SIMULTANEOUS NOAH'S DECONVOLUTION OF SHOT WAVEFORM AND MULTIPLE SIGNATURES

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Abstract

An extension of the Noah's method of deconvolution from the single channel to a two-channel case is presented. This extension is made to enhance decoupling between the shot waveform and the multiple reverberation train. In the two-channel case deconvolution consists of solving for small perturbations in the assumed shot waveform and reflectivity series.

Synthetic examples show that this method is capable of tracking both the shot waveform and primary reflectors through zones of marine data where primaries and multiples strongly interfere.

Introduction

Shot waveform estimation is one of the most persistent problems confronting the geophysicist wanting to suppress multiples from marine data. In a first step towards solving this problem, Riley (1976) introduced the notion of the Noah's seismogram. He showed that a useful approximation to the zero-dip-zero-offset reflection response could be obtained from this model. It turns out, however, that the Noah's model holds for slant-stacked data even if the zero offset assumption is relaxed. This is because slant stacks preserve the reverberation periods of multiples for a z-variable velocity earth (Morley and Claerbout, SEP-15, p. 191).

The proposed method of deconvolution is therefore intended for application to slant stacked data over areas where reflectivity is approximately one-dimensional. Admittedly, the restriction to small reflector dip is fairly severe, but there are real situations where this is not an unreasonable assumption. The data domain of zero dip and non-zero offset is still an area of processing in which satisfactory practical solutions need to be developed. It is also the area in which any improvements over the NMO-CDP STACK approach to multiple suppression is most likely to be made. Hopefully this model will also furnish insight into the general approach required for shot waveform and reflectivity estimation in more complicated situations.

The single channel case

The Noah's seismogram assumes that the observed upcoming wave $y(t)$ is the convolution of the shot waveform $b(t)$ with the earth's impulse response $c(t)$, corrected for surface multiples only.

In particular,

$$Y(z) = B(z)\, C(z)\, [1 - C(z) + C^2(z) - C^3(z) + \ldots] \tag{1}$$

or

$$Y = BC/(1 + C) \tag{2}$$

The algebraic inverse of (2) is

$$C = Y/(B - Y) \tag{3}$$

If the (gated) first multiple M is free of both noise and primaries, then we should have the relation

$$BM = P^2 \tag{4}$$

where P is the appropriately gated seafloor primary. Equation (4) may be better understood by noting that P^2 contains a factor of B^2 but that M contains only a single factor of B.

Riley (1976) used (4) to obtain a least square estimate of $\bar{B}$ to B by solving the problem

$$\bar{B} = \min_{B} ||BM - P^2|| \tag{5}$$

by Levinson recursion. He then obtained a single channel deconvolution by a further Levinson recursion on (3) with $\overline{B}$ substituted for B. Equation (5) not only gives the color of the shot waveform, but it also implicitly finds the basic delay time in the shot waveform and the overall scale factor between upcoming and downgoing waves. It does this using only the measured upcoming wave (and the assumption that the downgoing wave is consistent with the Noah's model).

The most attractive feature of this method is that stability is guaranteed since the algorithm is looking at the entire trace at once rather than recursively computing innovations in the output. Unfortunately, this approach only works for the one-dimensional case. F-K methods fail in two dimensions because the equations governing up- and downgoing waves are coupled according to Equation (6):

$$U''_{z''} + \frac{v}{2} U''^{t''}_{x''x''} = c'(x',z')\ D'[x'',z'',t''-(2z''/v'')]$$

$$D'_{z'} - \frac{v}{2} D'^{t'}_{x'x'} = 0 \qquad (6)^*$$

That is, F-K methods are inappropriate for (6) since we have a multiplication in ω but a convolution in k_x. This is because the reflectors, c', depend on x but are independent of ω.

Motivation for two-channel algorithm

The last section indicated that deconvolution in the presence of multiple reflections consists of two different steps. The first step calls for estimation of the shot waveform B. The second step involves deconvolving both the shot waveform estimate and the reverberation train of $1/(1 + C)$ from the observed seismogram. It is in this second step that a two-channel algorithm can be used to advantage. We can use it to exploit the fact that B and C enter into Y [Equation (2)] in fundamentally different ways - B enters as a direct convolution, whereas C enters as

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*The reader should refer to *Fundamentals of Geophysical Data Processing*, p. 258, for a more detailed discussion of these equations.

feedback. A simultaneous deconvolution of B and 1/(1 + C) should give a better result than trying to handle the two problems separately and ending up with crossfeed residuals between the two.

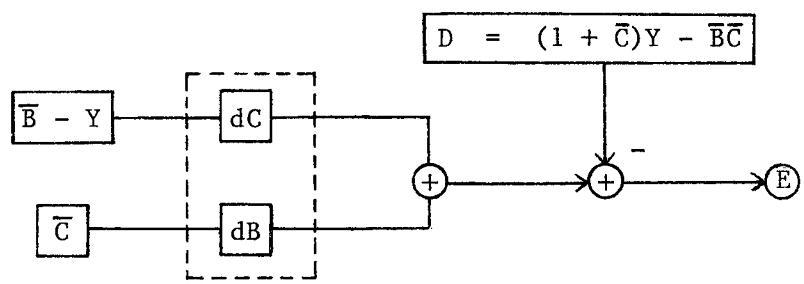
The algorithm

In most situations of interest it is reasonable to assume that we have approximations $\overline{B}$ and $\overline{C}$ to B and C. With this information we can cast the inverse problem in the form of a two-channel least squares problem in which the unknown filters are first order perturbations to the shot waveform and the reflectivity series. Equation (3) viewed in this context becomes a problem of finding:

$$\min_{dB,dC} \left|\left|(\overline{B} - Y + dB)(\overline{C} + dc) - Y\right|\right| \tag{7}$$

where $C = \overline{C} + dC$ and $B = \overline{B} + dB$.

The block diagram for this problem looks like



where we have ignored second order terms in dB and dC.

The normal equations for (7) are

$$\left[\begin{array}{ccc|cc}
R(0) & \cdots & R(nb) & r_{11}(nb+1)\cdots r_{11}(nc) & \\
 & & & r_{21}(nb+1)\cdots r_{21}(nc) & \\
\vdots & & \vdots & & \vdots \\
R(-nb) & \cdots & R(0) & & \\
\hline
r_{11}(-nb-1)\,r_{12}(-nb-1) & & & r_{11}(0) & \\
\vdots & & \vdots & & \ddots \\
 & & & & r_{11}(0)
\end{array}\right]
\left[\begin{array}{c}
dc(0) \\ db(0) \\ \vdots \\ dc(nb) \\ db(nb) \\ \hline dc(nb+1) \\ \vdots \\ dc(nc)
\end{array}\right]
=
\left[\begin{array}{c}
r_{1d}(0) \\ r_{2d}(0) \\ \vdots \\ r_{1d}(nb) \\ r_{2d}(nb) \\ \hline r_{1}d(nb+1) \\ \vdots \\ r_{1}d(nc)
\end{array}\right] \qquad (8)$$

$$\text{where}\quad R(\ ) = \begin{bmatrix} r_{11}(\ ) & r_{12}(\ ) \\ r_{21}(\ ) & r_{22}(\ ) \end{bmatrix}$$

$$r_{11} = r_{(\bar{b}-y)(\bar{b}-y)}, \quad r_{22} = r_{\bar{c}\bar{c}}, \quad r_{12} = r_{(\bar{b}-y)\bar{c}},$$

$$r_{21} = r_{\bar{c}(\bar{b}-y)}, \quad r_{1d} = r_{(\bar{b}-y)d} \quad \text{and} \quad r_{2d} = r_{\bar{c}d}.$$

nb is the length of db, and nc is the length of dc.

The constraint that $db(i) = 0$ for $i > nb$ is easily implemented by observing that

$$\left[\begin{array}{ccc|c} R(0) & \cdots & R(nc) & \\ \vdots & & \vdots & G^T \\ R(-nc) & \cdots & R(0) & \\ \hline & G & & 0 \end{array}\right] \left[\begin{array}{c} dc(0) \\ db(0) \\ \vdots \\ dc(nc) \\ db(nc) \\ \hline \lambda \end{array}\right] = \left[\begin{array}{c} r_{1d}(0) \\ r_{2d}(0) \\ \vdots \\ r_{1d}(nc) \\ r_{2d}(nc) \\ \hline 0 \end{array}\right] \tag{9}$$

with

$$G = \begin{bmatrix} 0 & \cdots & 0 & \overset{\overset{2(nb+1)}{\downarrow}}{1} & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \vdots & & & & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

which is solved by setting $db(i) = 0$ for $i > nb$.

This means that we just solve a regular Block Toeplitz problem of order nc as if there were no constraints at all.

Examples

Equation (9) is best solved by the Levinson-Wiggins-Robinson (LWR) algorithm (Robinson, 1967, and Wiggins and Robinson, 1965). This was done for the model of Figure 1. In this Figure, $\bar{B}$ was taken as unity and $\bar{C}$ was assumed equal to the true reflectivity without the last two spikes. The resulting two-channel deconvolution is clearly better than the single channel deconvolution.

This approach also appears to be an improvement over algorithms that depend on minimization of the total output power by variation of the shot waveform. Estevez (SEP-12, see Figure 2) was unable to invert a model similar to Figure 1 in which the first multiple coincided with a reflector.

In example (3) a very bad estimate of the shot waveform is improved considerably with a good reflectivity estimate. Example (4) shows the improvement obtained when $\bar{B}$ and $\bar{C}$ are formed by adding 5 percent noise to B and C. Note that in these examples the interpreter would be very hard pressed to even pick a gate for the first multiple.

A typical flow for deconvolution of shot waveform and multiple might consist of the following steps.

(1) Estimate $\bar{B}$ by $\bar{B} = \min_B ||BM - P^2||$, where P is the (appropriately gated) seafloor primary and M is the gated first multiple. This would have to be done in a zone where M really could be expected to be primary-free.

(2) Do a single channel deconvolution of Y using parsimony or one of the histogram-shaping norms to get an estimate of $C/(1 + C)$.

(3) Visually zero out the worst multiples in (2) to get $\bar{C}$.

(4) Solve the two-channel problem (7) to get a final estimate of C:

$$\hat{C} = \bar{C} + dC$$

(5) Keep repeating step (4) using the latest $\hat{B}$ and $\hat{C}$ as inputs to deconvolve successive traces.

Note that steps (1) through (3) could be replaced by any combination of steps that gives reasonable initial estimates for B and C.

Summary

We have shown that a simultaneous deconvolution of B and 1/(1 + C) can work better than a single channel deconvolution with an imperfect shot waveform estimate. There are often situations in practice where the interpreter cannot pick a clean gate for the first multiple because of the length of the shot waveform. This method could conceivably be combined with other shot waveform deconvolution programs to separate different effects due to the shot and reverberation processes. This could greatly accelerate the overall algorithm convergence.

REFERENCES

RILEY, D.C. and J.F. CLAERBOUT (1976), "Two-dimensional multiple reflections," *Geophysics* 41:4.

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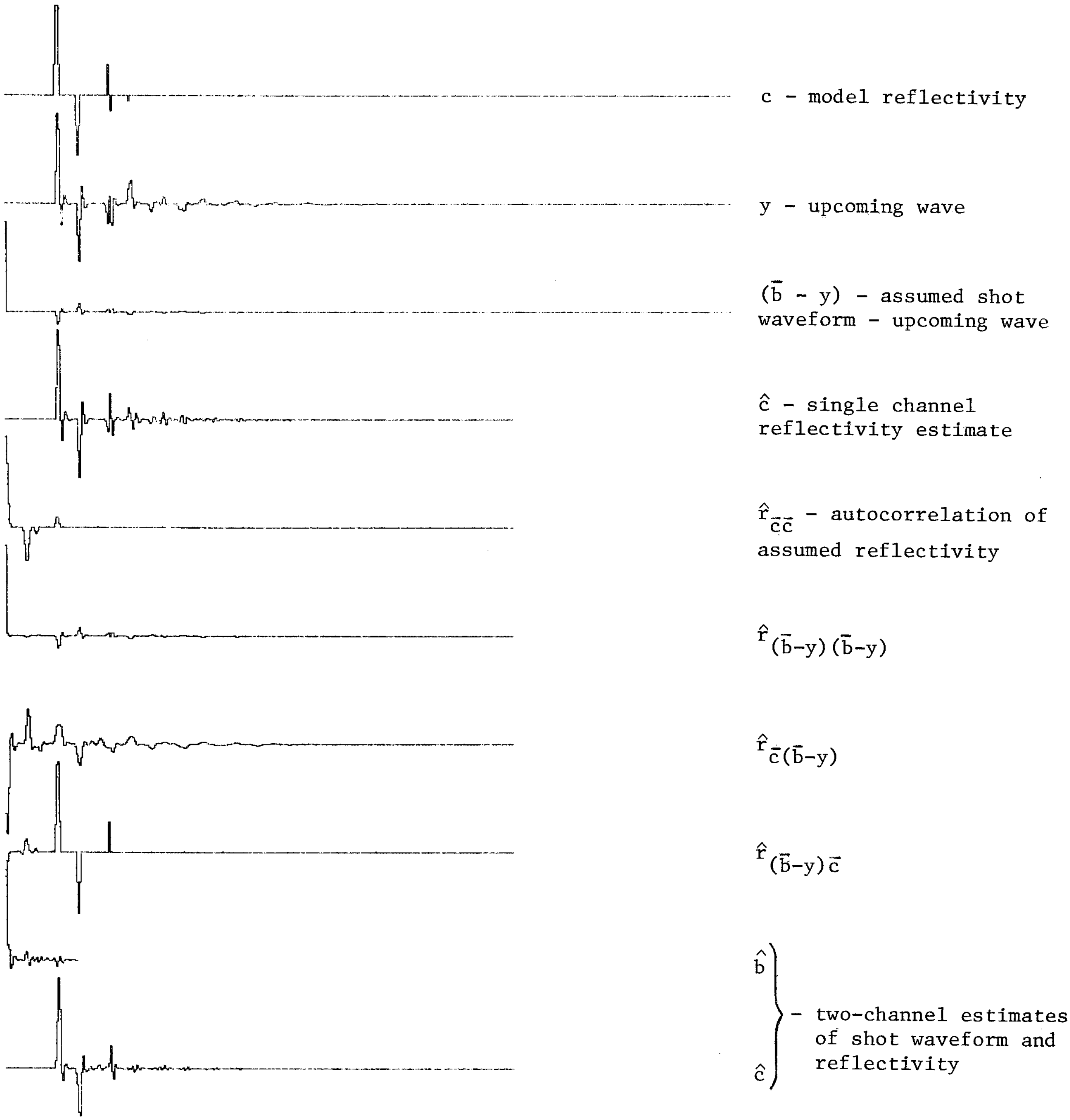
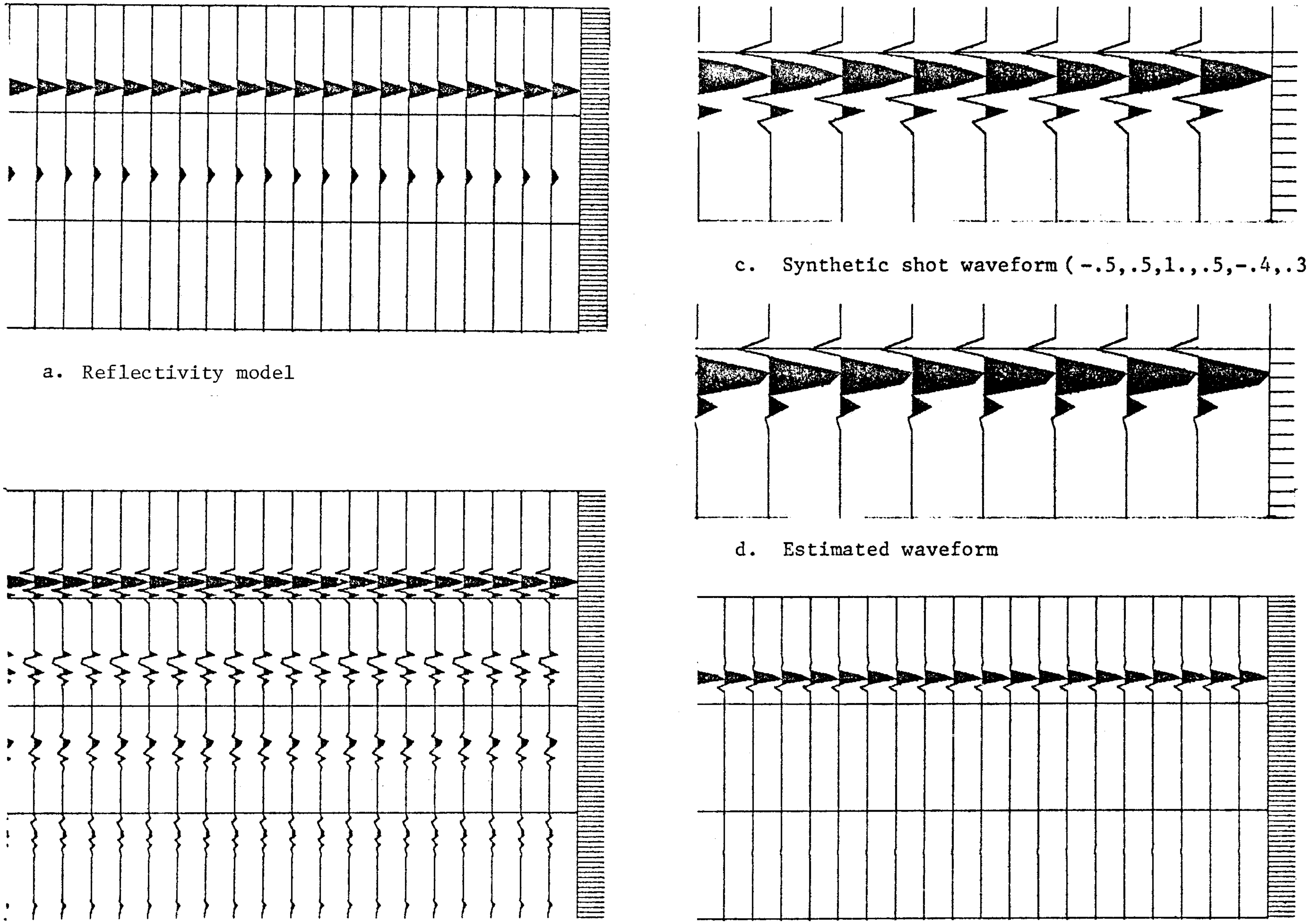


FIGURE 1.--Synthetic single and dual channel deconvolutions. $\bar{B}$ was assumed to be unity in both deconvolutions. In the two-channel case, $\bar{c}$ was taken as the true reflectivity without the last two spikes.



a. Reflectivity model

b. Synthetic seismogram

c. Synthetic shot waveform (-.5,.5,1.,.5,-.4,.3,-.2)

d. Estimated waveform

e. Estimated U (reflection coefficients)

FIGURE 2.(taken from Estevez, SEP-12, Figure 5.14)--This is a single channel algorithm designed to minimize the seismogram energy by variation of the shot waveform. Since the first multiple coincides with a reflector, the shot waveform estimate is seriously degraded.

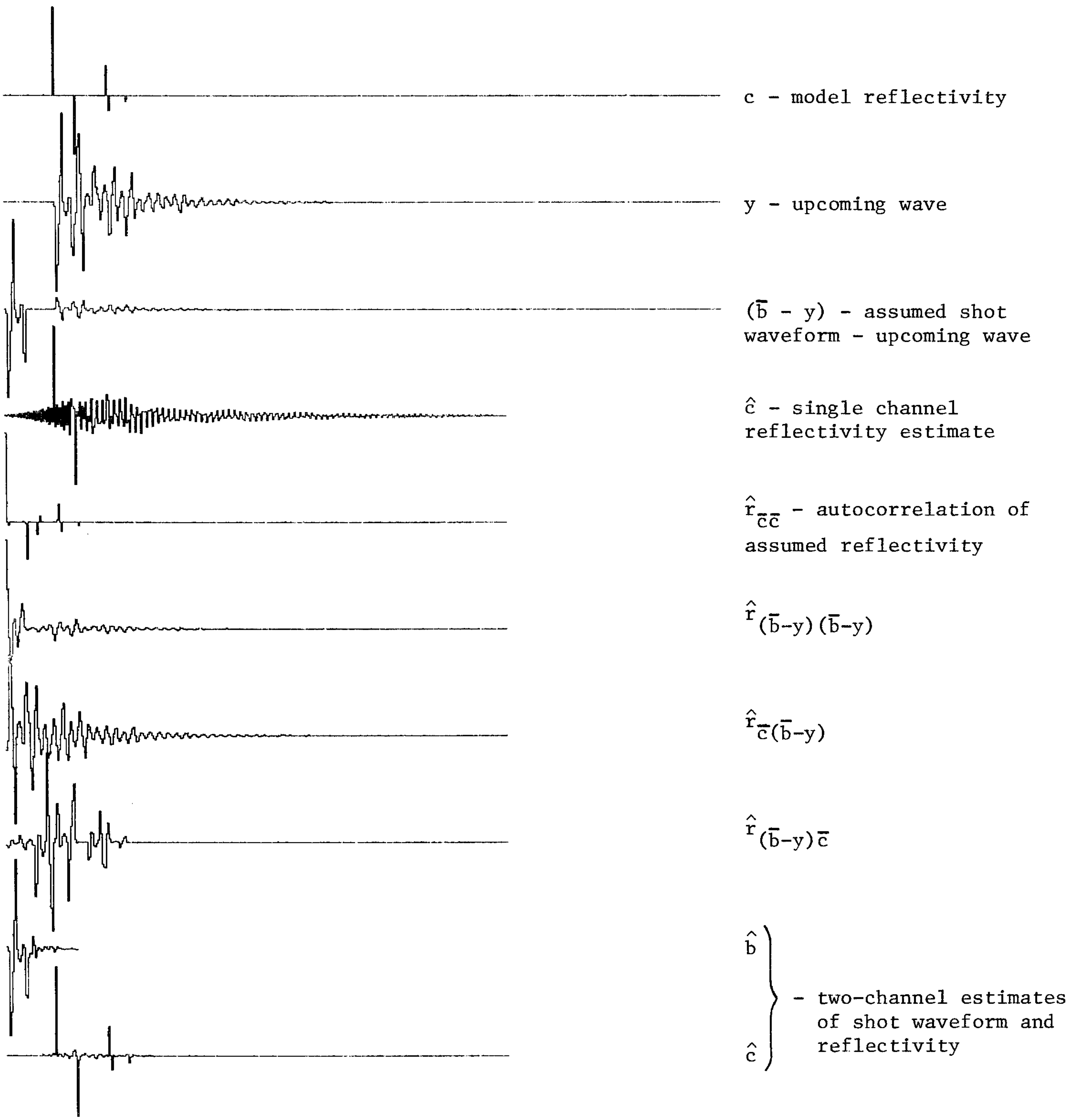


FIGURE 3.--One- and two-channel deconvolutions using a truncated bubble as an estimate to the true bubble pulse. The two-channel deconvolution eliminates the ringing seen in the single-channel solution.

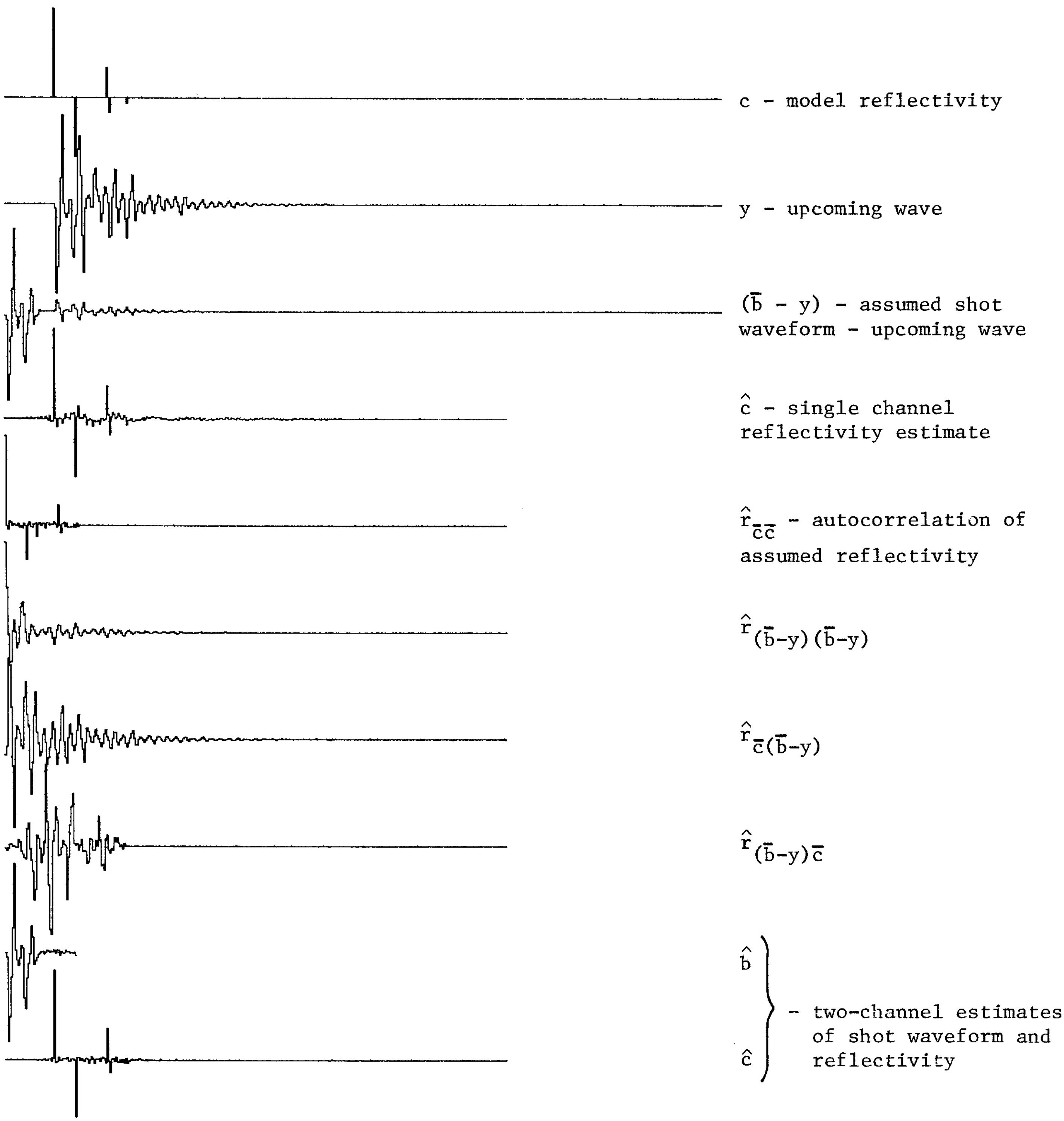


FIGURE 4.--Deconvolutions obtained when $\bar{B}$ and $\bar{C}$ are formed by adding 5 percent noise to B and C.