COMMON MIDPOINT MIGRATION

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The operations of stacking and migration may be viewed as parts of an overall process whose input is a surface recorded wavefield $P(y,h,z,=0,t)$, where y and h are midpoint and half-offset of the source and receiver locations respectively. The output is an image wavefield $P(y,h=0,z,t=0)$, which is proportional to the reflectivity $c(y,z)$. Stacking is generally considered to be the part of the process which collapses the offset dimension, while migration downward continues the wavefield to remove diffractions and move dipping events to their correct location.

In conventional processing the stacking is done first to reduce the volume of data, and is followed by a zero-offset migration scheme. The key assumption here is that the stacking actually does generate a zero offset section. Since stacking is based on a horizontally layered earth, one would expect problems with this approach for dipping events and diffractions.

This has led to processing schemes which attempt to remove these difficulties by migrating first to remove diffractions and dip effects and then stacking. In FGDP an equation (equation 11-3-18) for downward continuation (DC) of constant offset sections is presented.

Both of these approaches are approximations to an exact imaging method which stacks and migrates simultaneously. In this paper we examine the exact method in the Fourier domain, where the DC formulae are easier to derive. Our purpose is to derive constant offset migration equations which are valid for large dips and offsets. We present two such approximations in the Fourier domain. We have not as yet found time and space domain expressions for these operators.

To begin, we consider the wavefield in the coordinate system of the actual experiment, that is, in shot-receiver space. In this coordinate system the wave equation governs the downward continuation of the fields. The downward continuation operator in midpoint-offset coordinates can then be found by a simple coordinate transformation. This somewhat circuitous route for deriving the DC operator in midpoint offset coordinates is necessary because a CMP gather cannot be considered as a physical experiment, and hence the wave equation does not strictly apply.

Downward continuation in the shot geophone coordinate system means that we are lowering the entire experiment to a new datum level below the surface. To do this we first lower the geophones to the new datum level, and then lower the shots. The sequence of steps is shown in Figure 1. We define the recorded wavefield as $P(g,s,z_g=0,z_s=0,t)$, where g and s are the spatial locations of the geophones and shots, and z_g and z_s are their respective depths. The downward continuation of the geophones to a depth z in a constant velocity medium is given in the Fourier domain by the following equation:

$$P(k_g,k_s,z_g=z;z_s=0,\omega) = P(k_g,k_s,z_g=0,z_s=0,\omega)e^{i\frac{\omega}{v}(1-G^2)^{1/2}z} \qquad (1)$$

where k_g , and k_s are the duals of g and s , and G normalized geophone wavenumber (vk_g/ω) .

To downward continue the shots to the new level, one only need realize that the recorded wavefield is a continuous function of the source location and that changes in the wavefield due to changes in source location are governed by the wave equation. Hence, the downward continuation of the sources to a level z , when the receivers are already at that level is given by

$$P(k_g,k_s,z_g=z_1,z_s=z,\omega) = P(k_g,k_s,z_g=z_1,z_s=0,\omega)e^{i\frac{\omega}{v}(1-S^2)^{1/2}z} \qquad (2)$$

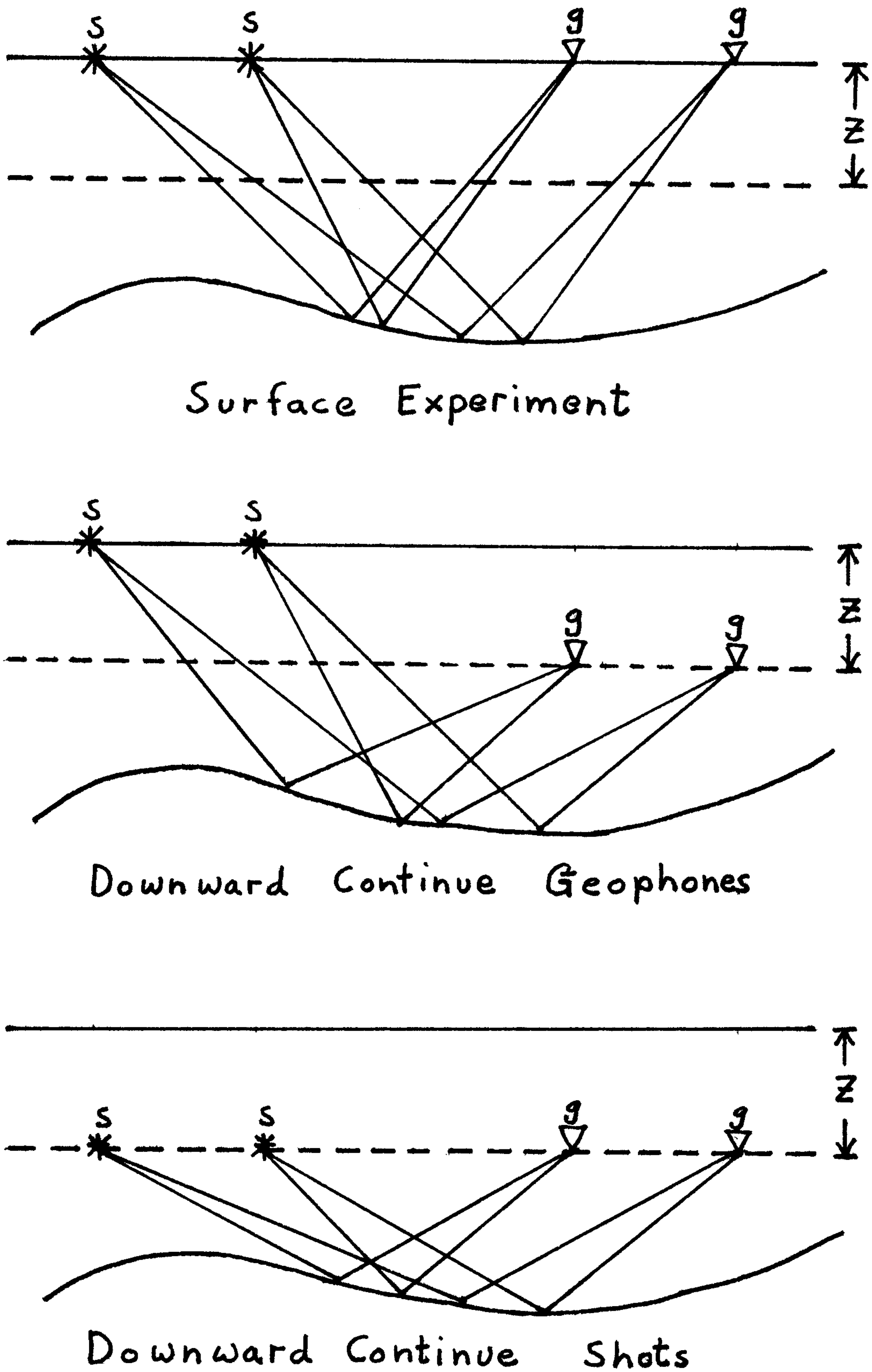

Figure 1. Downward continuation in shot - geophone space

where S is normalized shot wavenumber (vk_s/ω). The same result for DC of the sources can be achieved by invoking a reciprocity argument. Once the geophones are at the new datum, we can interchange the wavefields at the shots and geophones by reciprocity, and DC the "new" geophones. Then we once again interchange the wavefields to return to the original configuration. The net result is a DC operator for shots which is identical to the one above.

The shot and geophone DC operators can now be combined into one operator, which extrapolates the entire experiment to a depth z.

$$P(k_g,k_s,z,\omega) = P(k_g,k_s,0,\omega)e^{i\frac{\omega}{v}[(1-G^2)^{1/2} + (1-S^2)^{1/2}]z} \tag{3}$$

In this equation we dropped the distinction between z_s and z_g because we only consider moving the shots and geophones to the same level z.

The shot-geophone DC operator has the nice property that it is separable in the transform domain. That is, after the transform the geophone part of the operator can be applied, and then the shot part applied. This property is not true in the time domain where the differential equation has the form

$$P_{zzzz} + P_{gggg} + P_{ssss} - 2P_{ssgg} - \frac{4}{v^2} P_{zztt} + 2P_{zzgg} + 2P_{zzss} = 0 \tag{4}$$

This differential equation is obtained by noting that the argument of the exponential in equation (3) defines the dispersion relation of the DC operator and hence may be inverted in the normal manner. The presence of the fourth order differential in z indicates that there are four combinations of shots moving up or down, and geophones moving up or down. The Fresnel approximation of the DC operator

$$P_{zt} + \frac{2}{v} P_{tt} - \frac{v}{2} (P_{gg} + P_{ss}) = 0 \tag{5}$$

is also not separable, except by an approximate splitting method.

To obtain the DC operator in midpoint-offset coordinates we apply the transformation

$$g = y + h \quad , \qquad s = y - h \tag{6}$$

where y is midpoint, and h is half offset. In terms of the normalized wavenumbers the transformation is

$$G = \frac{Y + H}{2} \qquad S = \frac{Y - H}{2} \tag{7}$$

With this substitution the DC operator in frequency domain is

$$P(k_y, k_h, z, \omega) = P(k_y, k_h, 0, \omega) e^{i \frac{\omega}{v} \Phi (Y,H) z} \tag{8}$$

where

$$\Phi (Y,H) = \left[1 - \left(\frac{Y + H}{2}\right)^2\right]^{1/2} + \left[1 - \left(\frac{Y - H}{2}\right)^2\right]^{1/2} \tag{9}$$

In Figure 2 the dispersion relation of this operator is shown. This operator is no longer separable in the transform domain. This means that the operations of migration and stacking have to be done simultaneously and that a full three dimensional Fourier transform of the data is required to downward continue the wavefield. For economic reasons we would like the migration process to be independent of offset derivatives or wavenumbers. This would allow constant offset sections to be migrated independently of each other because the offset would enter only as a parameter in the process. We can check whether this is possible by forming a stationary phase approximation of the inverse Fourier transform of equation (8).

$$P(y,h,z,t) = \iiint_{-\infty}^{\infty} P(k_y, k_h, 0, \omega) e^{i \frac{\omega}{v} \Phi z} e^{-i(k_y y + k_h h - \omega t)} dk_y dk_h d\omega \tag{10}$$

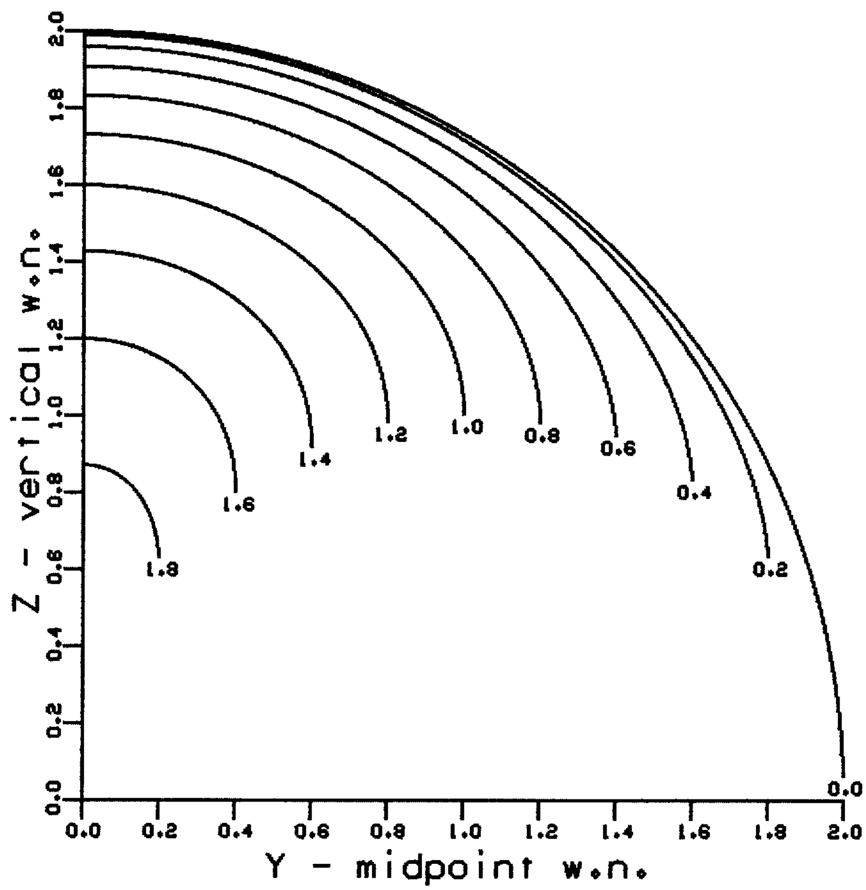

Figure 2. The dispersion relation for the exact downward continuation operator is shown. Each curve is a plot of normalized vertical wave-number versus normalized midpoint wavenumber, for a fixed offset wavenumber.

The main contributions to the integral occur when the derivative of the argument of the exponential is zero. This leads to three equations which define the locii of arrivals.

$$\left(\frac{\partial}{\partial k_y}, \frac{\partial}{\partial k_h}, \frac{\partial}{\partial \omega}\right)\left(\frac{\omega}{v}\Phi - k_y\frac{y}{z} - k_h\frac{h}{z} + \omega\frac{t}{z}\right) = 0 \tag{11}$$

Performing the differentation we have

$$\frac{Y + H}{s_1} + \frac{Y - H}{s_2} + \frac{4y}{z} = 0 \tag{12}$$

$$\frac{Y + H}{s_1} - \frac{Y - H}{s_2} + \frac{4h}{z} = 0 \tag{13}$$

$$\frac{1}{s_1} + \frac{1}{s_2} + \frac{vt}{z} = 0 \tag{14}$$

where

$$s_1 = \left[1 - \left(\frac{Y + H}{2}\right)^2\right]^{1/2} \quad \text{and} \quad s_2 = \left[1 - \left(\frac{Y - H}{2}\right)^2\right]^{1/2}$$

Eliminating Y, H, and ω from these three equations leads to

$$vt = \left[z^2 + (y - h)^2\right]^{1/2} + \left[z^2 + (y + h)^2\right]^{1/2} \tag{15}$$

This relation is an ellipse in y-z space which is perhaps more easily recognized if it is put in the form.

$$y^2 + \frac{z^2}{1 - \left(\frac{2h}{vt}\right)^2} = \left(\frac{vt}{2}\right)^2 \tag{16}$$

This implies that a point in h-t space migrates to an ellipse in the y-z space. Since the stationary phase approximation is equivalent to geometric optics, this result can be considered to be the non-zero offset, geometric optics method.

This result gives some encouragement to attempt to derive a wave operator which has this property of being independent of offset derivatives. We will approach this by finding some approximations to the DC operator Φ (equation 9), which separate into two parts: one which depends only on Y (the migration operator), and one which depends only on H (stacking operator).

Before proceeding with the analysis the DC operator in the transform space it is necessary to develop a few geometric relations of dipping beds with offset sources and receivers. Figure 3 depicts the situation to be considered. First we find a relationship between the ray angles at the source and receiver (γ_s and γ_g) and the dip angle (α) and the offset angle (β). From Figure 3 one can see that

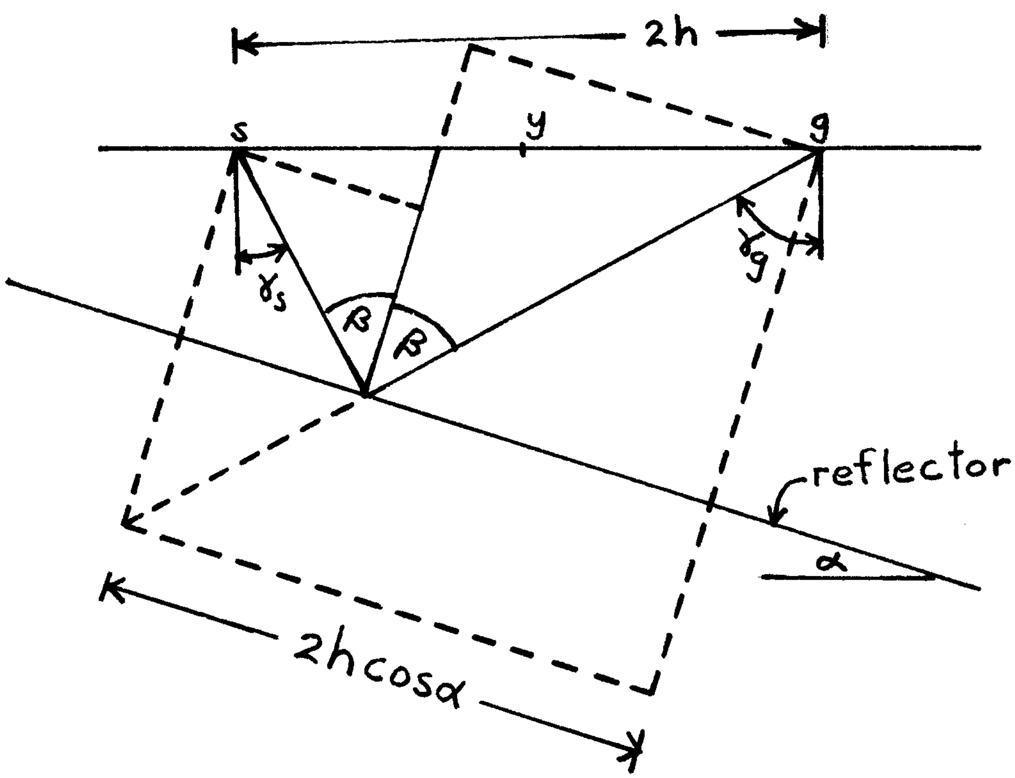

Figure 3. Illustrated is the geometry of an offset source and receiver over a dipping bed. The dashed lines are constructions to aid in the proof.

$$\gamma_g = \beta + \alpha \qquad \gamma_s = \beta - \alpha \tag{17}$$

The sine of the angle we identify as the normalized geophone wavenumber G , and similarly $S = -\sin \gamma_s$. In the latter case we use a minus sign so that both angles rotate in the same direction. Now applying the transformation of equation (7) we can obtain an interpretation of Y and H in terms of the dip and offset angles.

$$\begin{aligned} Y &= 2 \sin \alpha \cos \beta \\ H &= 2 \sin \beta \cos \alpha \end{aligned} \tag{18}$$

One final result to be obtained from Figure 3 is that

$$\sin \beta = \frac{2h}{vt} \cos \alpha \tag{19}$$

Now consider two limiting cases of the DC operator. The first is the zero dip limit and by equations 8 and 9, with $\alpha = Y = 0$, we have

$$\Phi\,(0,H) = 2\,[1 - (H/2)^2]^{1/2} \tag{20}$$

A stationary phase approximation of the integral

$$\iint_{-\infty}^{\infty} P(k_y, k_h, 0, \omega)\, e^{i \frac{\omega}{v} \Phi(0,H) z}\, e^{-i(k_h h - \omega t)}\, dk_h\, d\omega \tag{21}$$

results in the usual normal moveout time equation

$$vt = 2\,(z^2 + h^2)^{1/2} \tag{22}$$

Hence, equation 20 may be viewed as the Fourier domain counterpart of the stacking operator. If a dipping layer is present then the same operator may be used but with a change in the definition of H .

In this case

$$H' = \frac{v'k_h}{\omega} = 2 \sin \beta \tag{23}$$

where $v' = v/\cos \alpha$. In other words we simply correct the velocity by the cosine of the dip.

The second limiting case is that of zero offset. With $H = 0$, the DC operator becomes

$$\Phi\ (Y,0) = 2\ [1 - (Y/2)^2]^{1/2} \tag{24}$$

with $Y = 2 \sin \alpha$. In the same manner that a dip correction was applied in the stacking, we can apply an offset correction to the migration by changing the definition of Y to

$$Y' = \frac{v'k_y}{\omega}\ 2 \sin \alpha \tag{25}$$

with $V' = v/\cos \beta$. The dispersion relation for the migration operator becomes

$$k_z = 2 \frac{\omega}{v'} \left[1 - \frac{1}{4} \left(\frac{v'k_y}{\omega} \right)^2 \right]^{1/2} \tag{26}$$

The Fresnel approximation of this relation is

$$K_z = 2 \frac{\omega}{v'} \left[1 - \frac{1}{8} \left(\frac{v'k_y}{\omega} \right)^2 \right] \tag{27}$$

Converting to a differential equation and using a retard time coodinate to remove the shifting term we have

$$P_{zt} = \frac{v'}{4}\ P_{yy} = \frac{v}{4 \cos \beta} P_{yy} \tag{28}$$

If we assume a small dip - small offset situation then this equation is equivalent to the one given in FGDP (equation 11-3-18). First expanding the cos factor in a Taylor series we have

$$P_{zt} = \frac{v}{4} \; [1 + \frac{1}{2} \beta^2] \; P_{yy} \tag{29}$$

For small offset we have by equation (15)

$$\beta = \frac{2h}{vt}$$

Hence,

$$P_{zt} = \frac{v}{4} \left[1 + \frac{1}{2} \left(\frac{2h}{vt} \right)^2 \right] P_{yy} \tag{30}$$

There is unfortunately still at this time an unreconciled factor of 1/2 difference between this equation and equation (11-3-18) in FGDP.

Another approach to separating stacking from migration in the DC operator of equation (9), is to attempt to approximate H itself in the Fourier domain. The approach may seem a little ad hoc but there is an interpretation in the time-space domain. If H is approximated by a function that is independent of k_h and ω , then this is a slant stack in h-t space. For example, if

$$H = c$$

then

$$k_h = \frac{c}{v} \omega$$

and consequently

$$P_h + \frac{c}{v} P_t = 0$$

The characteristic of this equation is

$$t = \frac{c}{v} h$$

which is a slant stack trajectory. One can see that by approximating H we are biasing our migration about a particular offset defined by the Fresnel zone of the slant trajectory. This approach would appear to be distinctly better than the method of equation (30) which is biased about zero offset.

As a first guess we might consider taking H to be

$$H = \frac{2h}{vt} \tag{31}$$

By examining equations (15) and (16) one can see that this is a small dip approximation. A reasonable way to judge the accuracy of the approximation is to compare the group velocity of this operator with the elliptical one of the exact operator. In the appendix the procedure for computing group velocity curves is shown and in Figure 4 the results are shown. As one might expect the approximation fails at large dip angles, but it does perform well over a wide range of offsets.

As a second guess one might consider using equation (19) as an estimate of H

$$H = \frac{2h}{vt} \cos \alpha \tag{32}$$

and then approximate $\cos \alpha$ by $[1 - (Y/2)^2]^{1/2}$

$$H = \frac{2h}{vt} \left[1 - \left(\frac{Y}{2}\right)^2 \right]^{1/2} \tag{33}$$

The group velocity curve of this approximate is also shown in Figure 4. This approximation is valid over a wider range of dips than the previous one.

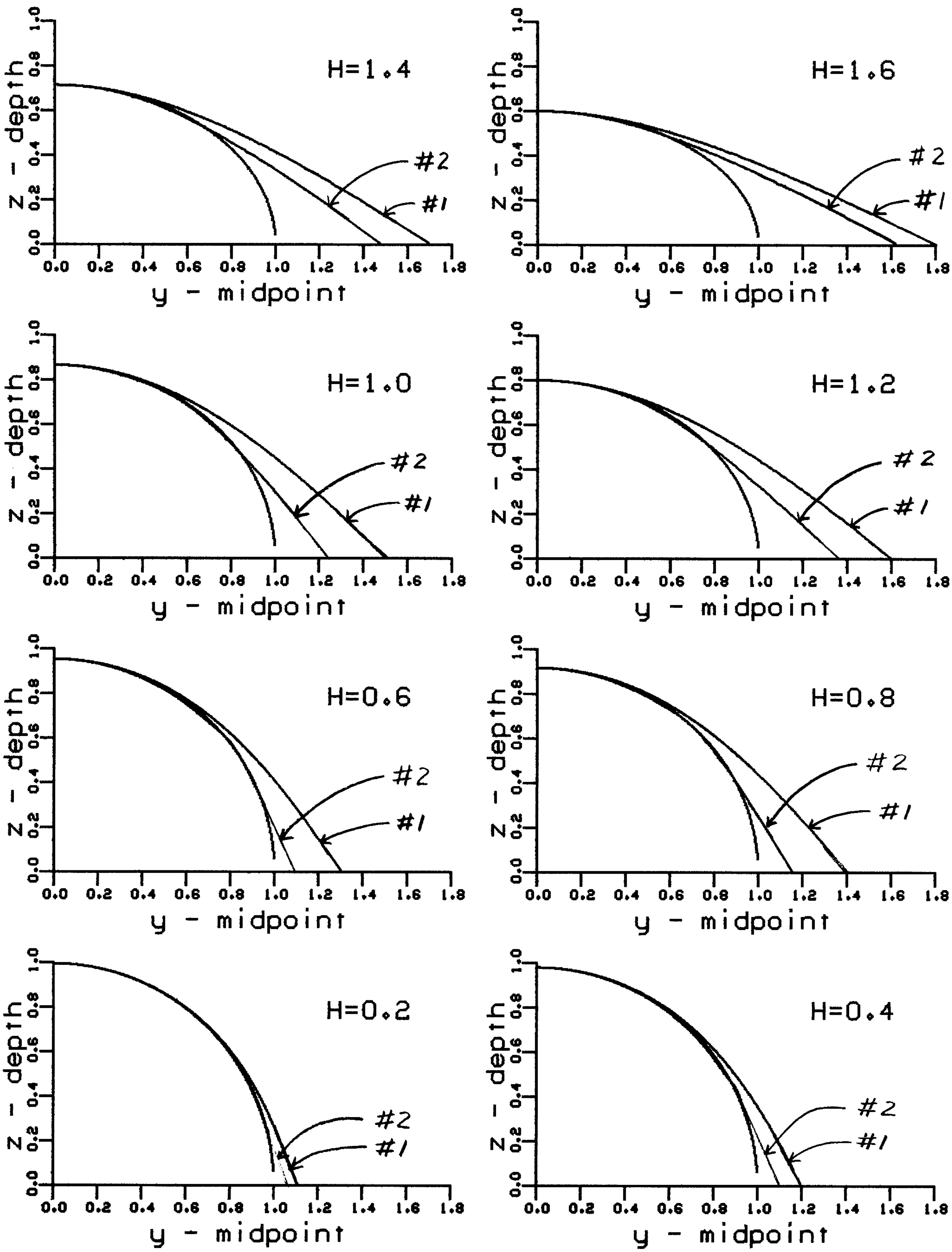

Figure 4. Group curves for two approximations of the exact downward continuation operator. Approximation #1 is that of equation (31), while #2 corresponds to equation (33). The unlabelled curve is the exact elliptical group curve (equation]6). Each frame is for a particular value of H = 2h/vt.

We believe there are other approximations along the same lines as equations (31) and (33) which will increase the range of dip angles of the constant offset migration. The next step in the analysis will be to convert the Fourier domain approximations into time-space domain migration scheme. For now however, they are only useful in f-k migration methods.

Appendix

To compute the group velocity curves of the migration operator

$$P(y,h,z,t) = \iint_{-\infty}^{\infty} P(y,k_h,0,\omega) e^{iz\left\{\frac{\omega}{v}\Phi\,[Y,H(y)]-k_y\frac{y}{z}+\omega\frac{t}{z}\right\}} dk_y d\omega \qquad \text{(A1)}$$

we use a stationary phase approximation. Differentiating the argument of the exponential with respect to k_y and ω , we have

$$\frac{\partial}{\partial k_y} = 0 \rightarrow \Phi' - \frac{y}{z} = 0 \qquad \text{(A2)}$$

$$\frac{\partial}{\partial \omega} = 0 \rightarrow \Phi - Y\,\Phi' + \frac{t}{z} = 0 \qquad \text{(A3)}$$

The derivative of Φ is

$$\Phi' = \frac{\partial}{\partial Y}\Phi = -\frac{\frac{1}{4}(Y+H)(1+H')}{\left[1-\left(\frac{Y+H}{2}\right)^2\right]^{1/2}} - \frac{\frac{1}{4}(Y-H)(1-H')}{\left[1-\left(\frac{Y-H}{2}\right)^2\right]^{1/2}} \qquad \text{(A4)}$$

where $H = H(Y)$ and $H' = \frac{\partial}{\partial Y} H(Y)$

Equations (A2) and (A3) may be solved to form parametric equations for y and z .

$$y = \frac{\Phi' vt}{Y\,\Phi' - \Phi} \quad \text{and} \quad z = \frac{vt}{Y\,\Phi' - \Phi} \qquad \text{(A5)}$$

In order to compare these curves to the exact elliptical curves (equation (16)) we have to consider the effect of the stacking. This may be done by considering equation (A5) at $Y = 0$.

$$y = 0 \quad \text{and} \quad z = \frac{vt}{-\Phi\,[0,H(0)]} \tag{A6}$$

where

$$\Phi\,[0,H(0)] = 2\left\{1 - \left[\frac{H(0)}{2}^{2}\right]\right\}^{1/2}$$

For the two approximations considered in this paper $H(0) = 2\,\frac{2h}{vt}$, hence

$$z = -\frac{vt}{2}\left[1 - \left(\frac{2h}{vt}\right)^{2}\right]^{-1/2}$$

If we now consider the exact relation at $y = 0$ we have

$$z_e = -\frac{vt}{2}\left[1 - \left(\frac{2h}{vt}\right)^{2}\right]^{1/2} \tag{A7}$$

The stacking operation which brings equations (A6) and (A7) into agreement is

$$z' = z\left[1 - \left(\frac{2h}{vt}\right)^{2}\right] \tag{A8}$$

The curves in Figure 4 are plots of z' versus y .