SLANT STACKS AND INTERVALS OF OPTIMUM STACKING

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Since starting working with the slant stacks, we have had the feeling that stacks corresponding to different values of p (ray parameter $p = \sin\theta/v$) will enhance different time intervals of the final stacked section. This is because we are limited by several factors among which are: (1) The finite character of the data, especially along the x-axis. Besides the finite cable length, we could add here the muting that sometimes has to be applied to the data in order to get rid of unwanted arrivals (refractions, etc.). (2) Aliasing problems because of carrying the sum too far out off the Fresnel Zone at some points. (3) The need for an anti-aliasing window which, especially at early times, tends to restrict the sum to a region inside the Fresnel Zone.

All these factors define intervals of optimum stacking along the time axis, within which the stacking corresponding to a given p will produce its most reliable results. Intuitively we may guess that larger values of p will yield better results at earlier times and vice versa. The idea then will be to produce several stacks (for different p-values), whose intervals of optimum stacking overlap and, on the whole, cover the full time axis. Afterwards, all these stacks could be combined linearly, applying weighting functions to each one, with maximum values along these intervals.

To get at least a feeling of how these intervals could be defined quantitatively, let us study the case of a constant-velocity, flat-layered medium.

Since in all the following considerations the concept of the Fresnel Zone is going to be used frequently, let us try to find how this zone varies as we stack down the time axis. For simplicity, we will refer to the angle of propagation θ instead of the ray parameter p. For the more general case of velocity variable media, these results can be expressed in terms of p, recalling that $\sin\theta = pv$.

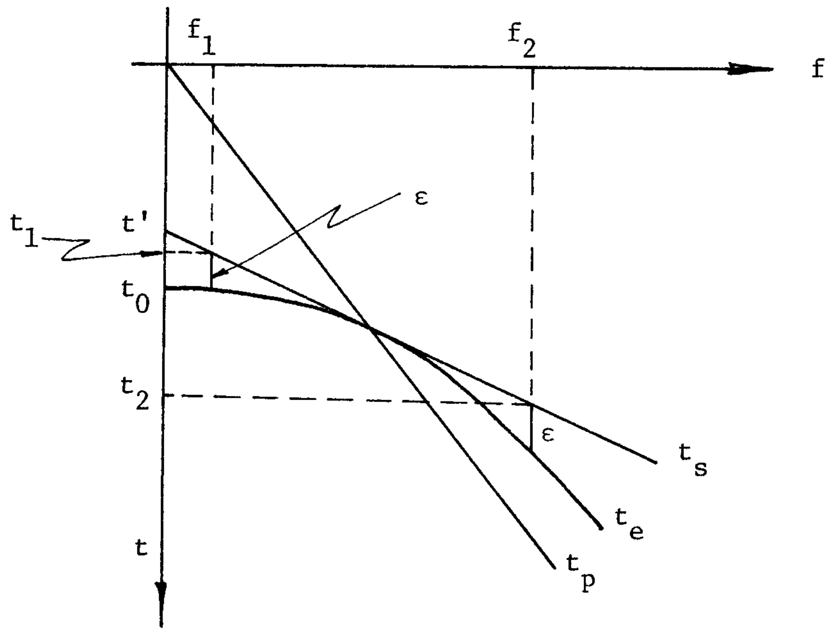

FIGURE 1.

We will start by reviewing the equations used in the stacking process:

$$t_p = f/pv^2 = f/v \sin\theta , \tag{1}$$

defines the direction of propagation;

$$t_s = t' + p f = t' + (f \sin\theta/v) , \tag{2}$$

defines the direction of stacking; and

$$t_e^2 = t_0^2 + (f^2/v^2) , \tag{3}$$

defines a given event (flat reflector). Additionally we have

$$t' = t_0 \cos\theta . \tag{4}$$

The Fresnel Zone is then defined along the stacking path t_s as a region bounded by coordinates (f_1,t_1) and (f_2,t_2) such that the vertical distance ε (units of time) between t_s and t_e equals half the period of the source waveform. In order to define the boundaries of this zone, what we then need are equations for f_1 and f_2 (or t_1 and t_2) as functions of t' and ε. From Fig. 1 and Eqs. (2) and (3), after some algebra, we get

$$f_1(t') = (v/\cos^2\theta)\{\sin\theta(\varepsilon + t') - [\varepsilon(\varepsilon + 2t')]^{1/2}\}, \qquad (5)$$

$$f_2(t') = (v/\cos^2\theta)\{\sin\theta(\varepsilon + t') + [\varepsilon(\varepsilon + 2t')]^{1/2}\}. \qquad (6)$$

Further, we can make one of the two following assumptions: (1) We consider that, for a fixed f, ε increases linearly with time: $\varepsilon = \varepsilon_0 + bt'$ (ε_0, b-const.), so that $Q = \varepsilon/t'$ = const. (since $bt' >> \varepsilon_0$ most of the time), in which case (5) and (6) become

$$f_{1,2} = (v/\cos^2\theta)\{[1 + (1/Q)]\sin\theta \mp [1 + (2/Q)]^{1/2}\}\varepsilon . \qquad (7)$$

For larger t' these equations become straight lines, as shown in Fig. 2, and could define a "natural" anti-aliasing window.

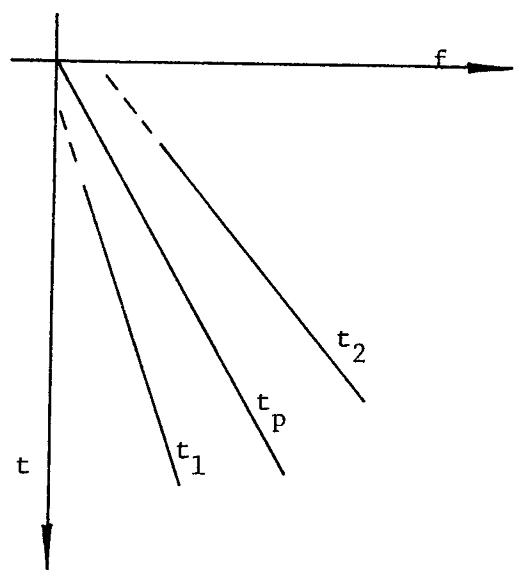

FIGURE 2.

(2) We might also consider ε = const. with time, in which case (5) and (6) become (assuming $\varepsilon << t'$)

$$f_{1,2} \cong (\varepsilon v/\cos^2\theta)[t'\sin\theta \mp (2\varepsilon t')^{1/2}] . \qquad (8)$$

The shapes of these curves are illustrated in Fig. 3.

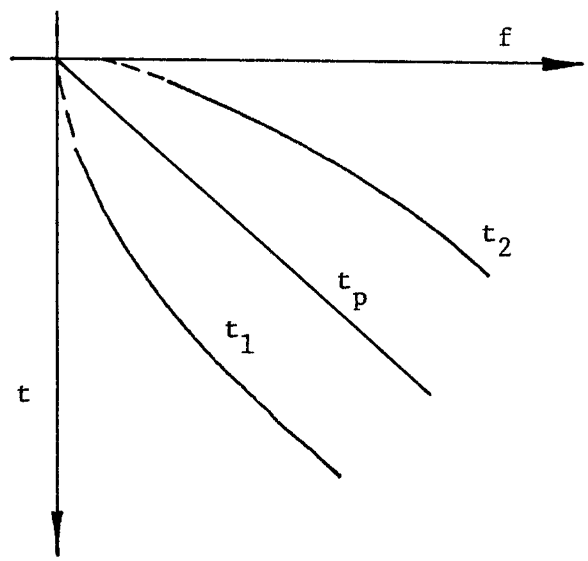

FIGURE 3.

The first approach (Q = const.) will probably model better primary arrivals. For one of our waveform estimation techniques, however, where only paths within the water layer are considered, we should take the second approach (ε = const.).

Following these ideas, let us consider that portion of the stacking area that is inside the Fresnel Zone and is bounded by the given data (the shaded area in Fig. 4). This region defines an interval in time along the stacking axis $\Delta t' = t_2' - t_1'$ for which the summation will include only given data. If we want to compute this interval quantitatively, then we need inverse equations to (5) and (6). That means t' as a function of f along the Fresnel curves:

$$t_{1,2}' = (1/v \tan^2\theta)\{\varepsilon v + f \sin\theta \mp (\varepsilon v/\cos\theta)[1 + (2 f \sin\theta/\varepsilon v)]^{1/2}\}. \quad (9)$$

For a given θ and a fixed offset f, these solutions give two values of t along the stacking axis t' as illustrated in Fig. 5.

Replacing f by F_1, the initial offset of the gather, and by F_2, the final (farthest) offset, in Eq. (9), we can then find the coordinates

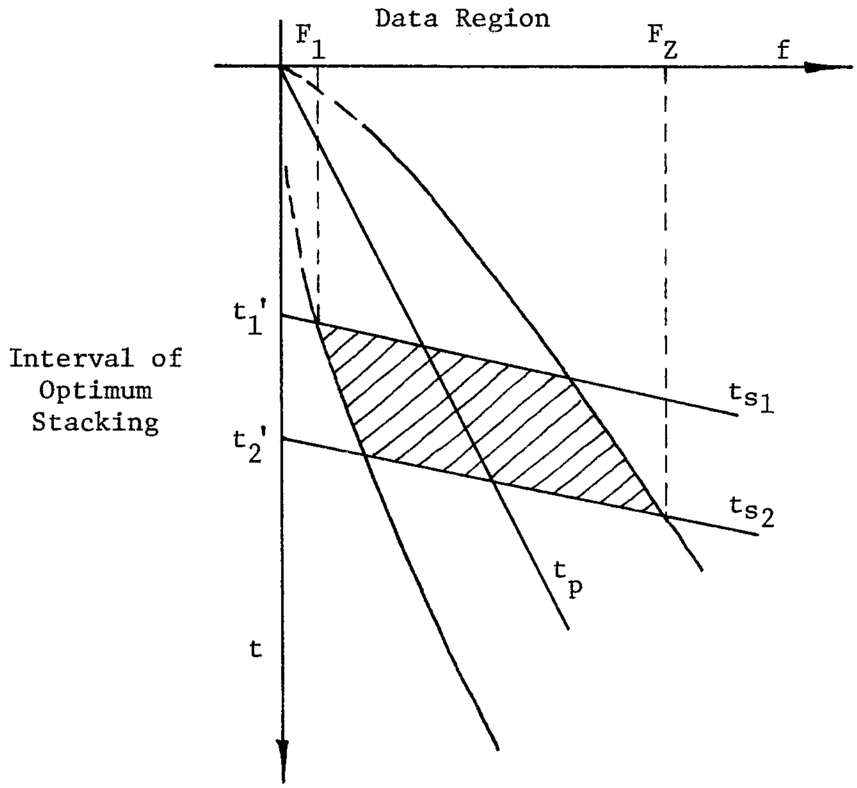

FIGURE 4.

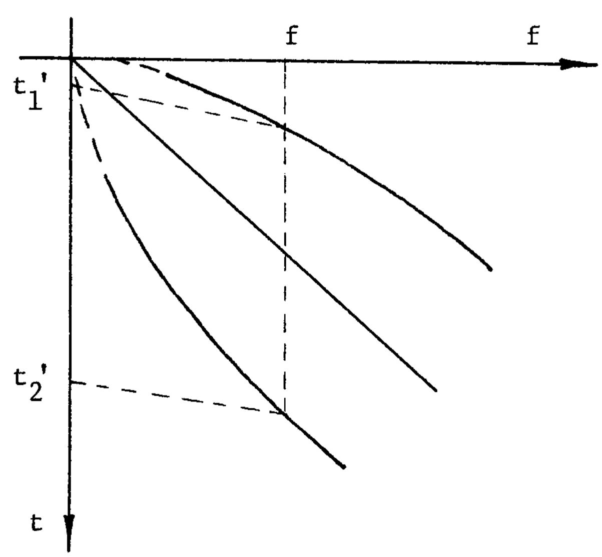

FIGURE 5.

t_1' and t_2' which define the interval of optimum stacking in Fig. 4.

As an example of practical interest, let us consider data for which $F_1 = 200$ m, $F_2 = 2550$ m (48 traces 50-m apart), and $\varepsilon = 0.05$ sec (for a 100-msec period waveform). The results for two different velocities $v = 1500$ m/sec and $v = 2000$ m/sec, are shown in Tables 1 and 2. Such tables eventually will allow us to choose the most convenient propagation angle (p-value) for a given time interval of interest. We should mention here that these intervals of optimum stacking could be increased by extrapolating traces at the left of F_1 and the right of F_2, but this would imply a good knowledge of velocity.

We could also define an interval of optimum stacking in relation to the anti-aliasing window, as that interval of the time axis within which our window includes the whole Fresnel Zone but does not go too far off it (shaded area in Fig. 6).

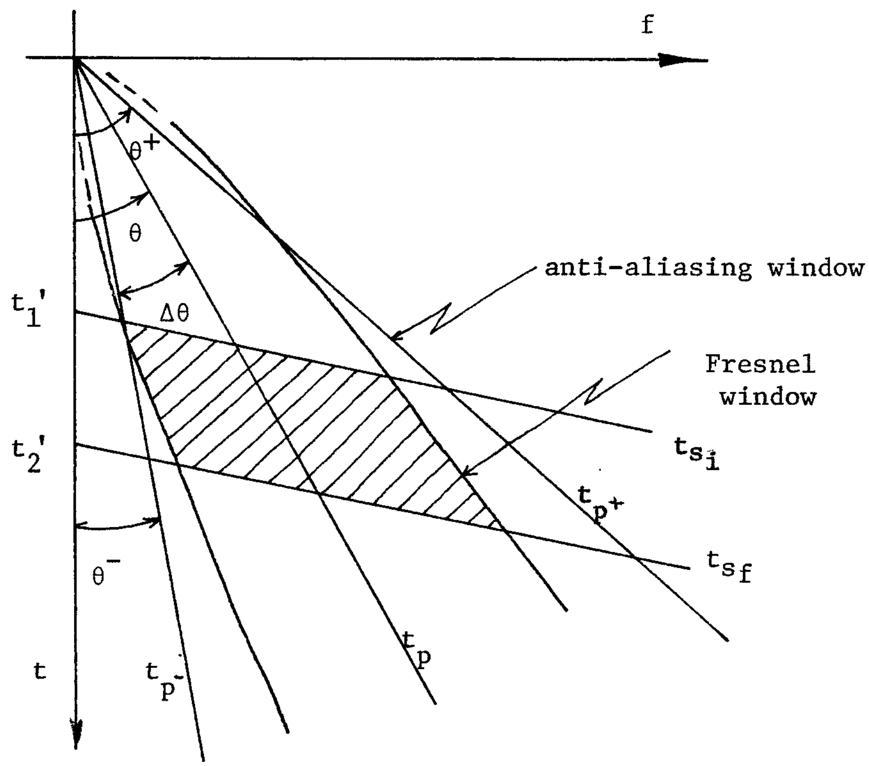

FIGURE 6.

Without going through the algebra, we can find a relation that links t' and the distance δ between t_s and t_e at the boundaries of the stacking lines, given by the anti-aliasing window (Fig. 7):

TABLE 1 (v = 1500 m/sec)

p (sec/m) $\times 10^{-4}$	θ (deg)	t_1' (sec)	t_2' (sec)	Dt' (sec)
0.58	5	15.99	8.63	0.00*
1.16	10	4.62	5.26	0.64
1.73	15	2.30	3.70	1.40
2.28	20	1.40	2.80	1.40
2.82	25	0.95	2.15	1.20
3.33	30	0.68	1.68	1.00
3.82	35	0.50	1.30	0.80
4.29	40	0.39	1.00	0.60

*No solution

TABLE 2 (v = 2000 m/sec)

p (sec/m) $\times 10^{-4}$	θ (deg)	t_1' (sec)	t_2' (sec)	Dt' (sec)
0.44	5	15.30	5.75	0.00*
0.87	10	4.30	3.60	0.00*
1.29	15	2.09	2.57	0.48
1.71	20	1.25	1.94	0.69
2.11	25	0.84	1.50	0.66
2.50	30	0.60	1.17	0.57
2.87	35	0.44	0.91	0.45
3.21	40	0.34	0.70	0.36

*No solution

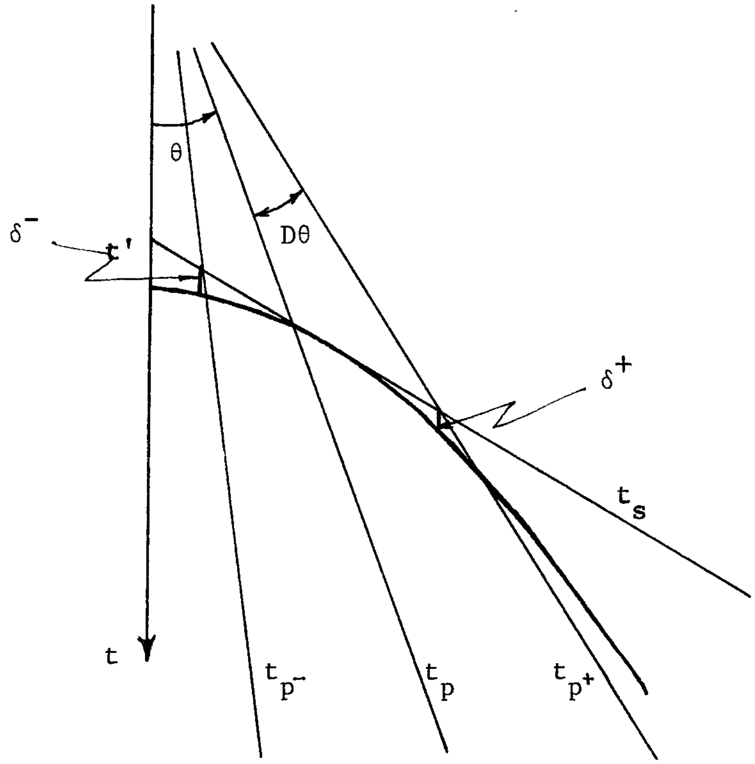

FIGURE 7.

$$t'^{\pm} = \left(\cos\theta(1-\sin\theta\,\sin\theta^{\pm})\Big/\{[1+\sin\theta^{\pm}(\sin\theta^{\pm}-2\,\sin\theta)]^{1/2}-\cos\theta\}\right)\delta^{\pm}, \quad (10)$$

where $\theta^{+} = \theta + \Delta\theta$ and $\theta^{-} = \theta - \Delta\theta$.

When $\delta^{-} = \varepsilon$ (half the waveform's period), the window will include the entire Fresnel Zone. This condition defines our t_1' in Fig. 6. As the stack proceeds to later times, δ becomes larger than ε and aliasing will be introduced. We must then attempt to define a lower boundary t_2' (Fig. 6) by setting $\delta^{+} = r\varepsilon$ $(r > 1)$, such that within the interval from t_1' up to t_2' we have a tolerable amount of aliasing. Although at present I am not very sure what the value of r should be, I tried to produce a table for $r = 2$ (that means $\delta^{+} = 2\varepsilon$, the full waveform's period). This will set sort of a minimal interval of optimum stacking.

As it turns out, $t'^{+} \approx t'^{-}$ in relation (10), so by making $\delta^{+} = r\varepsilon$ we merely set $t_2' = r\,t_1'$ (for a fixed θ and $D\theta$). The corresponding values are displayed in Table 3.

If tables like 1 and 2 allow us to choose the best value of θ (or p) for a given time interval, tables like 3 will allow us to choose the more convenient window for a given θ. Finally, we may remark that the anti-aliasing

TABLE 3

θ (deg)	$D\theta$ (deg)	t_1' (sec)	t_2' (sec)	Dt' (sec)
10	10	3.24	6.48	3.24
15	10	3.12	6.24	3.12
15	15	1.42	2.84	1.42
20	10	2.95	5.90	2.95
20	15	1.34	2.68	1.34
20	20	0.78	1.56	0.78
25	10	2.75	5.50	2.75
25	15	1.25	2.50	1.25
25	20	0.73	1.46	0.73
25	25	0.48	0.96	0.48
30	10	2.51	5.02	2.51
30	15	1.14	2.28	1.14
30	20	0.67	1.34	0.67
30	25	0.44	0.88	0.44
30	30	0.32	0.64	0.32
35	10	2.25	4.50	2.25
35	15	1.03	2.06	1.03
35	20	0.60	1.20	0.60
35	25	0.40	0.80	0.40
35	30	0.30	0.60	0.30

window does not have to be symmetrical and that in some cases we might choose $\theta^+ = \theta + \Delta\theta_1$ and $\theta^- = \theta - \Delta\theta_2$, where $\Delta\theta_1 \neq \Delta\theta_2$. In any case, relation (10) can easily be extended to incorporate such an eventuality.

As an illustration, I include in Fig. 8 three different stacks of real data. For the computation, I have assumed a velocity of 2000 m/sec, which is a lower bound of the actual velocities in the section. According to Table 2, for an angle of propagation of 20° (shallow depths), we shall expect an interval of optimum stacking that goes from 1.25 to 1.94 sec. As can be seen from Fig. 8(c), this prediction correlates pretty well with the shown section. Figures 8(a) and 8(b) correspond to angles of 10° and 15°. Here we are considering greater depths where the velocities are on the order of 4000-5000 m/sec; thus, Table 2 is no longer a good approximation, and we see that the intervals of optimum stacking are higher (in time) than the predicted values.

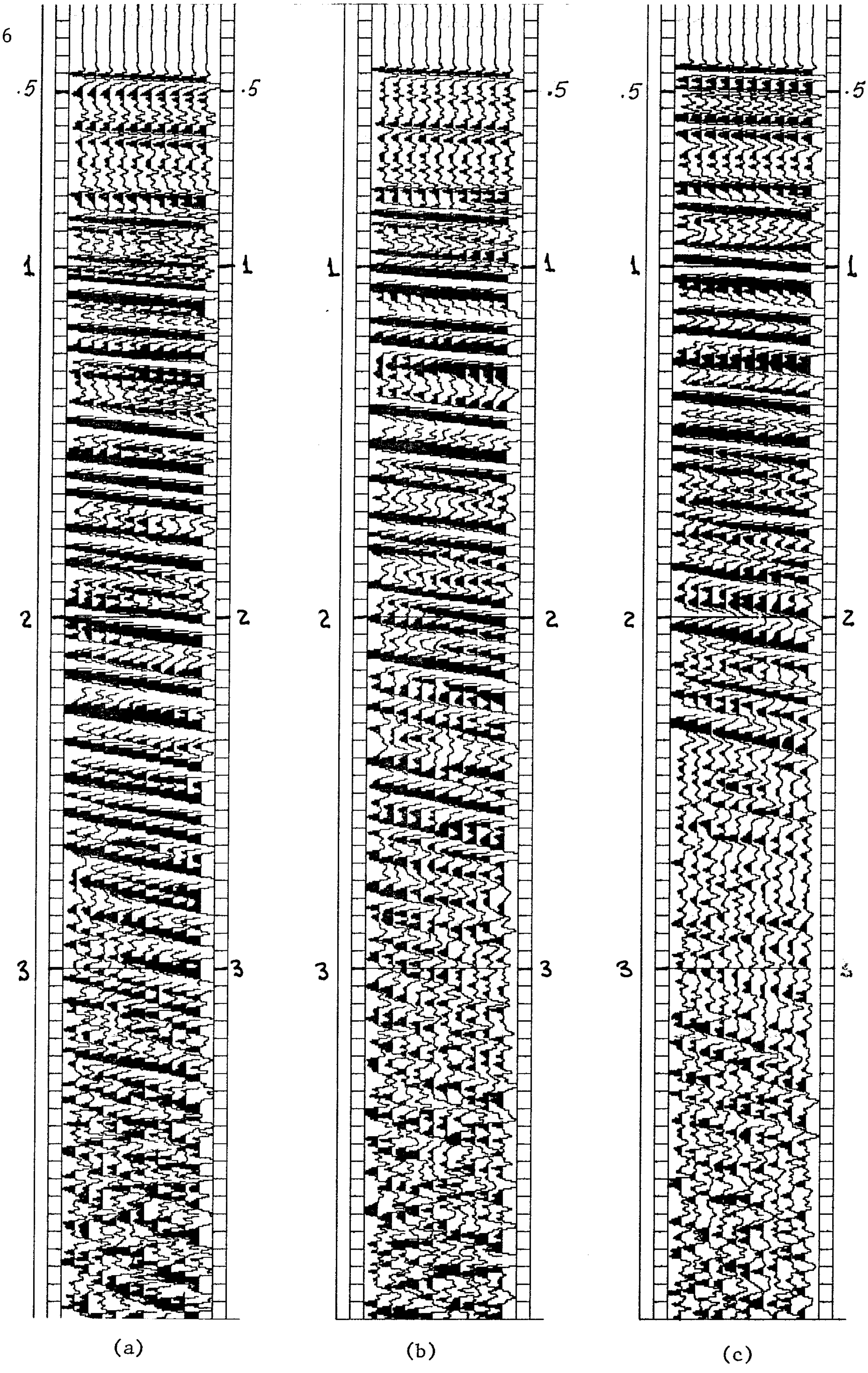

FIGURE 8.—(a) Slant stack for v = 2000 m/s, θ = 10° ($p = 0.87 \times 10^{-4}$ s/m);
(b) Slant stack for v = 2000 m/s, θ = 15° ($p = 1.29 \times 10^{-4}$ s/m);
(c) Slant stack for v = 2000 m/s, θ = 20° ($p = 1.71 \times 10^{-4}$ s/m).