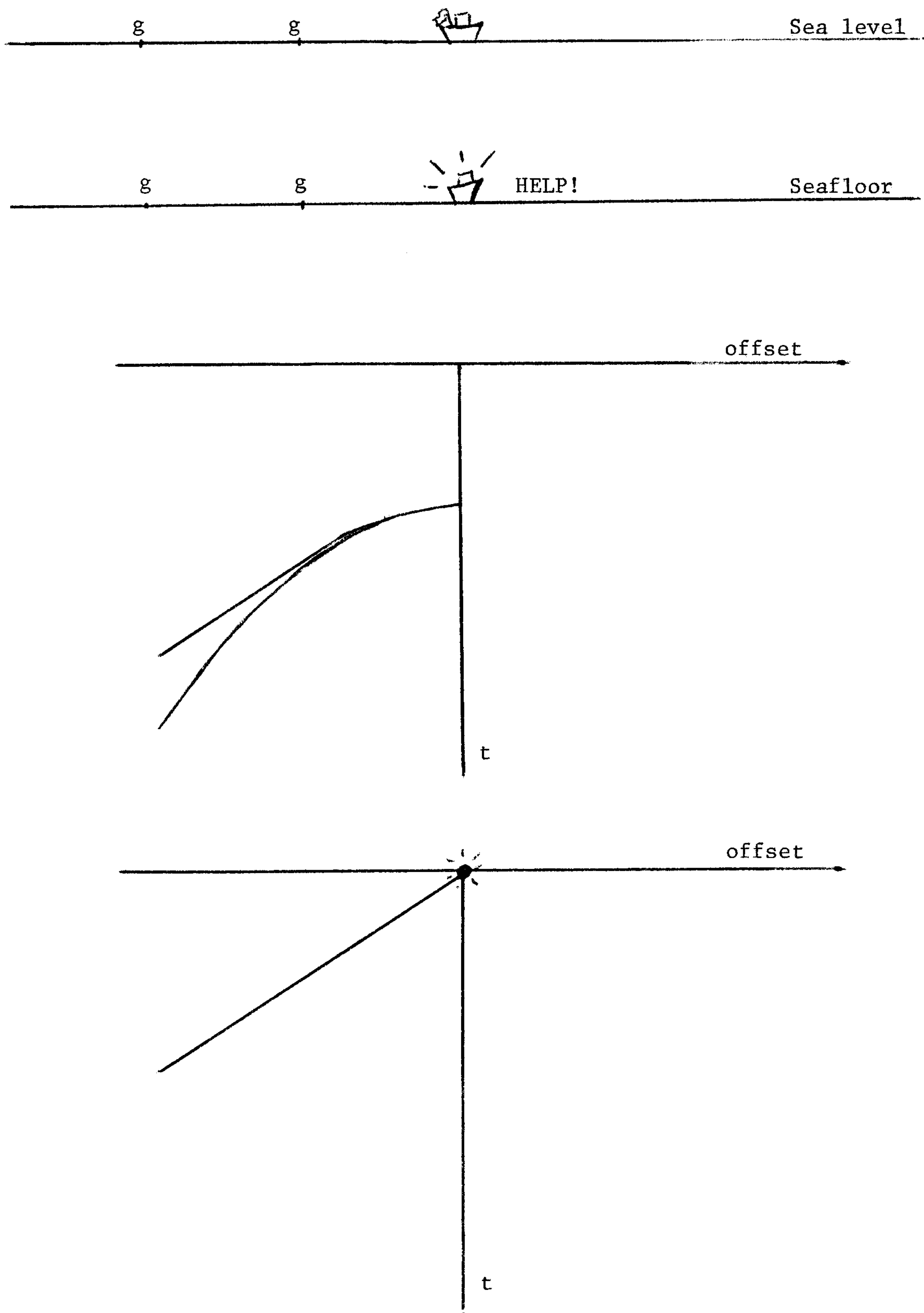


FIGURE 2.--Top: Exploding ship recording geometry. Two common shot gathers are recorded, one at the sea floor and the other at sea level. Middle: Sea level (z=0) recording. Bottom: Seafloor seismogram composed of an arm connected to a zero-offset, zero-time spike.

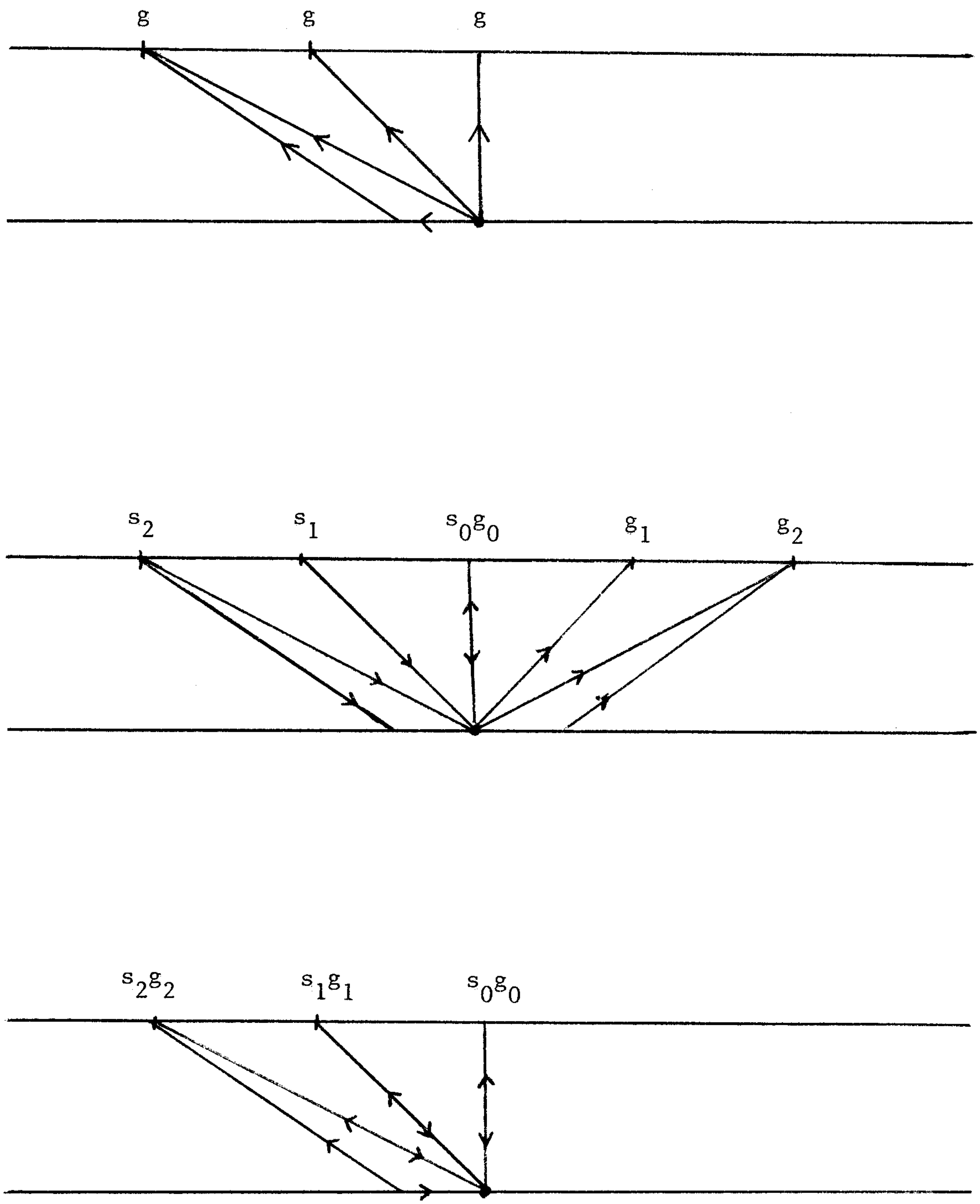


FIGURE 3.--Top: Common shot recording geometry. Reflective ray paths connect the shot to every geophone. Refractive ray paths, on the other hand, only connect the far offset phones. Middle: Common midpoint geometry. Bottom: Zero-offset ray path diagram.

geometries. Their respective seismograms are displayed in Figure 4. Again, from top to bottom, are the common-shot, common-midpoint, and zero-offset cases.

The most striking thing about these seismograms is that they all have the same pattern. In fact, if the explosive-reflector time axis was indexed with one-way rather than two-way traveltime, then the three seismograms would be identical except for the labeling of the horizontal position axis.

The similarity of the impulse responses of Figure 4 leads one to expect that the wave equations for the three geometries will also resemble one another. This is true within factors of two:

$$U_z(G,\omega,z) = i \frac{\omega}{v(z)} (1 - G^2)^{1/2} U(G,\omega,z) \qquad \text{Explosive-reflectors} \qquad (1)$$

$$U_z(Y,\omega,z) = i \frac{\omega}{v(z)} (1 - Y^2)^{1/2} U(Y,\omega,z) \qquad \text{Zero-offset} \qquad (2)$$

$$U_z(H,\omega,z) = i \frac{\omega}{v(z)} (1 - H^2)^{1/2} U(H,\omega,z) \qquad \text{Common-midpoint} \qquad (3)$$

where G, Y, and H, are wave numbers for geophone, midpoint, and offset coordinates, respectively. The three wave equations can be derived from the double square root equation.

The algorithm for modeling in all three recording geometries is now easily stated. Given the wavefield from the previous z-step we want the wavefield at the next z-level. To get this, add in the appropriate source functions for the scatterers at this z-level. If there is no refracting interface there, use spikes as source functions - otherwise, sources like that of Figure 1 need to be summed in. After adding the two wavefields, the next step is to use the telescope equation form of the appropriate continuation equation (1-3) to project the sum to the next z-level. This process should be repeated until the surface is reached.

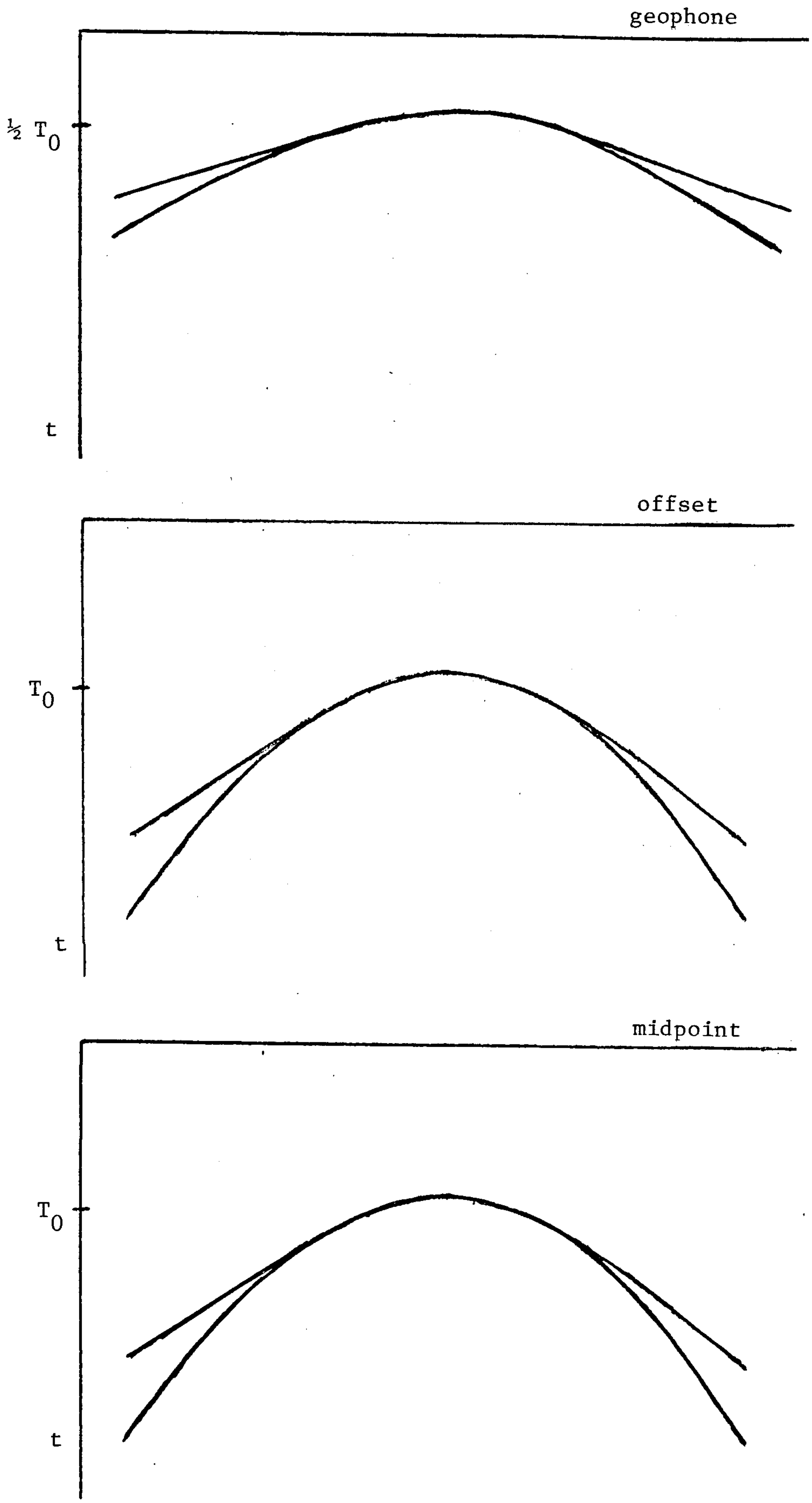


FIGURE 4.--Top: Common shot seismogram for the recording geometry at the top of Figure 3. Middle: Common midpoint seismogram. Bottom: Zero-offset recording corresponding to the diagram at the bottom of Figure 3.

*Analytic forms for the source function*

Our wave equation should be implemented as a telescope equation (Claerbout, *Fundamentals of Geophysical Data Processing*, p. 190) in any modeling procedure so that the algorithm is accurate for waves traveling just shy of 90 degrees to the vertical. This means that the procedure should be in the Fourier domain. An analytic expression for the source function in the frequency domain would therefore be of great use.

A time-space domain expression for the source function for a scatterer at position $x_0$ is

$$p(x,t) = c_1(x_0)\ \delta(t)\ \delta(x - x_0) + c_2(x_0)\ e^{-\varepsilon t}\ H(t) \left[ \left( t + \frac{x - x_0}{v} \right) + \left( t - \frac{x - x_0}{v} \right) \right]$$

where $H(t)$ is the Heaviside function used to insure causality. The coefficients $c_1(x_0)$ and $c_2(x_0)$ control the amplitudes of the reflection and refraction respectively. A slow amplitude decay along the branches of the refractive portion of the source function is included because the wave traveling along the refractive interface decays like $r^{-1.5}$ in a two-dimensional world. This result can be obtained by considering the asymptotic properties of the head wave expression from Cagniard's method.

The two-dimensional Fourier transform of this source function is to be used in a modeling procedure. The continuous transform is given by Equation (4).

$$P(k,\omega) = c_1(x_0)\ e^{-ikx_0} + v\ c_2(x_0)\ e^{-ikx_0} \left[ \frac{1}{i(\omega + vk) + \varepsilon} + \frac{1}{i(\omega - vk) + \varepsilon} \right] \tag{4}$$

This expression was checked by implementing it in the SEP computer. After multiplication by $e^{-i\omega t_0}$, the product was Fourier transformed. The result, Figure 5, is offered as evidence that Equation (4) is the right transform for this application.

To properly model head wave amplitudes is much trickier. The decay is like r in the far field, so a frequency domain convolution would have to be implemented. The decay with distance is a function of distance, and this should prove difficult and cumbersome to include in a modeling procedure.

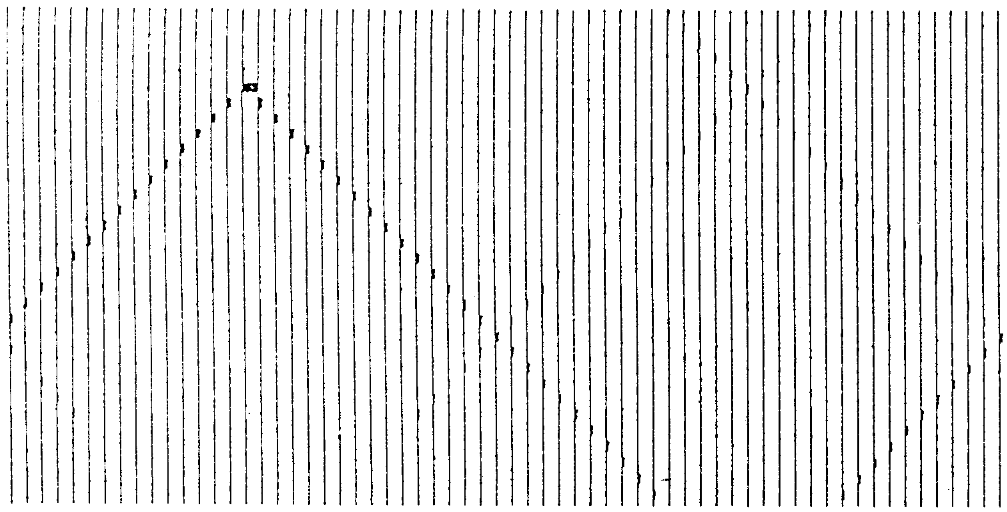

FIGURE 5.--Fourier domain generated source function shifted downward in time so that the causality of the function can be seen. Parameters: $c_1 = 0.60$, $c_2 = 0.20$, $\varepsilon = 0.02$, $v = 0.50$, $nx = nt = 64$, $x = t = 1.0$.

REFERENCES

CLAERBOUT, J. F., "The Double Square Root Equation," SEP-15, pp. 73-80.

CLAYTON, R., "Common Midpoint Migration," SEP-14, pp. 21-36.

HILL, D. P., "Velocity Gradients and Anelasticity from Crustal Body Wave Amplitudes," *Journal of Geophysical Research*, 76:14, May 1971, pp. 3309-3325.