EXTRAPOLATION OFF THE SIDES OF A COMMON MIDPOINT GATHER

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Two kinds of common midpoint gather are (1) the field data gather, which is a function of half offset h and (ray) traveltime t, and (2) the slant stacked gather, which is a function of Snell's parameter p and the wavefront traveltime t'. As a practical matter it is probably more important to be able to extend the (h,t) gather than to extend the (p,t') gather. Regretably, the easier problem, and the only one we can solve exactly, is the extension of the (p,t') gather. So this will be our present subject.

The double square root operator for p-gathers is

$$DS(z,p) = -\frac{i\omega}{v}\{[1 - (Y - pv)^2]^{1/2} + [1 - (Y + pv)^2]^{1/2}\} \tag{1}$$

The downward continuation equation is

$$Q(z,p) = Q(0,p)\ \exp\int_0^z DS(z,p)\ dz \tag{2}$$

Now we formally make the statement that the earth image implied by data at p equals the image implied by $p + dp$.

$$Q(z_o,p+dp) = Q(z_o,p) \tag{3}$$

In (3) we have used z_o to denote the depth of the image. Restatement of (3) at $z=0$ is achieved by substituting (2) into both sides of (3).

$$Q(0,p+dp)\ \exp \int_0^{z_o} DS(z,p+dp)\ dz \quad = \quad Q(0,p)\ \exp \int_0^{z_o} DS(z,p)\ dz$$

$$Q(0,p+dp) \quad = \quad Q(0,p)\ \exp \left[-dp \int_0^{z_o} \frac{d}{dp}\ DS(z,p) \right] dz \tag{4}$$

Equation (4) suggests that we differentiate Equation (1) with respect to p. Although we could proceed with the analysis for all dips Y, our objectives are mainly instructional, and it will suffice to work with low order terms of a power series in Y. Doing the differentiation, we get

$$\frac{d}{dp}\ DS \quad = \quad i\omega 2\ \frac{pv}{(1 - p^2v^2)^{1/2}} \quad + \quad i\omega 2\ \frac{pv}{(1 - p^2v^2)^{3/2}}\ Y^2 \tag{5}$$

Inserting in (4), we obtain the final result. It is the solution to the differential equation

$$\left.\frac{\partial Q}{\partial p}\right)_{z,t'} \quad = \quad -i\omega \left\{ \int_0^{z_0} \left[\frac{2pv\ dz}{(1 - p^2v^2)^{1/2}} \quad + \frac{2pv\ dz}{(1 - p^2v^2)^{3/2}}\ Y^2 \right] \right\}\ Q \tag{6}$$

The coefficient of Y^0 seems to provide the usual moveout corrections.