TWO TYPES OF MIGRATED TIME SECTIONS

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Introduction

The double square root equation (see Claerbout, SEP-15, pp. 73-80) represents the complete operator to downward continue shots and receivers into the earth. In addition to collapsing diffraction hyperbolae and migrating dipping energy in the midpoint dimension, the double square root equation also performs the normal moveout correction in the offset dimension during the downward continuation (see Clayton, SEP-14, p. 29). Depending on the purpose of the processing (imaging, multiple suppression, etc.) it may be advantageous to separate the migration from the moveout correction. In fact, the standard procedure in seismic data processing is to first perform the normal moveout correction and stack the data into zero offset and then migrate the stacked section.

An alternative type of stack is a common midpoint (cmp) slant stack where the stacking is done along a linear moveout trajectory (see Ottolini, SEP-15, pp 97-108). In this type of stack, a hyperbolic event will stack into zero offset at some time t' which is dependent upon the slope of the stacking trajectory. t' is always less than or equal to the zero offset traveltime (or t' for a zero slope slant stack) and their difference is defined as the slant moveout time. In migrating cmp slant stacks, it may or may not be desirable to correct for the slant moveout. Whether or not the moveout time is corrected for in the migrated time section depends upon how the depth to time conversion is defined. For moveout corrected time sections the vertical traveltime to depth transformation is used in the double square root equation

$$\tau \;=\; 2\int_0^z \frac{dz}{v(z)} \tag{1}$$

To leave the moveout time unchanged, the following time to depth conversion is used in the double square root equation

$$\tau = 2\int_0^z \frac{[1 - p^2 v^2(z)]^{1/2}\, dz}{v(z)} \qquad (2)$$

where p is the slope of the linear moveout summation trajectory. Both types of time sections are useful in various applications and their differences are summarized in the following table

moveout-corrected	not moveout-corrected
flat events will move in time	flat events do not move in time
velocity sensitive	velocity insensitive
good for velocity estimation	good for multiple suppression
image is independent of p	image is dependent on p
	velocity estimation from linearly moved-out cmp gathers

There exists a similar situation with cmp (hyperbolic) stacks. If the energy is stacked into some non-zero offset instead of zero offset, there would be some residual moveout on the stacked sections. We would now have the same choices as above, which are to correct for the moveout during the migration or to leave it unchanged. The remainder of this paper will go into detail on the migrated time sections for cmp slant stacks.

Moveout-corrected time sections

We will first consider the transformation given in Equation (1). Quickly summarizing Claerbout's derivations in SEP-15 (pp. 92-95), we have from the double square root equation and for flat beds

$$P_z = \frac{-2i\omega}{v}(1 - H^2)^{1/2} P \qquad (3)$$

where $H = vk_h/(2\omega)$ and k_h is the offset wavenumber. In the slant frame, $t' = t + 2ph$ which implies

$$k_h = k_{h'} - 2p\omega'$$

For a slant stack, $k_{h'} = 0$, and so Equation (3) becomes

$$P_z = \frac{-2i\omega'}{v} (1 - p^2v^2)^{1/2} P \qquad (4)$$

To simplify the following concepts, let us assume a constant velocity. We will generalize to a stratified medium later. Upon choosing a slant angle p, a hyperbolic event from a flat bed at depth z will be stacked into a time t' given by

$$t' = \frac{2z(1 - p^2v^2)^{1/2}}{v} \qquad (5)$$

where v is the correct medium velocity. Now consider an entire p-stack which will consist of one flat event at t'. Using Equation (1) with our estimate of the velocity $\hat{v}$ to rescale the depth axis, Equation (4) becomes

$$P_{\hat{\tau}} = -i\omega' (1 - p^2\hat{v}^2)^{1/2} P$$

where $\hat{v}$ is the migration velocity and $\hat{\tau} = 2z/\hat{v}$. Inverse Fourier transforming with respect to ω yields

$$P_{\hat{\tau}} = (1 - p^2\hat{v}^2)^{1/2} P_{t'} \qquad (6)$$

Equation (6) is simply a shifting equation with the solution

$$P = P[t' + (1 - p^2\hat{v}^2)^{1/2} \hat{\tau}]$$

Hence a point situated at t_0' and $\hat{\tau} = 0$ will be shifted to

$$\hat{\tau} = \frac{t_0'}{(1 - p^2\hat{v}^2)^{1/2}} \qquad (7)$$

after migration as shown in Figure 1.

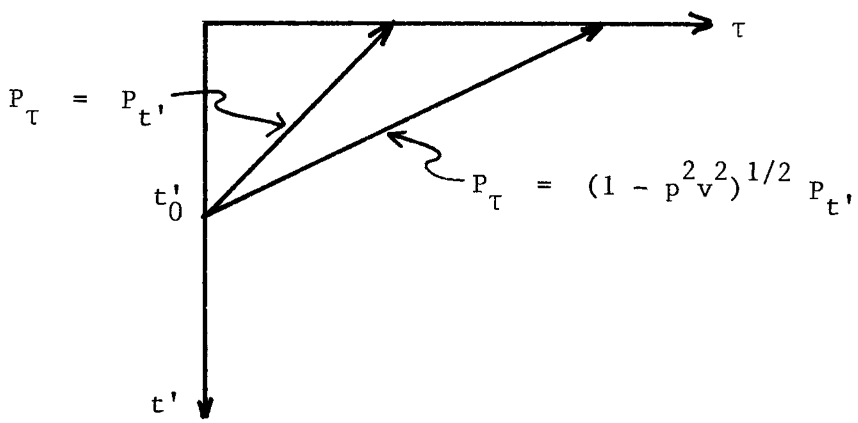


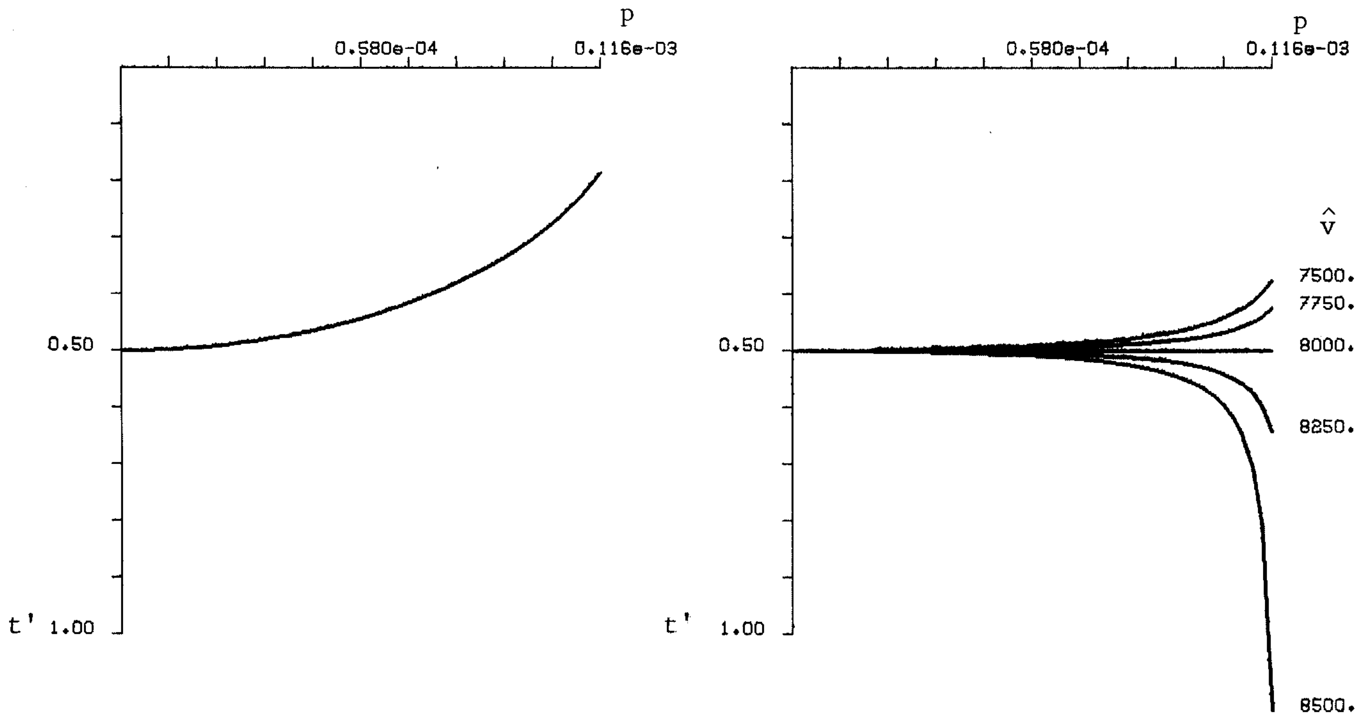
FIGURE 1.

Combining Equations (5) and (7) we obtain the relation between the post-migration τ $(\hat{\tau})$ and the correct τ (τ):

$$\hat{\tau} = \left[\frac{1 - p^2\hat{v}^2}{1 - p^2v^2}\right]^{1/2} \tau \tag{8}$$

Thus, if $\hat{v} = v$, then $\hat{\tau} = \tau$ and is independent of p. Otherwise, $\hat{\tau}$ will vary as a function of p.

Consider an example where the true media velocity is 8000 ft/sec down to a reflector at 2000 feet. Prior to migration, a cmp p-gather would look like Figure 2a. After migrating all of the cmp slant *stacks* with Equation (6) we regroup them back into cmp p-gathers shown in Figure 2b. The migration velocity $\hat{v}$ is shown on the right. For $\hat{v} = v = 8000$ ft/sec, the resulting p-gather is independent of p as it should be. For $\hat{v} < v$, the p-gathers appear to be under-migrated with the larger p-values not being pushed down far enough. Similarly, for $\hat{v} > v$, the larger p-values are overmigrated. Note that the effect of having $\hat{v} > v$ is much more pronounced than having $\hat{v} < v$.



a. pre-migration b. post-migration

FIGURE 2.

It is a simple matter to estimate v from any of the migrated p-gathers. Solving Equation (8) for v:

$$v = \frac{1}{p}\left[1 - (1 - p^2\hat{v}^2)\left(\frac{\hat{\tau}}{\tau}\right)^2\right]^{1/2} \tag{9}$$

τ can be measured from the original migrated slant stack for $p = 0$. $\hat{\tau}$ is the migrated time for some particular value of p using a migration velocity of $\hat{v}$ as shown in Figure 2b.

We can easily generalize this result to stratified media. In a stratified medium, an event at a depth z will stack into a time t' given by

$$t' = 2\int_0^z \frac{[1 - p^2v^2(z)]^{1/2}\,dz}{v(z)} \tag{10}$$

After migration with a velocity of $\hat{v}$, this event will be located at a depth $\hat{z}$ given by [see Claerbout, SEP-15, p. 94, Equation (18)]

$$t' = 2\int_0^{\hat{z}} \frac{[1 - p^2\hat{v}^2(z)]^{1/2}\, dz}{\hat{v}(z)} \qquad (11)$$

Letting

$$\hat{\tau} = 2\int_0^{\hat{z}} \frac{dz}{\hat{v}(z)}$$

Equation (11) becomes

$$t' = \int_0^{\hat{\tau}} [1 - p^2\hat{v}^2(\tau)]^{1/2}\, d\tau \qquad (12)$$

Using Equation (1) to rewrite (10) as

$$t' = \int_0^{\tau} [1 - p^2 v^2(\tau)]^{1/2}\, d\tau$$

and equating t' with Equation (12) we obtain

$$\int_0^{\tau} [1 - p^2 v^2(\tau)]^{1/2}\, d\tau = \int_0^{\hat{\tau}} [1 - p^2\hat{v}^2(\tau)]^{1/2}\, d\tau$$

Assume that $\hat{v} = v$ up to the j-1st reflector at τ_{j-1} and we want the velocity v_j between τ_{j-1} and τ_j. Also assuming that v_j is constant within the interval, we get

$$(1 - p^2 v_j^2)^{1/2}\, \Delta\tau = (1 - p^2\hat{v}_j^2)^{1/2}\, \Delta\hat{\tau} \qquad (13)$$

where $\Delta\tau = \tau - \tau_{j-1}$ and $\Delta\hat{\tau} = \hat{\tau} - \tau_{j-1}$. Since the migration velocity is correct down to τ_{j-1}, $\hat{\tau}_{j-1}$ is equal to τ_{j-1} and is independent of p. Solving Equation (13) for v_j.

$$v_j = \frac{1}{p}\left[1 - (1 - p^2\hat{v}_j^2)\left(\frac{\Delta\hat{\tau}}{\Delta\tau}\right)\right]^{2\quad 1/2} \qquad (14)$$

Again $\Delta\tau$ can be measured directly from the pre-migrated slant stack for $p = 0$. $\Delta\hat{\tau}$ is the time difference between the migrated j-lst reflector and the j-th reflector measured at some non-zero value of p.

Once the correct velocity is found a final migrated section can be made by simply superimposing all of the individually migrated cmp slant stacks, thus acquiring a higher signal to noise ratio.

Moveout-uncorrected time sections

Let us now consider a migrated time section using Equation (2) as the depth to time conversion. Rewriting Equation (4) using

$$\frac{d\tau}{dz} = \frac{2[1 - p^2v^2(z)]^{1/2}}{v(z)}$$

from Equation (2) we obtain

$$P_\tau = -i\omega' P$$

Or, in the time domain

$$P_\tau = P_{t'} \qquad (15)$$

which is a simple time shift independent of v as we downward continue in τ. If $\Delta\tau$ is set equal to $\Delta t'$, then the energy migrates along a 45-degree line in (t',τ) space as shown in Figure 1 and the timing relationship between flat events remains unchanged. Note that if we worked in a retarded time frame defined by

$$t' = t + 2\int_0^z \frac{[1 - p^2v^2(z)]^{1/2}\, dz}{v(z)} = t + \tau$$

then Equation (15) becomes

$$P_\tau = 0$$

which implies that the flat events remain stationary in time during the downward continuation. Thus, flat events on a pre-migrated cmp p-gather will remain unchanged on the post-migrated p-gather (i.e., after resorting the migrated p-*stacks* back into p-*gathers*) (see Figure 2a). Velocity estimation can be performed on the p-gathers using the procedure described by Claerbout in SEP-15 (pp. 91-92). It really does not make any difference in the final result if we use a retarded frame or not. Using a retarded frame just simplifies the algorithm.

Perhaps the most useful application of this type of time section is in the suppression of multiples where it is critical to maintain correct timing relationships between events (see Morley and Claerbout, SEP-15). Another application is in measuring rms and interval velocities from cmp gathers using the tangency method described by Claerbout (SEP-14, pp. 13-16; SEP-15, pp. 61-66). Claerbout showed that the velocity between two events (Figure 3) is given by

$$v^2 = \frac{1}{p\,\frac{dt}{df}} \tag{16}$$

where f is offset and dt/df is measured between the tangent points of the line $t' = t + pf$ with the two events. In a linearly moved-out coordinate system, as in Figure 3b, the tangency points become the tops of the skewed hyperbolae, and the resulting velocity estimation is given by

$$v^2 = \frac{1}{p\left(p + \frac{dt'}{df}\right)} \tag{17}$$

Migrating the linearly moved-out cmp gathers will focus the energy at the tops, thereby increasing the reliability of the velocity estimations. The migration equation is derived from Equation (4). However, since we are not downward continuing slant *stacks*, $k_{h'}$ is not set equal to zero in Equation (4) and the downward continuation operator is

$$P_z = \frac{-2i\omega'}{v}\left[1 - \left(\frac{vk_{h'}}{2\omega'} - pv\right)^2\right]^{1/2} P \tag{18}$$

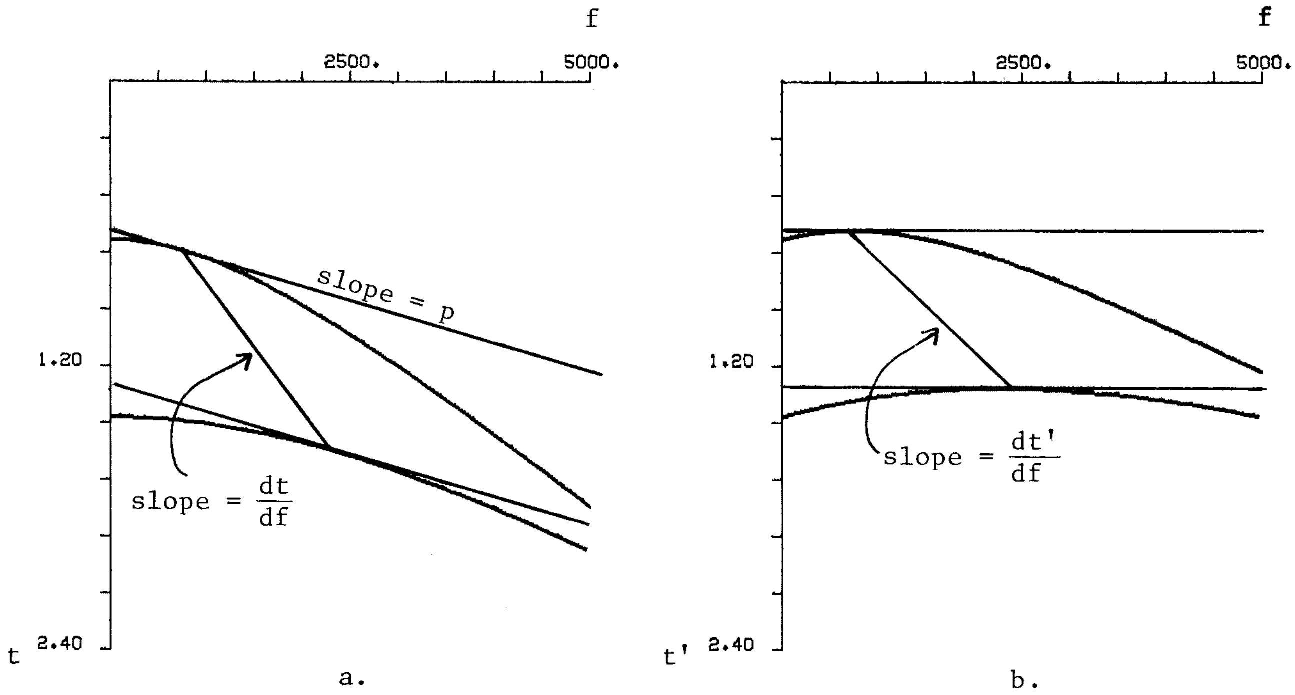


FIGURE 3.

Using the depth to time conversion in Equation (2), this becomes:

$$P_\tau = -i\omega' \left[\frac{1 - \left(\frac{vk_{h'}}{2\omega'} - pv \right)^2}{1 - p^2v^2} \right]^{1/2} P \tag{19}$$

The top of the skewed hyperbola is where $k_{h'} = 0$ and in this case Equation (19) becomes

$$P_\tau = -i\omega' P \tag{20}$$

which again is a simple shift in t independent of velocity as we step down in τ. Physically, it does not make sense to downward continue a cmp gather by itself. However, if the medium is vertically stratified we can consider the gather to be a common shot gather which is valid to downward continue. It appears that the main advantage of using the migrated gathers is to estimate velocity is on noisy records where the hyperbola tops cannot be picked reliably.

In summary, we have two different choices to make in creating migrated time sections from data in a slant frame. If we want to make absolutely certain that timing relations for flat events are not disturbed, then Equation (2) is the proper depth to time conversion. If we want to create several migrations of cmp p-stacks and have identical images for later superposition, then Equation (1) is the proper conversion.