Estimating the Shifting Function

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We have shown examples of migration in which the slowness function $s(x,z)$ in the equation

$$P_{zt} = s(x,z)\, P_{tt} \tag{1}$$

is non-zero for some small range of z (Schultz, SEP 5, p. 66-78). If $s(x,z)$ is zero except for the range z_1 to $z_1 + \Delta z_s$, then we can write the solution to (1)

$$P(x,\, z_1 + \Delta z_s,\, t) = P(x,\, z_1,\, t + \tau_s(x)) \tag{2}$$

where

$$\tau_s(x) = \int_{z_1}^{z_1+\Delta z_s} s(x,z)\, dz \tag{3}$$

Note that although τ_s is a function of both x and z, the thin lens approximation allows us to separate the dependence. We therefore have the shifting function τ_s as a function of x only, but applied to the wave field at some z value which is close to z_1.

Figure 1 illustrates this concept. Note that the migration velocity, $\bar{v}$, is not a constant. We are implicitly migrating using a stratified media slant frame described elsewhere in this report.

From this point in the discussion onward, we will be concerned with the estimation of $\tau_s(x)$ rather than $s(x)$. $\tau_s(x)$ is the time-shifting of the wave field and can be estimated directly from the relative time shifts between traces, while some knowledge of the nature of the functional dependence of s on z is needed to use equation (3) to obtain s from τ_s.

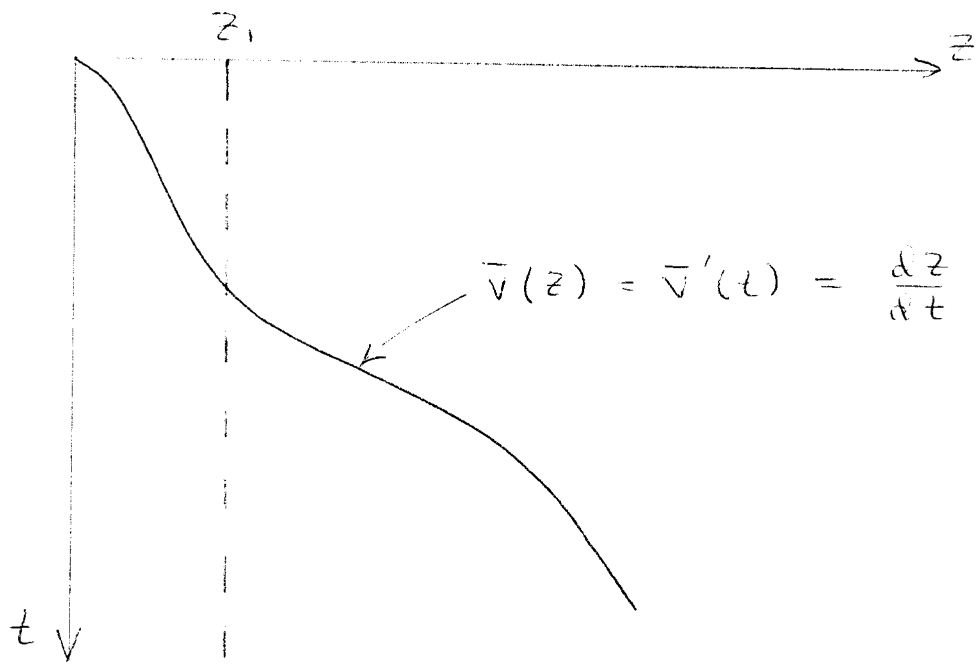

Fig. 1. z,t plane showing migration velocity $\bar{v}(z) = \bar{v}'(t)$ and depth z_1 where shifting $\tau_s(x)$ is to be done.

An interesting correction occurs to the time shift, τ_s , when incidence is at some angle θ from the vertical. If we have a depth range Δz_s over which the slowness is independent of Δz , our time shift will be for vertical incidence,

$$\tau_s(x, \theta=0) = s(x)\, \Delta z_s$$

For an angle $\theta \neq 0$, we can imagine the effective depth range to go to $\Delta z_s \sec\theta$. This gives an angle dependent time shift to be

$$\tau_s(x,\theta) = \tau_s(x, \theta=0) \sec\theta \qquad (4a)$$

or

$$\tau_s(x,p) = \tau_s(x, p=0)\,(1 - p^2\bar{v}^2)^{-1/2} \qquad (4b)$$

where θ and $\bar{v}$ are the local values at $z = z_1$.

As a final reminder that we are using the thin lens approximation, the time shifts that we observe from trace to trace which are caused by transmission effects are, in fact, given by an integration of the slowness over the ray path.

$$\Delta \tau_s(x,z) = \int_{\text{path to }(x,z)} s(x,z)\, dr \qquad (5)$$

We are first making use of the thin lens approximation which allows us to integrate over a straight line ray path some angle θ from the vertical (instead of a curved ray path); and second, we are specifying that the slowness function, s , (and therefore also the time shift τ_s) be non-zero only over some range z_1 to $z_1 + \Delta z_s$. The particular model we are then restricting ourselves to is shown in figure 2.

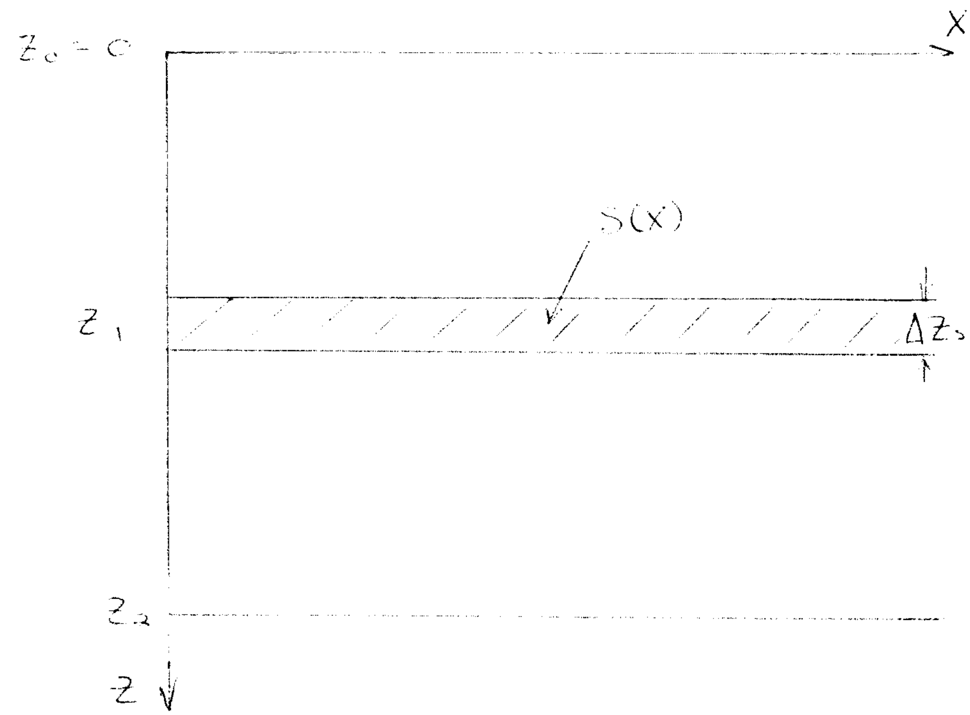

Fig. 2. The earth model we are considering. All the shifting takes place at z_1 and over some (unknown) range Δz_s .

At some depth around z_2 are coherent reflecting horizons which will show the effects of the transmission distortions at z_1 .

There are (among others) two approaches we may now use to estimate τ_s . The first involves correlating the traces with some window around a reflector at z_2 . The depth z_2 must be close enough to depth z_1 such that we may claim commutivity between the diffraction and the shifting. We will then have three contributions to the total time shift for each trace (the downgoing path at z_1 , the upcoming path at z_1 , and the reflector topography at z_2) which we will try to resolve by the redundancy in the data (stacks over more than one p value). The second involves correlating the traces with a window over reflectors at all depths greater than z_1 , in an attempt to average out the shifts due to the downgoing wave path and the reflector topography. We will investigate these separately.

Let us consider the former option with a correlation window about a reflector at some z_2 near z_1 . Do we need to downward continue the data? Figures 3 to 5 show why this is essential in order to minimize error. Figure 3 shows the region through which energy will pass on its way to the single geophone at the surface, assuming the gathers have been slant stacked over some θ and that reflector dip and irregularities perturb the wave path by no more than 15°. No distortion of the wave paths propagating through the region at depth z_1 is due to the commutivity assumption.

Figure 4 shows that this region of energy acceptance for the geophone is significantly narrower after downward continuation of the slant stacked data to depth z_1 . Notice, in particular, the increased resolution in sensing the shifting function at z_1 , with excellent resolution for the upcoming path and improved resolution for the downgoing path. If downward continuation proceeds to z_2 (ignoring the shifting that

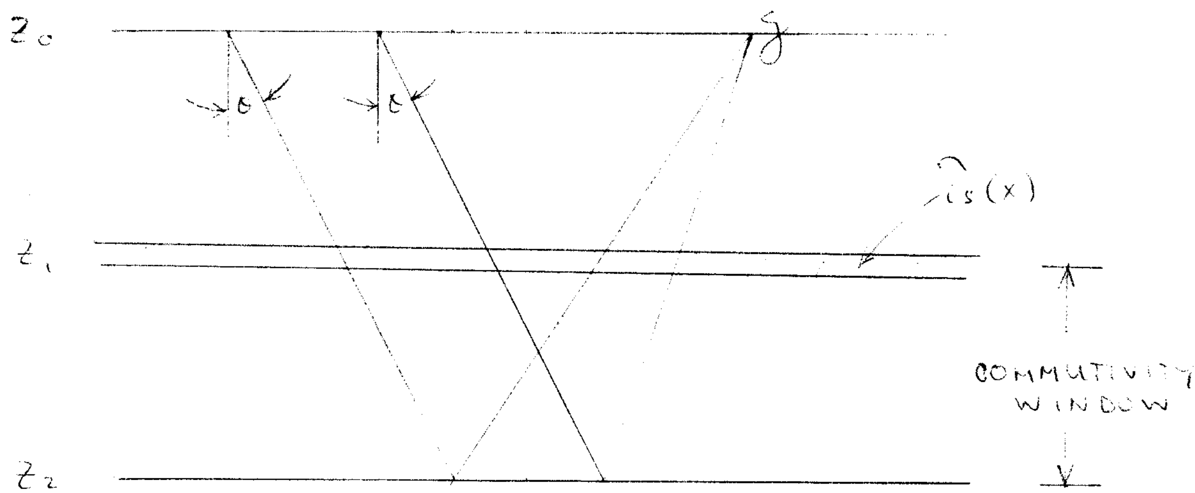

FIG. 3

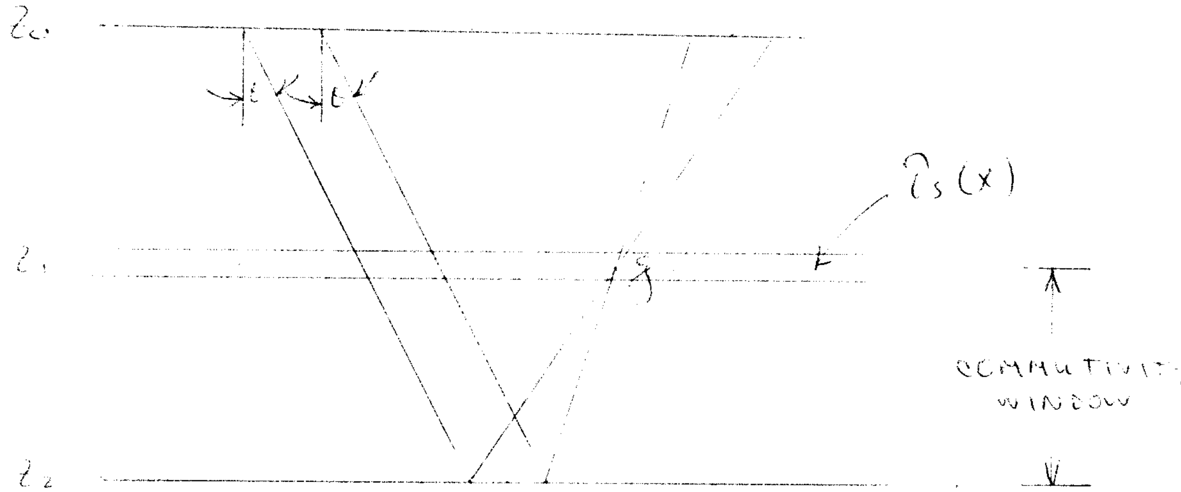

FIG. 4

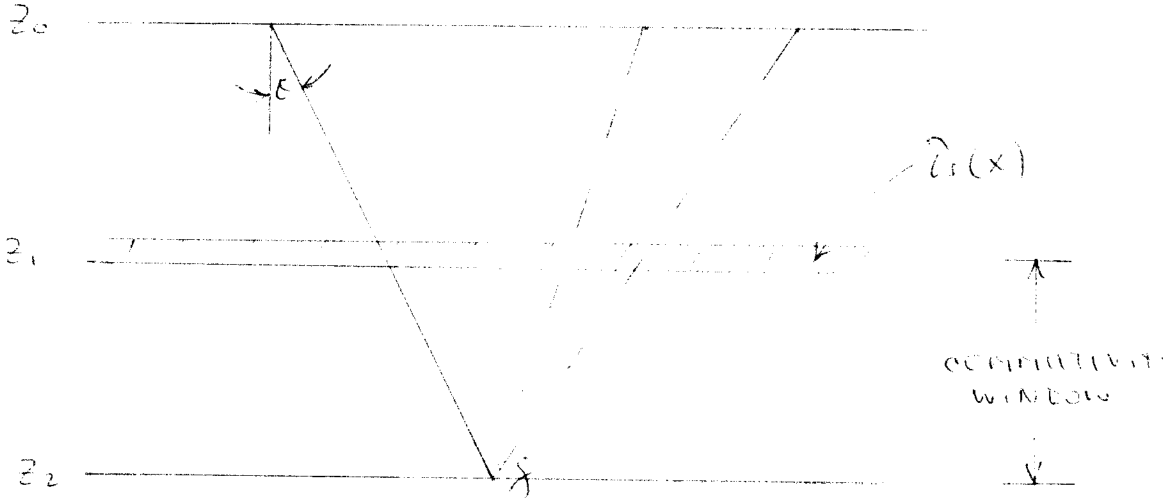

FIG. 5

Fig. 3. Beam of acceptable energy for slant stacked sections of angle θ . The downgoing wave maintains its plane wave character until reflection. Diffracting effects of $\tau_s(x)$ at z_1 are not shown because of the commutivity assumption.

Fig. 4. Beam of acceptable energy for data downward continued to z_1 . Note the improved resolution of the shifting function $\tau_s(x)$.

Fig. 5. Beam of acceptable energy for data downward continued to z_2 without shifting at z_1 . Dashed lines show resolution loss of $\tau_s(x)$ because of overmigration. The total resolution of $\tau_s(x)$ for both wave paths is the same as in Fig. 4.

should be done at z_1 -- our commutivity assumption allows this), the resolution in sensing the shifting function is not increased (however, maximum sensitivity now lies with the downgoing path), but we have the advantage of being not farther than $(z_2 - z_1)$ away from either of the two transmission shifting operations.

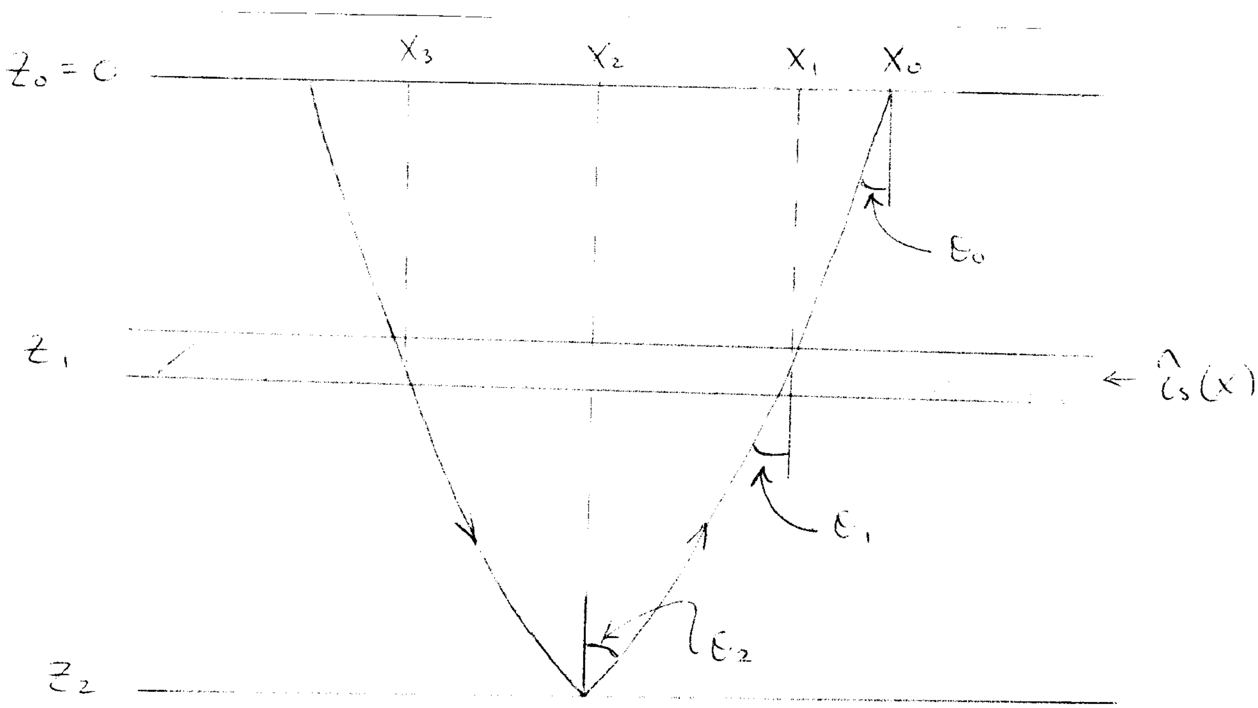

Fig. 6. The schematic wave paths calculable from the migration velocity, $\bar{v}(z)$, and for a particular p value.

Figure 6 shows a possible ray path that we can calculate from the migration velocity, $\bar{v}(z)$. The surface geophone coordinate is x_0, and x_1, x_2, x_3 are the horizontal coordinates (unprimed frame; i.e., normal x and z coordinates). Notice that x_1, x_2 and x_3 can be calculated from $\bar{v}(z)$, x_0, and p with the relations

$$x_1 = x_0 - \int_0^{z_1} \tan\theta(z)\, dz$$

$$= x_0 - \int_0^{z_1} \frac{p\, \bar{v}(z)\, dz}{(1-p^2\bar{v}^2(z))^{1/2}} \qquad (6)$$

$$x_2 = x_0 - \int_0^{z_2} \tan\theta(z)\, dz$$

$$= x_0 - \int_0^{z_2} \frac{p\, \bar{v}(z)\, dz}{(1-p^2\bar{v}^2(z))^{1/2}} \qquad (7)$$

$$x_3 = x_0 - \int_0^{z_2} \tan\theta(z)\, dz - \int_{z_1}^{z_2} \tan\theta(z)\, dz$$

$$= x_0 - \int_0^{z_2} \frac{p\, \bar{v}(z)\, dz}{(1-p^2\bar{v}^2(z))^{1/2}} - \int_{z_1}^{z_2} \frac{p\, \bar{v}(z)\, dz}{(1-p^2\bar{v}^2(z))^{1/2}} \qquad (8)$$

Although the above calculations assume a non-diffracted ray path, we are minimizing diffraction effects by downward continuation to z_2 .

Let us now assign a time shifting as a function of the x coordinate to the reflector at z_2 in order to include reflector topography. The total time shift, T , for the trace at x_0 as a function of the stacking parameter, p , and referenced to some standard trace, is

$$T(x_0, p) = \tau_s(x_1)\sec\theta_1 + \tau_s(x_3)\sec\theta_1 + \tau_r(x_2)\sec\theta_2 \qquad (9)$$

where θ_1 and θ_2 are local to z_1 and z_2 respectively. Or,

$$\begin{aligned} T(x_0, p) = {} & \tau_s(x_1)\,(1-p^2\bar{v}^2(z_1))^{-1/2} \\ & + \tau_s(x_3)\,(1-p^2\bar{v}^2(z_1))^{-1/2} \\ & + \tau_r(x_2)\,(1-p^2\bar{v}^2(z_2))^{-1/2} \end{aligned} \qquad (10)$$

Let us now represent the positions x_1 , x_2 and x_3 by the indices i , k and j respectively. We have

$$\tau_s(x_1) \to S_i \tag{11a}$$

$$\tau_s(x_3) \to S_j \tag{11b}$$

$$\tau_r(x_2) \to R_k \tag{11c}$$

$$T(x_0,p) \to T_{ijk} \tag{11d}$$

and we note that $k = (i+j)/2$.

As a simplification to the following analysis, let $\bar{v}(z)$ vary slowly from z_1 to z_2 so that

$$(1 - p^2\bar{v}^2(z_1))^{-1/2} \cong (1 - p^2\bar{v}^2(z_2))^{-1/2}$$

We now have the set of equations

$$T_{ijk} = S_i + S_j + R_k \tag{12}$$

and we are interested in the homogeneous solutions

$$T_{ijk} = 0 \tag{13}$$

to determine any intrinsic ambiguities.

Using a technique common in statics analysis (see, for example, Taner et al, Geophysics, v. 39, #4, p. 441-463, 1974), we represent the variables by a power series. We define

$$S_i = a_0 + a_1 i + a_2 i^2 + \ldots \tag{14a}$$

$$S_j = b_0 + b_1 j + b_2 j^2 + \ldots \tag{14b}$$

$$\begin{aligned} R_k &= c_0 + 2c_1 k + 4c_2 k^2 + \ldots \\ &= c_0 + c_1(i+j) + c_2(i+j)^2 + \ldots \end{aligned} \tag{14c}$$

We adjust the coefficients of (14) so that it will be a solution to (13) by identifying coefficients of $i^n j^m$.

$$a_0 + b_0 + c_0 = 0 \quad (n=0, m=0) \tag{15a}$$

$$a_1 + c_1 = 0 \quad (n=1, m=0) \tag{15b}$$

$$a_1 + b_1 = 0 \quad (n=0, m=1) \tag{15c}$$

$$a_2 + c_2 = 0 \quad (n=2, m=0) \tag{15d}$$

$$b_2 + c_2 = 0 \quad (n=0, m=2) \tag{15e}$$

$$c_2 = 0 \quad (n=1, m=1) \tag{15f}$$

and

$$a = b = c = 0 \quad \text{for} \quad n + m \geq 2 \tag{15g}$$

We have

$$a_0 = b_0$$

from self-consistent geometry, and

$$c_0 = - (a_0 + b_0) = - 2 a_0 \tag{16a}$$

$$b_1 = a_1 \tag{16b}$$

$$c_1 = - a_1 \tag{16c}$$

which gives

$$S_i = a_0 + a_1 i \tag{17a}$$

$$S_j = a_0 + a_1 j \tag{17b}$$

$$R_k = - 2 a_0 - a_1 (i+j) \tag{17c}$$

Therefore, the determination of time shifts τ_s and τ_r will be uncertain up to a linear term, and calculation of time shifts of adjacent traces will show an insensitivity to long wavelength variations with x of the shifting functions, τ .

Let us now turn our attention to a second option for estimating these time shifts. In this case the correlation window will be the entire trace below z_1 and an attempt will be made to average out all the transmission effects with the exception of that due to the geophone (i.e., transmission distortions on the upcoming wave path). In this way our time shift measurements will reflect the geophone "static" and no other, so that a formal solution to equation (9) will not be necessary. The attempt is to average in such a way that the $\tau_s(x_3)$ and $\tau_r(x_2)$ terms in (9) will be zero.

Since we are enhancing the effect of the geophone static, the logical depth for downward continuation in this case is z_1 . Figure 4 shows that resolution of the upcoming wave shifting function will be optional at this depth. In addition, the lesser resolution at the downgoing wave shifting and at the reflector will serve to enhance any attempt made to average these effects to zero.

In addition to a large time window to average the downgoing wave static, several p values will be available (ranging from $p_{max} \Rightarrow \theta_{max}$ to $p_{min} \Rightarrow \theta_{min}$) in an attempt to average the effects of reflector topography. Figure 7 shows ray paths, with common receiver positions at z_1 , for two p values with an implied set of intermediate p values. The darkened lines show the regions over which averaging will take place.

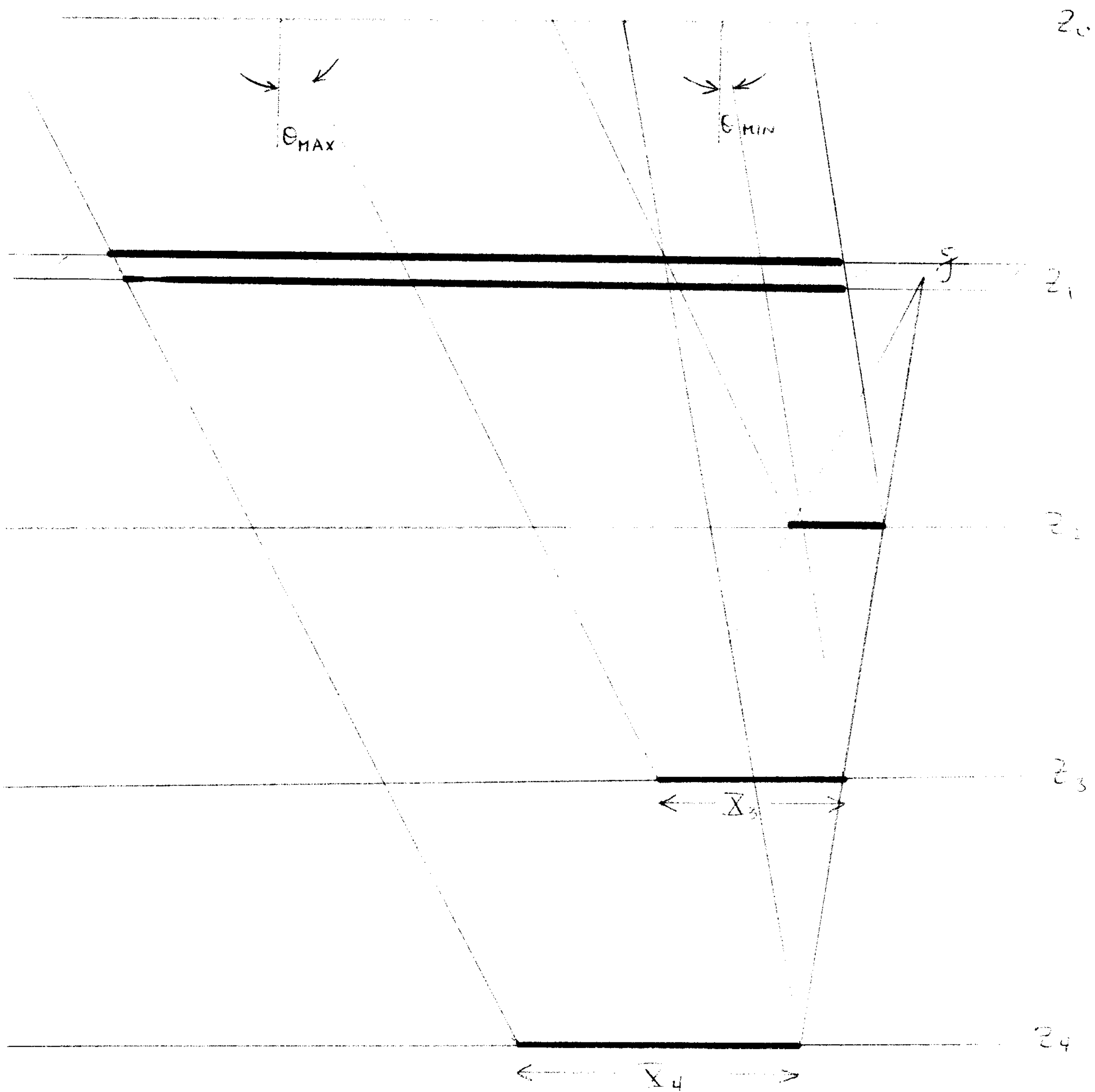

Fig. 7. Ray paths for common geophone traces in the two extreme slant sections. We assume a continuum of p values between p_{max} and p_{min} and a continuum of reflectors between z_2 and z_4. The darkened lines show the regions over which averaging will be done. Note that the data has been downward continued to z_1 to increase resolution of $\tau_s(x)$ for the upcoming wave.

Let us quickly estimate the effectiveness of this averaging by assuming a continuum of p values between p_{max} and p_{min}. The zone of averaging is different for each reflector and the transmitter at z_1, see Figure 7, but let us turn our attention to any one of those regions of horizontal dimension X. When two adjacent traces in a given p section are correlated, the result will be sensitive to the slope of the topography. Assuming a single Fourier component for topography,

$$\text{topo} = A \sin \frac{2\pi x}{\lambda_x}$$

where λ_x is the horizontal topographic wavelength. Then,

$$\text{slope} = \frac{2\pi A}{\lambda_x} \cos \frac{2\pi x}{\lambda_x}$$

Now,

$$\text{aver slope} = \frac{\int_0^X \frac{2\pi A}{\lambda_x} \cos \frac{2\pi x}{\lambda_x} \, dx}{\int_0^X dx}$$

$$= \frac{A}{X} \sin \frac{2\pi X}{\lambda_x}$$

but since

$$\text{max slope} = \frac{2\pi A}{\lambda_x}$$

we have

$$\frac{\text{aver slope}}{\text{max slope}} = \frac{\lambda_x}{2\pi X} \sin \frac{2\pi X}{\lambda_x}$$

$$\cong \frac{\lambda_x}{2\, X \sqrt{2}} \cong .1 \, \frac{\lambda_x}{X} \qquad (18)$$

Equation 18, as we expected, shows that the averaging is most effective when the region of averaging, X , is large compared to the wavelength, λ_x , of the topography. This result again shows a difficulty in dispatching long wavelength fluctuations in dip, and just as in the previous estimation approach (equations 17), they will have to be separated from the shifting estimate by other means.

Neither of these two methods have yet been tried on field data, but my suspicion is that the second method using a larger correlation window will be more practically feasible. I expect the larger window will aid in noise reduction, and that the averaging will eliminate the dependence of the estimate accuracy on the accuracy of $\bar{v}(z)\big|_{\text{all } z} = \tilde{v}(z)\big|_{\text{all } z}$. The second method requires only that $\bar{v}(z)\big|_{z<z_1} \cong \tilde{v}(z)\big|_{z<z_1}$.