Source Waveform Ambiguity with Surface Reflection Coefficient

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We have called the "Noah Seismogram" the seismogram which would have been recorded if the free surface interface were changed to a perfect absorber. All free surface bounce multiple reflections are absent on a Noah seismogram, hence our interest in the general question of computing Noah seismograms from observed seismograms. The most significant practical difficulty has to do with determination of the shot waveform. Use of a slightly erroneous shot waveform yields a Noah seismogram with surface multiples incompletely removed. This observation led me to hypothesize that we could regard the error as being in the free surface condition rather than in the source waveform. To understand this more precisely, consider Figure 1.

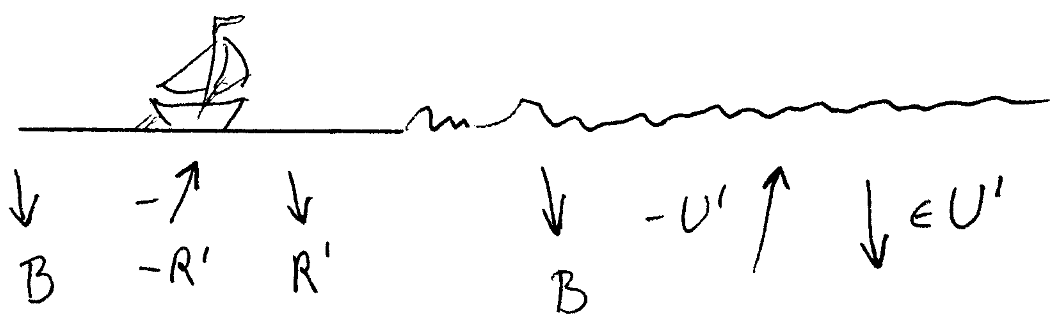

Figure 1. A perfectly reflecting free surface on the left and an imperfectly absorbing ($0 < |\varepsilon| < 1$) surface on the right.

To convert the reflecting surface seismogram R' to the imperfectly absorbing surface seismogram U' we equate the ratios of up to downgoing waves.

$$\frac{R'}{B+R'} = \frac{U'}{B+\varepsilon U'} \tag{1}$$

Clear fractions and solve for U'

$$(B+\varepsilon U')\,R' = (B+R')\,U'$$

$$B\,R' = [\,B + (1-\varepsilon)R'\,]\,U'$$

$$U' = \frac{B\,R'}{B+(1-\varepsilon)R'}$$

$$U' = \frac{[\,B/(1+\varepsilon)\,]\,R'}{[\,B/(1+\varepsilon)\,]+R'} \tag{2}$$

In equation (2) the quantity ε may be regarded as a Z transform polynomial, as are B , U' and R' . The denominator of (2) isn't necessarily minimum phase, so FFT's rather than time recursions may be necessary to compute (2).

Now suppose that R' is given and an estimate $\hat{B}$ of the shot waveform B is at hand. Then we can set $\varepsilon = 0$ and use (2) to compute U' . If in the resulting U' the multiples have not been completely suppressed, we may presume either (1) $\varepsilon = 0$ but $\hat{B} \neq B$, or (2) $\hat{B} = B$ but $\varepsilon \neq 0$. Thus, we can take the imperfect Noah seismogram, presume we know the source waveform $B = \hat{B}$, and set out to estimate the non-zero free surface reflection coefficient ε .

With this estimate $\hat{\varepsilon}$, we can improve our estimate of the shot waveform B by considering (2) . From any correct R' and U' we can have a multitude of ε_j and B_j such that

$$\frac{B_j}{1+\varepsilon_j} = \frac{\hat{B}}{1+\hat{\varepsilon}} \quad ; \text{ many } j \tag{3}$$

Thus, if we want $\varepsilon_j = 0$, then

$$B_j = \frac{\hat{B}}{1+\hat{\varepsilon}} \tag{4}$$

What we have accomplished so far is to convert the problem of estimating a source waveform on a real seismogram to the problem of estimating the free surface reflection coefficient on an imperfect Noah seismogram on which the source waveform is known. The next step is to consider departures from the vertical incidence plane layer model. With the non-diffracting slant wave stack models (SEP-7, p. 1-7) we can now wonder if shot waveform variation along the survey line is isomorphic to free surface reflectivity variation.