SLANT STACK MIGRATION AND VELOCITY ANALYSIS: EQUATIONS FOR PROGRAMS

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In principle exact migration before velocity analysis is not possible. However, in principle velocity analysis can be done simultaneously with migration so that velocity is determined as it is required for downward continuation. This is described in SEP-14, pp. 13-15, 73-81. A more conventional approach is to assume a velocity $\hat{v}(z)$ and use it for moveout and migration, then by measuring residual moveout to bootstrap to a better velocity. This will be done here. SEP researchers have been unable to provide equations for migrating *constant offset sections* which are exact for all offsets and dips (although approximations are found in FGDP). Luckily, exact equations are easily found for migration of slant stacked sections. The slant stacks have several other theoretical attractions such as the ability, in principle, to migrate and suppress diffracting multiple reflections and the ability to systematically approach the problem of lateral velocity variation.

Less grandiose is our present objective, namely to provide the equations required for a bootstrap velocity estimation which will lead to migrations and final stacks which are exact for offset and dip angles up to 90 degrees. Since the technology for towing geophone streamers of length L is obviously much less expensive than for drilling holes to depth L, these accurate equations should have some utility. Hopefully, their extreme angle accuracy will be self-evident by improved coherence of fault plane reflections. Less dramatic but possibly of greater significance in petroleum prospecting may be the more accurate determination of velocity which is possible with these wide angle waves.

We will begin by forgetting about migration and developing the equation defining the moveout as a function of slant angle which applies to slant stacks in a layered medium. Then we will get the equation which shows how residual moveout determines a better velocity. Next we will see how the migration equation automatically does the moveout correction along with the migration. Finally, the migration equation will be expressed in a form in which it is useful for producing migrated time sections.

A ray trace to predict moveout for slant stacks in stratified media.

Referring to Figure 1 we see the x-t plane and x-z plane interpretation of the traveltime of a wavefront generated by a surface source moving along the x-axis with speed 1/p.

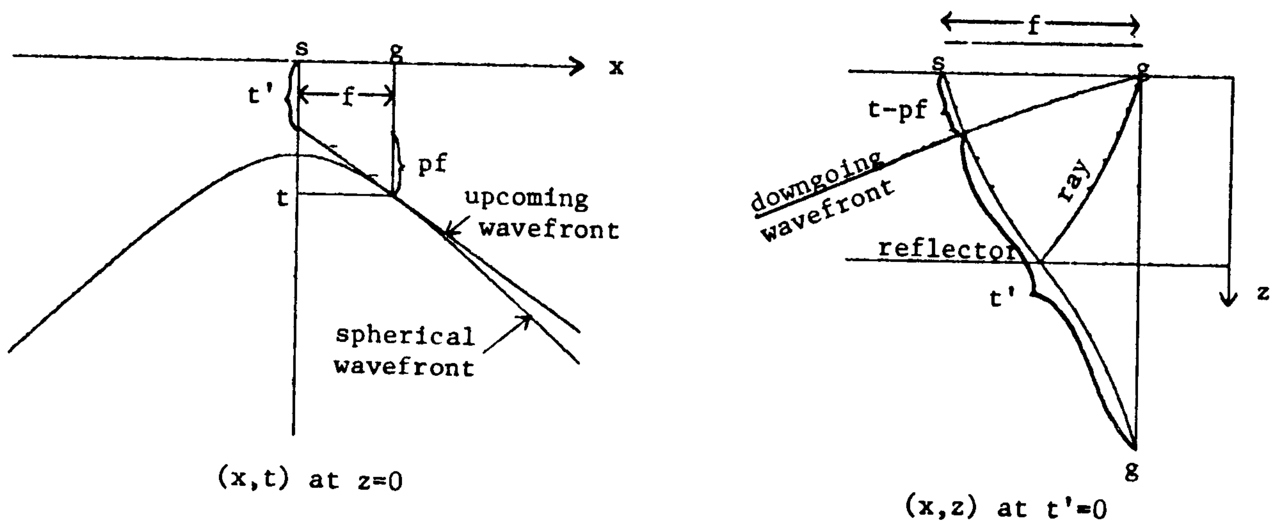

FIGURE 1.--An arbitrary stepout p (slope in x-t plane) is chosen tangent to a hyperbolic arrival. When projected back to zero offset the traveltime t' is exactly that for the downgoing wavefront in the figure to travel down the ray to the reflector and back up to the surface geophone at g.

The shot to geophone reflected spherical wave traveltime t exceeds the Snell wavefront traveltime t' by the time p(g-s) = pf that it takes for the moving surface source to go from s to g. Thus,

$$t' = t - pf \tag{1}$$

For a ray originating at the origin in a stratified medium we have the self-evident equations.

$$x = \int_0^z \tan\theta \, dz \tag{2a}$$

$$t = \int_0^z \frac{dz}{v \cos\theta} \tag{2b}$$

For a wave to go down to the reflector and then return we double (2a) and (2b) and insert into (1) getting

$$t' = 2\int_0^z dz \left(\frac{1}{v \cos\theta} - p \tan\theta \right) \tag{3}$$

Using $pv = \sin\theta$ this reduces to the simple, exact equation for determining slanted time t' as a function of slant angle p.

$$t' = 2\int_0^z \frac{[1 - p^2 v(z)^2]^{1/2}}{v(z)} \, dz \tag{4}$$

Equation (4) may be used to prepare transparent overlays for data analysis in much the same way that hyperboloidal overlays are commonly used to analyze common midpoint gathers. With constant velocity these overlays are a family of ellipses in the (t',p) plane. Note that opposite to the familiar situation with hyperboloids the ellipses are narrow when the velocity is fast and wide when it is slow.

How the time residuals may be used to estimate velocity

In applied seismogram analysis it is often convenient to avoid reference to actual depth z whenever a vertical two-way traveltime τ can be used. This is because τ is more directly measurable. Obviously the transformation between them is

$$\tau = 2\int_0^z \frac{dz}{v(z)} \tag{5}$$

Using $d\tau/dz$ from (5) to change variables in Equation (4) we get

$$t' = \int_0^\tau [1 - p^2 v(\tau)^2]^{1/2} \, d\tau \tag{6}$$

Let us see how to use Equation (6) to bootstrap the velocity. Suppose a velocity model $\hat{v}(\tau)$ has been found to fit the traveltime of the j-1 st coherent reflector as a function of p but the model does not yet fit the j-th reflector. Then subtracting (6) for the true velocity v from (6) for the currently estimated velocity $\hat{v}$ gives

$$\Delta t'(p) = \int_{\tau_{j-1}}^{\tau_j} \left\{ [1 - p^2 v(\tau)^2]^{1/2} - [1 - p^2 \hat{v}(\tau)^2]^{1/2} \right\} d\tau \qquad (7)$$

where τ_j is the vertical incidence traveltime to the j-th interface, and $\Delta t'(p)$ is the observed time t' less the t' predicted by the transparent overlay based on velocity $\hat{v}$. Let $v(\tau)$ be a constant within the layer and do a perturbation analysis, say

$$v = \hat{v} + \Delta v$$

Then (7) becomes

$$\Delta t' = (\tau_j - \tau_{j-1}) \left. \frac{\partial}{\partial v} (1 - p^2 v^2)^{1/2} \right|_{v=\hat{v}} \Delta v \qquad (8)$$

$$= (\tau_j - \tau_{j-1}) \frac{-p^2 \hat{v}\, \Delta v}{(1 - p^2 \hat{v}^2)^{1/2}}$$

So the new velocity estimate v is

$$v = \hat{v} - \frac{(1 - p^2 \hat{v}^2)^{1/2}}{p^2 \hat{v}\, (\tau_j - \tau_{j-1})} (t'_{observed} - t'_{predicted}) \qquad (9)$$

where p is taken to be the p at which $\Delta t'$ is measured.

How slant migration does moveout correction

The downward continuation operator will not be derived here. (See for example SEP-14, pp. 59-61.) It is

$$\frac{dP}{dz} = -i \frac{\omega}{v} \left\{ [1 - (Y+H)^2]^{1/2} + [1 - (Y-H)^2]^{1/2} \right\} P \qquad (10)$$

where

$$Y = \frac{v\ k_y}{2\omega} \quad \text{(like sine of dip)} \tag{11a}$$

$$H = \frac{v\ k_h}{2\omega} \quad \text{(like sine of offset)} \tag{11b}$$

and where k_y and k_h are spatial frequencies in the direction of midpoint and half offset. Let us now review why it is that slant stacking amounts to a rejection in $P(\omega,k_h)$ of all frequency components except those for which $p = k_h/2\omega$.

Define

$$h = h' = \frac{(g - s)}{2} \tag{12a}$$

$$t = t' + 2ph' \tag{12b}$$

Let

$$P'(h',t') = P(h,t) \tag{13}$$

The chain rule for partial differentiation gives

$$\frac{\partial P'}{\partial h'} = \frac{\partial P}{\partial h}\frac{\partial h}{\partial h'} + \frac{\partial P}{\partial t}\frac{\partial t}{\partial h'} \tag{14a}$$

which in the frequency domain is

$$i\ k_{h'}\ P' = i\ k_h\ P + 2p\ (-i\omega)\ P \tag{14b}$$

Slant stacking is really identical with linear moveout (12b) followed by Fourier analysis and selection of the zero spatial frequency component $k_{h'} = 0$. Thus from (11b) and (14b) with $k_{h'} = 0$ we see that slant stacking gives

$$H = \frac{v\ k_h}{2\omega} = vp \tag{15}$$

At the moment we are not interested in dipping beds so we may set Y = 0. This plus (15) into (10) yields

$$\frac{dP}{dz} = -i\ 2\ \omega \frac{(1 - p^2v^2)^{1/2}}{v} \quad P \tag{16}$$

which integrates to

$$P(z,\omega) = P(0,\omega)\ \exp\left\{-i\ 2\ \omega \int_0^z \frac{[1 - p^2v(z)^2]^{1/2}}{v(z)} dz\right\} \tag{17}$$

Now let us suppose that the surface observation $P(0,\omega)$ is $e^{i\omega t'}$ representing a single impulsive arrival at time t'. Next Fourier transform the downward continued wave into the time domain.

$$P(z,t) = \int e^{-i\omega t}\ \ e^{i\omega t'}\ \ \exp\left\{-i\ 2\ \omega \int_0^z \frac{\cos\theta}{v}\, dz\right\}\ d\omega$$

$$= \delta\left(-t + t' - 2\int_0^z \frac{\cos\theta}{v}\ dz\right)$$

The migrated section is seen at zero travel t = 0. It will be a delta function at the z value given by

$$t' = 2\int_0^z \frac{[1 - p^2v(z)^2]^{1/2}}{v(z)}\ dz \tag{18}$$

which is identical with Equation (4). In a constant velocity medium this would be for a depth given by combining (5) and (18)

$$\tau = \frac{2z}{v} = \frac{t'}{(1 - p^2v^2)^{1/2}} \tag{19}$$

Thus energy which was at t' has been pushed by migration to a later time $\tau \geq t'$ by a cosine division. If the velocity of migration in (19) equals the velocity of the earth in Equation (6) then the cosine multiplication of the slant stack (6) is compensated by the cosine division of the migration (19). The conclusion is that correctly migrated slant stacks show coherent energy at time τ which does not depend on stepout p. If there is observed residual moveout then velocity may be re-estimated with (9).

Equation for producing migrated time sections

As we said earlier most data analysts prefer a time section to a depth section. It preserves the familiar timing relationships of multiple reflections. It is not distorted by incorrectly estimated velocities. Beginning with Equation (10), multiplying by $dz/d\tau$ from (5), and including (11) we obtain

$$\frac{dP}{d\tau} = -\frac{i}{2}\left\{[\omega^2 - v(\tau)^2(\frac{k_y}{2} + p\omega)^2]^{1/2} + [\omega^2 - v(\tau)^2(\frac{k_y}{2} - p\omega)^2]^{1/2}\right\} P \tag{20}$$

This equation is integrated downward where at t = 0 the migrated time section is given by $P(\tau,y)$.