II. SINGLE CHANNEL MAXIMUM ENTROPY SPECTRAL ANALYSIS FROM AUTOCORRELATION MEASUREMENTS

The most important application of maximum entropy spectral analysis (MESA) has been in estimating a power spectrum from partial knowledge of its autocorrelation function, $R(n)$. In this situation, the measurement equations are

$$R(n) = \int_{-W}^{W} P(f) \; e^{i2\pi fn\Delta t} \, df \, , \quad (-N \leq n \leq N) \, .$$

Here, Δt is the sampling period and $W = 1/(2\Delta t)$ is the Nyquist foldover frequency. We note that $R(n) = R^*(-n)$, so that n could be restricted to range from 0 to N. However, it is mathematically more convenient to assume the $\pm N$ range for n.

As indicated in section I, maximum entropy spectral analysis is not limited to autocorrelation information for the estimation of power spectra. However, the past development of MESA theory has been based on autocorrelation measurements for two main reasons. First, spectral estimation from the autocorrelation function has been the classical approach since Wiener[5] discovered (or defined) the fourier transform relation between the power spectrum and the autocorrelation function. Direct implementation of Wiener's work calls for estimation of the autocorrelation function first, followed by fourier transformation to get the power spectrum. These autocorrelation values can be easily estimated from digital time series data by calculating average lag products. Thus, from both an historical and practical point of view, it was natural for second order statistics of a stationary time series to be presented in terms of the autocorrelation function for maximum entropy spectral analysis.

Second, it is computationally very easy to solve the maximum entropy variational problem in terms of the autocorrelation measurements. Much of the practical success of MESA is due to the highly fortunate fact that, starting with autocorrelation lag values, the computation of maximum entropy spectra is not appreciably more complex than the computation of conventional spectral estimates. In fact, the depth and breadth of this subject are the result of the analytical elegance given to the maximum entropy theory by the autocorrelation measurements. When general measurement constraint equations are considered, the variational approach is far more difficult and not so well understood.