PREDICTING MULTIPLE REFLECTION ARRIVALS ON SLANT STACKS

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Noah's waveform estimation, as described in our previous reports (SEP-8, p. 202), requires a precise gating of the primaries and first-order multiple reflections. Since picking up the primary gate is not a major problem, the situation reduces to predicting correctly the multiple arrivals.

In the case of a flat sea bottom, the problem is trivial and the first-order multiple should arrive at twice the primary time, unless the location of $t = 0$ was not well-defined in the records; and even in this case, the inconvenience can be easily corrected.

If the sea bottom is irregular, the travel time associated with each bounce can vary from path to path, especially at large angles of propagation (stacking). The situation is illustrated in Figs. 1 and 2. Figure 1 shows a section corresponding to a plane wave that propagates from left to right at 15°, as indicated in the upper-left corner. The section was produced using an antialiasing window of ±15° and a tapering function along the stacking line as suggested by P. Schultz (SEP-9). It is displayed in the original slanted coordinates.

Later a program designed to pick up primary and multiple arrivals from this section was run under the assumption that the multiple arrived at twice the primary travel time. The resulting gates are shown in Fig. 2. Toward the flat portion of the data the program did a reasonable job, but in the left dipping part it missed the multiple practically everywhere.

In order to get the proper result, a correction has to be made to account for both factors: the slant propagation and the dipping of the sea bottom. The migration slanted theory previously developed (SEP-5, p. 20, and SEP-8, p. 222) correctly models these arrivals, and it is only a matter of running the forward problem, using the primary gate as input, to predict the multiple arrivals. But if we do not want to get involved with the general theory, a simple ray approximation analysis will suffice. Let us assume that the two bounces of the multiple's path occur close enough to consider a single dipping angle α. The geometry corresponding to an angle of propagation θ is then depicted in Fig. 3.

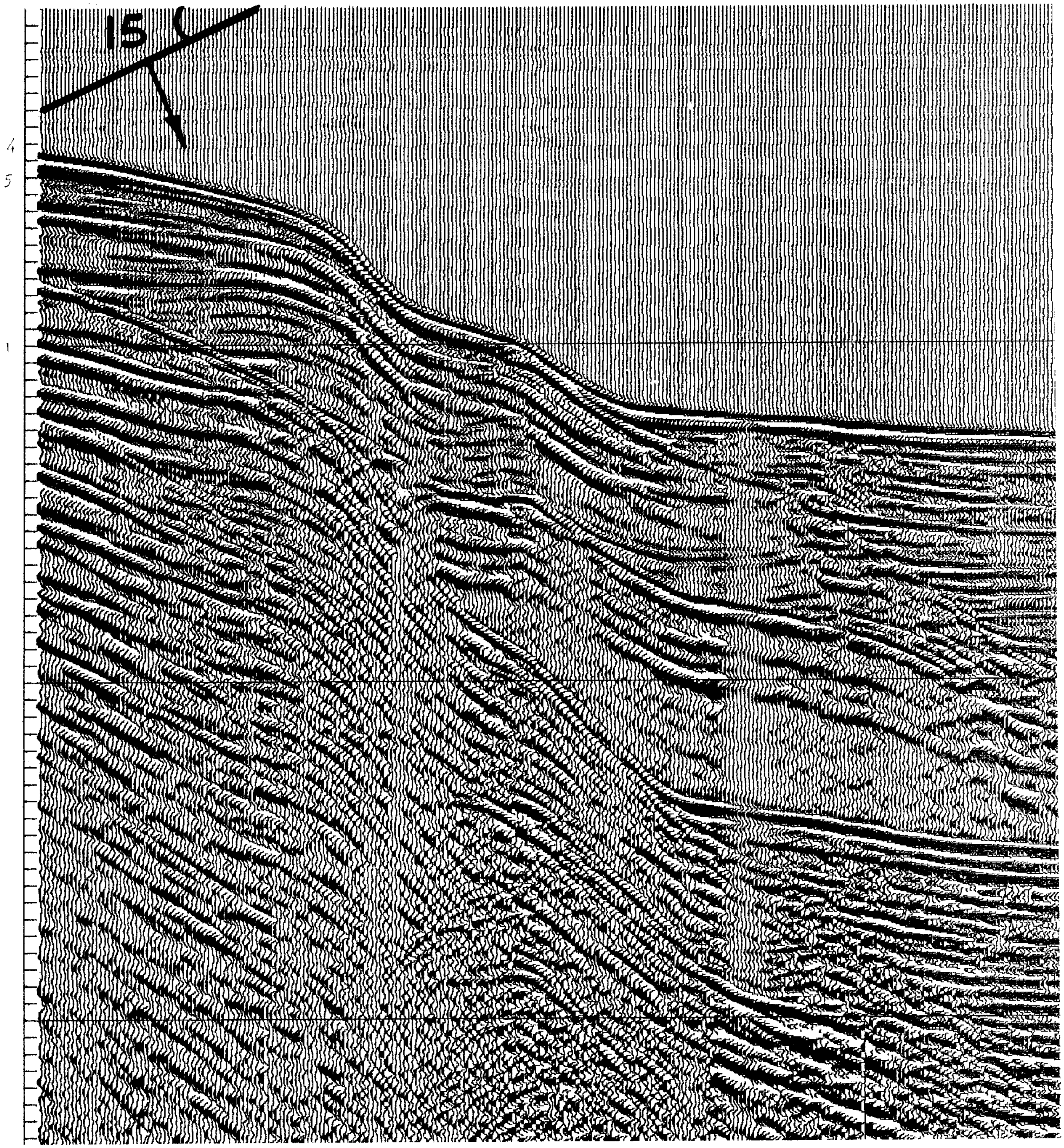

FIGURE 1.——Slant plane wave section for $p = 1.73 \times 10^{-4}$. The direction of propagation is shown in the upper left corner. An anti-aliasing window of $\pm 15°$ and a tapering function (cosinus bell) along the stacking line were applied.

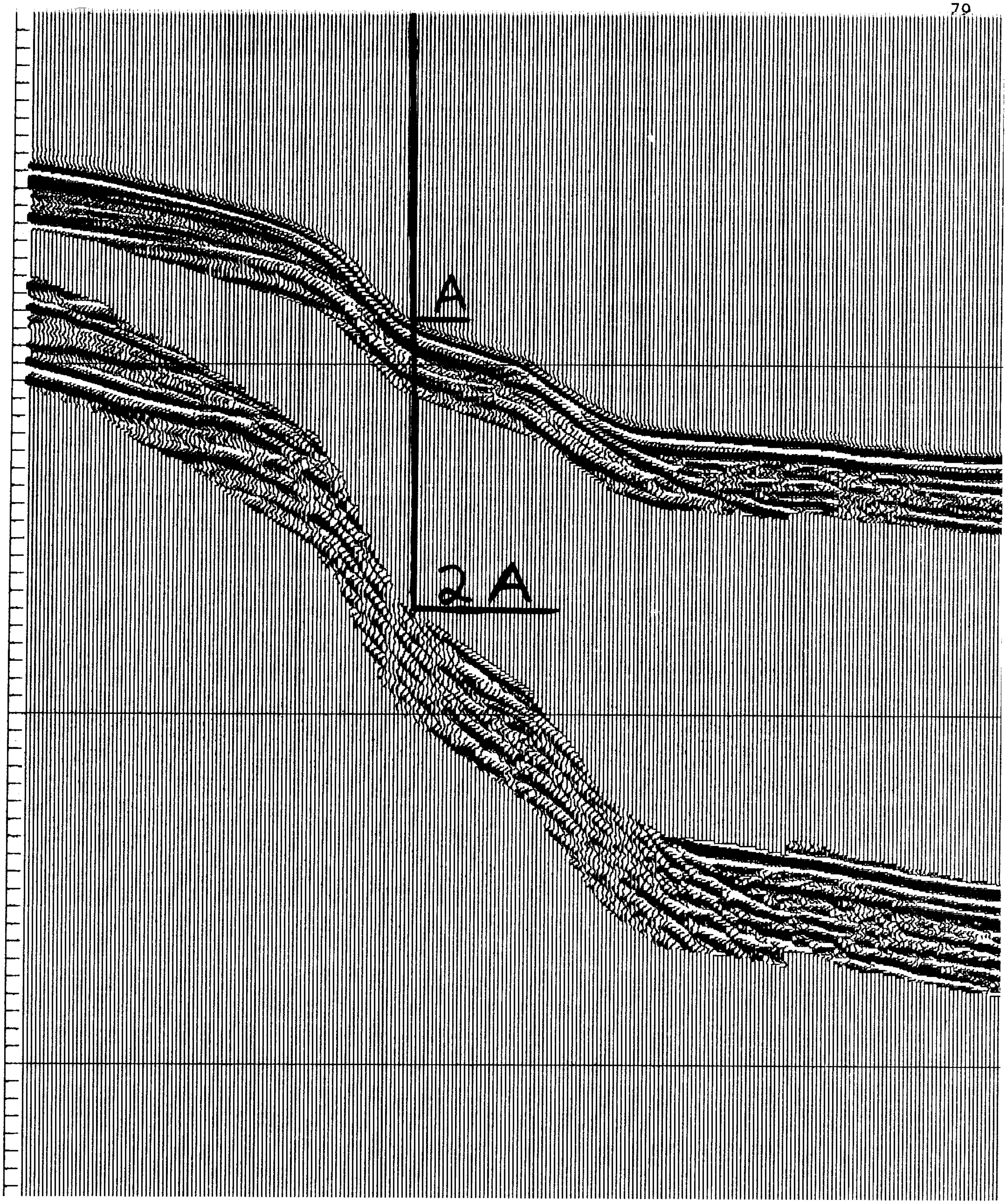

FIGURE 2.——Primary and multiple gates assuming vertical propagation and no dip.

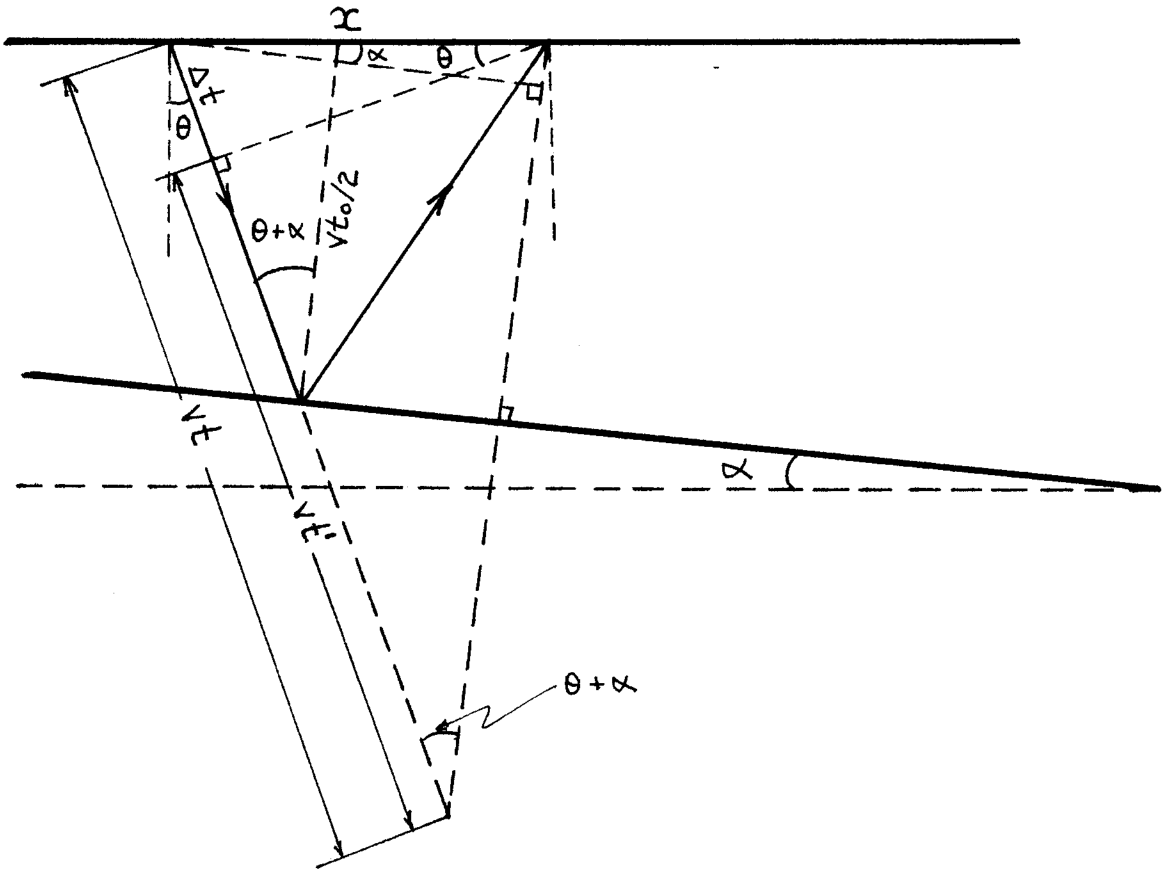

FIGURE 3.

The idea then is to compute the travel time associated with the multiple's path O_1ABCD (multiple at location D) as a function of the primaries' travel times associated with the paths O_1AB and $O_2C'D$ (primaries at locations B and D, respectively). Of course, we will have to compute as well the horizontal distance between the two primaries' locations BO_2.

From triangle BO_2D' we have:

$$BD' = [\cos\theta/\cos(\theta+2\alpha)]O_2D' , \qquad (1)$$

and from triangle BDD':

$$BD = [\sin(\theta+3\alpha)/\cos\alpha]BD' . \qquad (2)$$

If now we call $t_{p_1} = O_1B'/v$, the travel time up to the primary in location B(path O_1AB);

$t_{p_2} = O_2D'/v$, the travel time up to the primary in location D(path $O_2C'D$);

$t_{m_2} = (O_1B' + BD')/v$, the travel time up to the multiple in location D (path O_1ABCD);

$Dx = BO_2$, the horizontal distance between the two primaries' location B and O_2;

then from expressions (1) and (2) we obtain

$$Dx = \frac{\cos\theta \, \sin(\theta + 3\alpha)}{\cos\alpha \, \cos(\theta + 2\alpha)} v t_{p2} , \quad (3)$$

and

$$t_{m_2} = t_{p_1} + \frac{\cos\theta}{\cos(\theta + 2\alpha)} t_{p2} . \quad (4)$$

Notice that when $\alpha = 0$, expression (4) becomes $t_{m2} = t_{p1} + t_{p2}$ as expected.

These results were computed in the so-called "interpretation frame," but in many instances we may like to work in the slanted frame. The implied coordinate transformation is summarized in Fig. 4, where t is associated with the interpretation frame and t' with the slanted one.

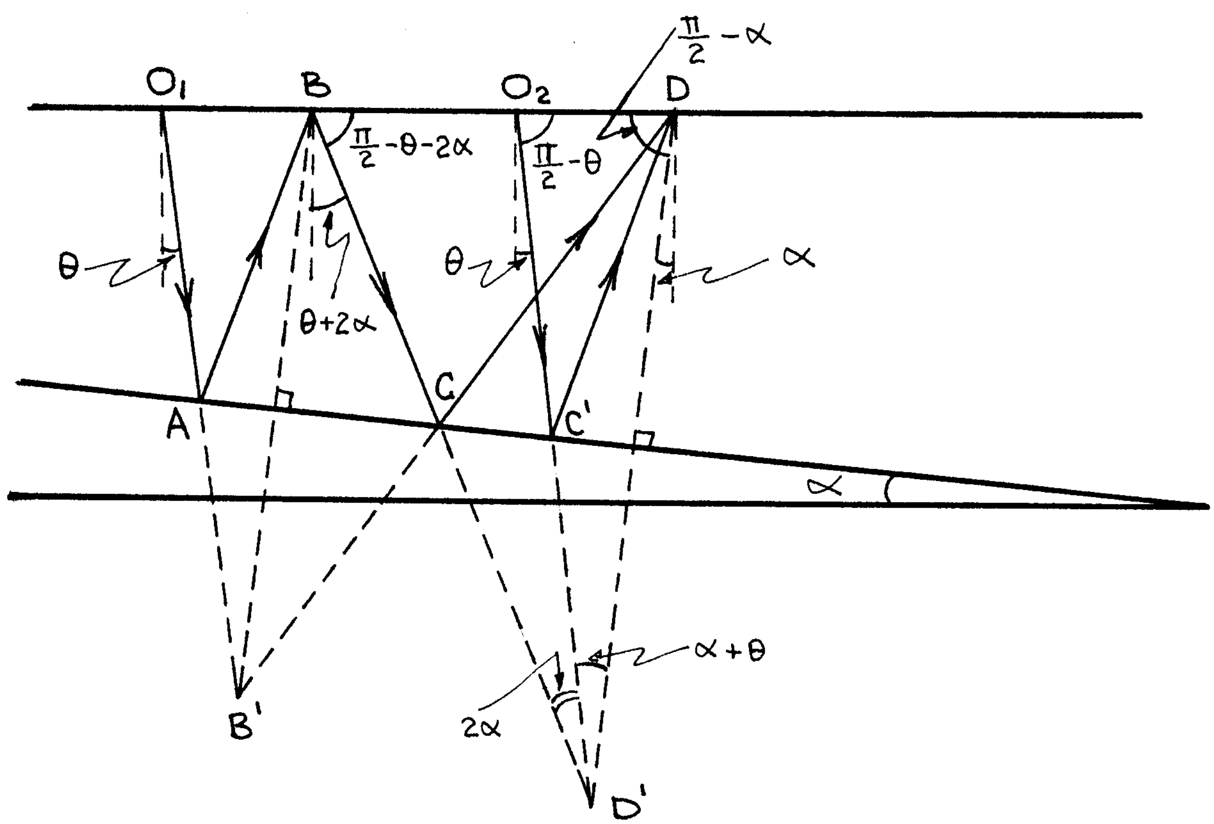

FIGURE 4.

According to Fig. 4, we have the following relation:

$$t = t' + \Delta t = [1 + \tan\theta \tan(\theta + \alpha)]\, t' . \tag{5}$$

As a matter of future reference, I will include

$$x = [\sin(\theta + \alpha)/\cos\alpha]\, v t , \tag{6}$$

$$x = \frac{\sin(\theta + \alpha)}{\cos\alpha}[1 + \tan\theta \tan(\theta + \alpha)]\, v t' , \tag{7}$$

$$t_0 = \cos(\theta + \alpha)\, t. \tag{8}$$

Substituting (5) into (3) and (4), we finally set

$$Dx = \frac{\cos\theta \sin(\theta + 3\alpha)}{\cos\alpha \cos(\theta + 2\alpha)}[1 + \tan\theta \tan(\theta + \alpha)]\, v t'_{p_2} . \tag{9}$$

$$t'_{m_2} = t'_{p_1} + \frac{\cos\theta}{\cos(\theta + 2\alpha)} \frac{1 + \tan\theta \tan(\theta + \alpha)}{1 + \tan(\theta + 2\alpha)\tan(\theta + 3\alpha)}\, t'_{p_2} . \tag{10}$$

Taking into account the slanted propagation but not the dip ($\alpha = 0$), we get the gates shown in Fig. 5. The result is much better than the one in Fig. 2, but still we miss the multiple in those places where the dip is significant. Finally in Fig. 6 is the result as computed according to relations (9) and (10).

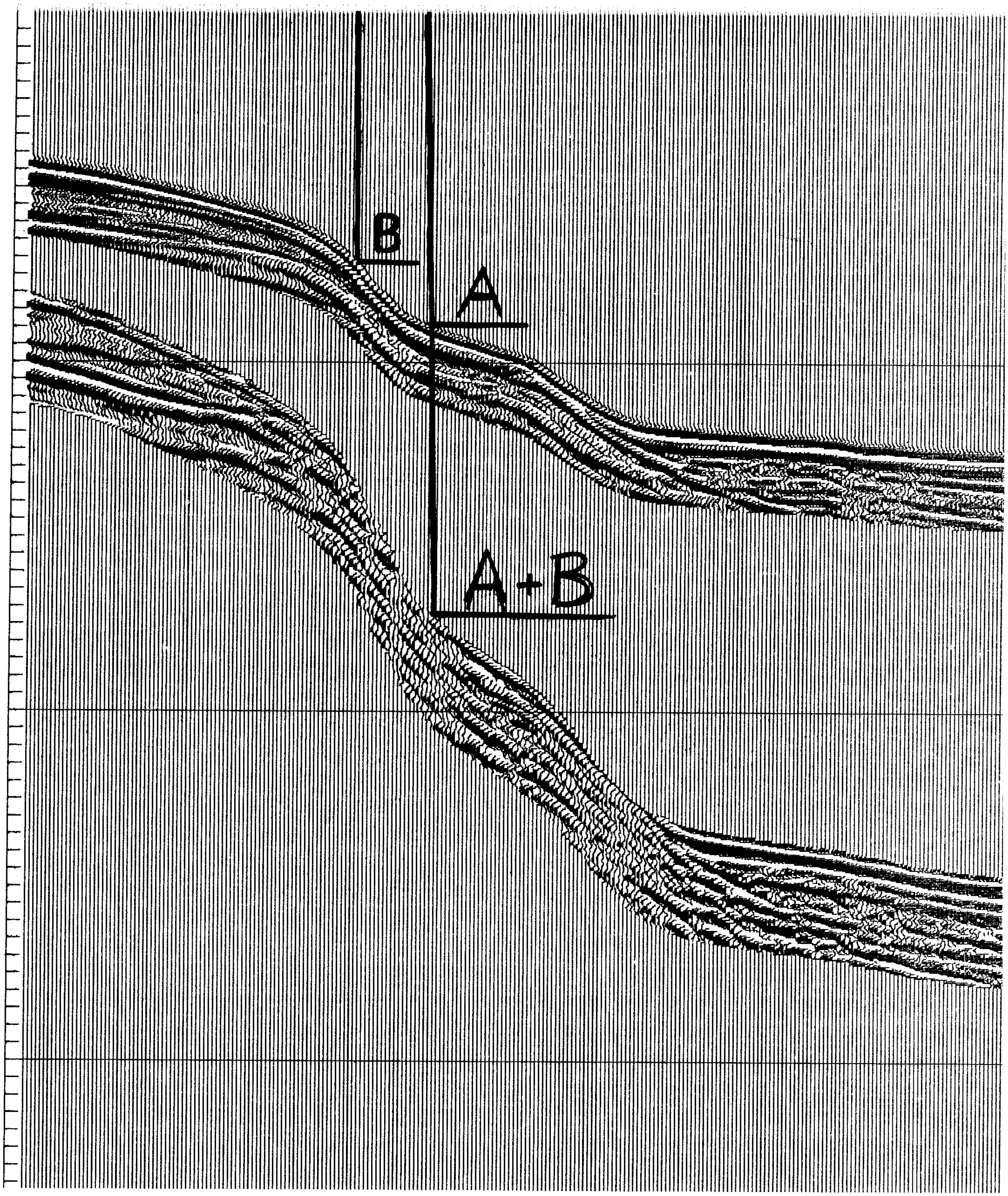

FIGURE 5.——Primary and multiple gates taking into account the slanted propagation but not the dip.

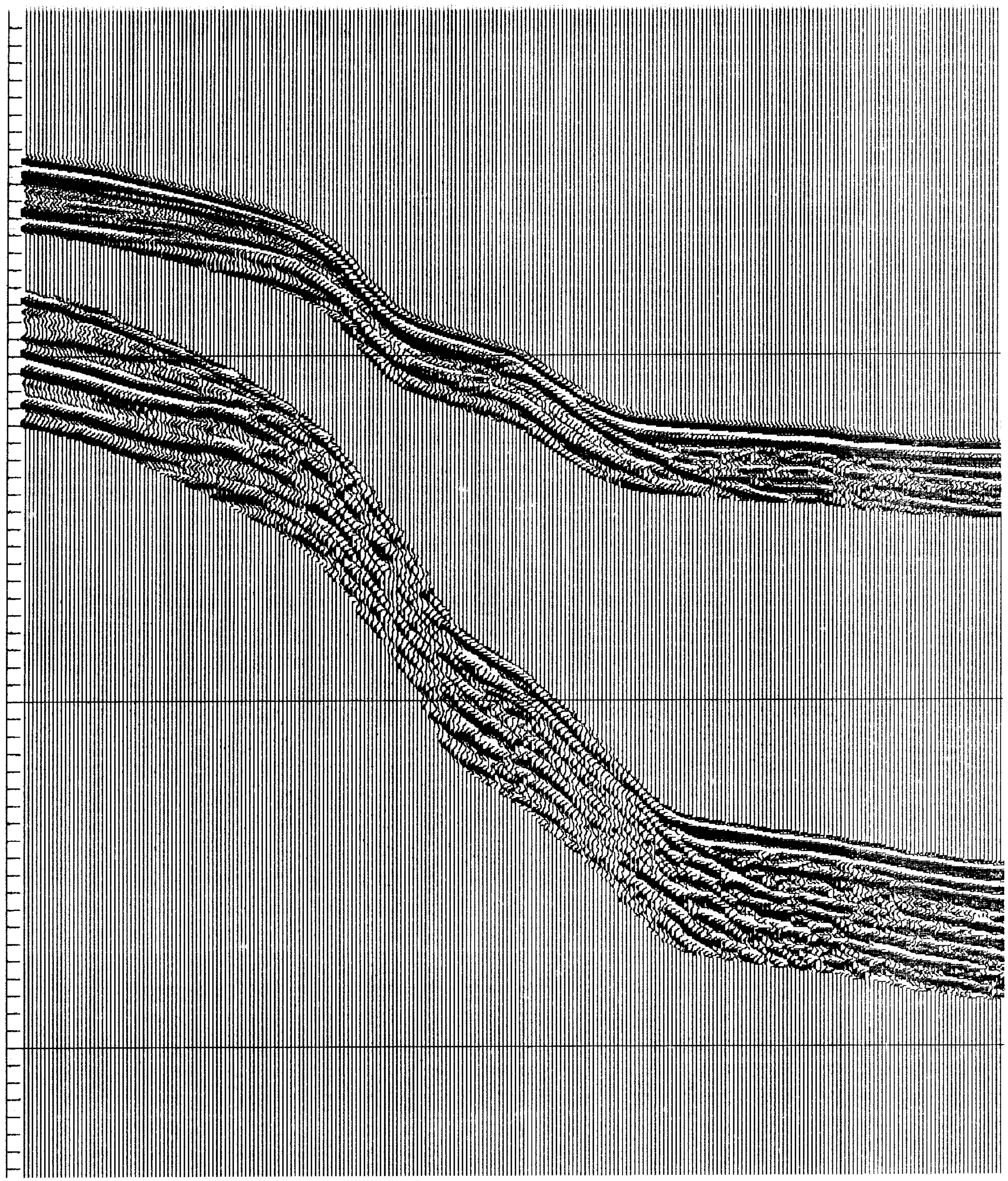

FIGURE 6.——Primary and multiple gates with both the slanted and dip corrections incorporated.