X-Outer Migration Example

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In SEP Report 2, page 109, we developed an explicit migration calculation technique, and on page 127 of the same report, we implemented the x-outer method for memory storage allocation during migration. The theory of the method was completely explained, but the results were not completed in time for that report. Now we will show the one section which was migrated and review the practical considerations of choice of gridding parameters. Figure 1 shows the CDP section on the bottom and the migrated version on the top. This section was supplied to us by Digicon, Inc. With left-right reversal it currently appears in their Geophysics advertisement. One of the disadvantages of the x-outer method is the big square of NT by NT zeros which have to be appended on both left and right sides of the section. In Figure 1 none of the left hand square is shown and only a part of the right hand square is shown. It will be noted that after migration this square becomes filled with quasi-semi-circular arcs due to the abrupt truncation of the section. Studying these arcs, especially near $t = 1$ sec reveals several aspects of wave equation migration. First, we usually make the Fresnel Approximation for 15° dips rather than going to 45° dips. Thus, a semicircle is approximated by some other figure which fits well only at the bottom. The fit becomes increasingly worse at steeper propagation angles. Of course, it is hard to recognize a semicircular arc at a vertical exaggeration of about four, but near $t=1$ on the right hand edge the departure from semicircles is quite apparent. Because our main objective was to test the x-outer algorithm we did not bother with

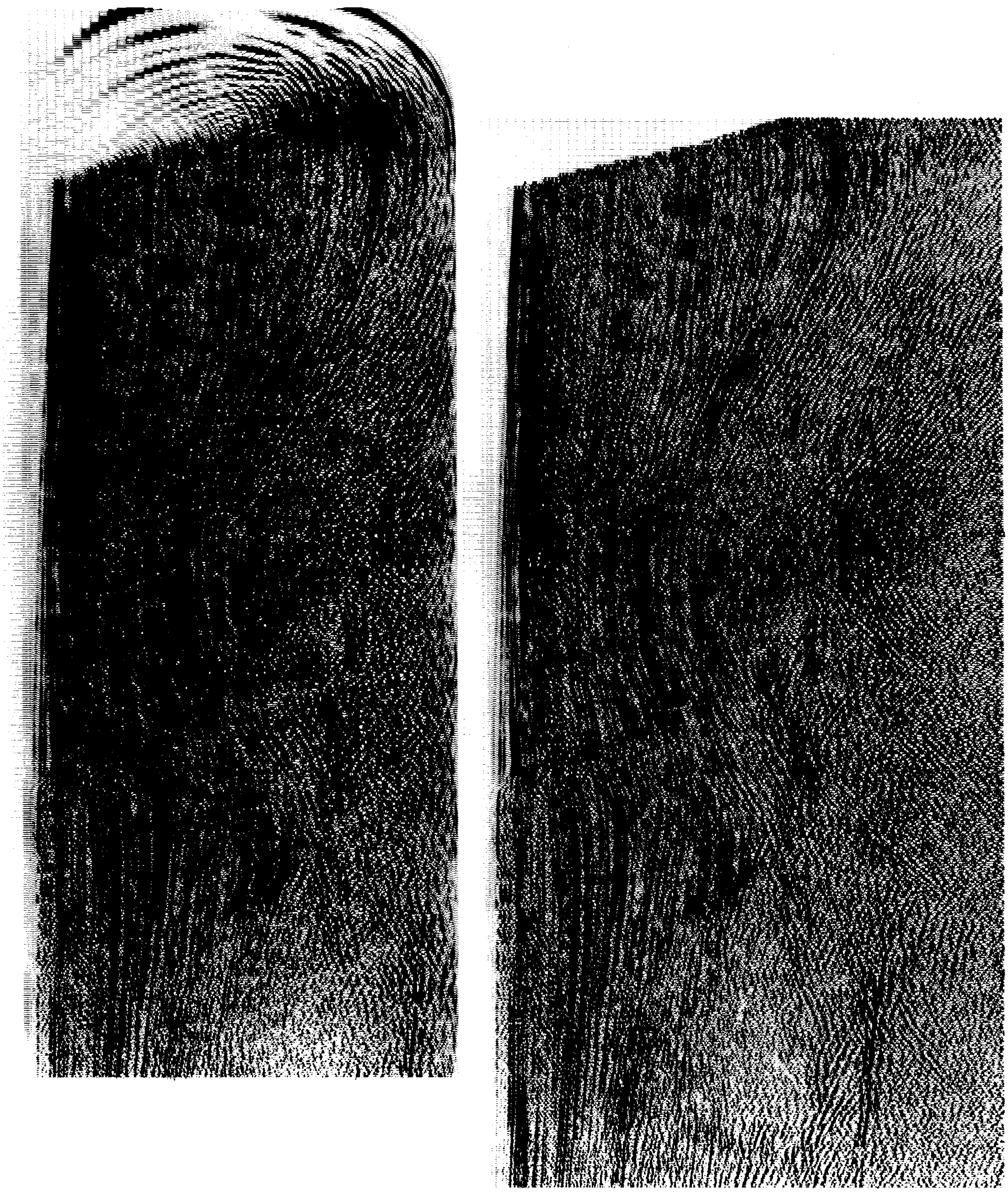

Fig. 1. CDP section (right) -- migrated (left).

the detailed velocity information with which we had been supplied. We migrated the section at a constant uniform velocity of 6000 ft/sec. Hence, the theoretical result should be seimcircles.

Two reasons for departure from semicircularity are: first, an approximation in the differential equations, the Fresnel-like approximation (documented by Schultz, SEP, vol. 1, p. 41); and second, departure of difference equations from differential equations at short wavelengths. This latter effect is quite pronounced around 1 sec on the right hand edge where the quasi-semi-circular waveform breaks up into 100 millisec segments. Clearly the size of the z-axis mesh corresponds to 100 millisec of travel time. Now let us go into the considerations which led up to this choice. Refer to SEP, Report 2, page 128. We see the equation

$$\frac{NT}{NZ} = \frac{\Delta t''}{\Delta t'} = 32\ a\left(\frac{\Delta y}{v\,\Delta t'}\right)^2 \qquad (1)$$

where

$\Delta t'$ = data time sample interval = 4 ms

$\Delta t''$ = depth interval measured in units of two-way time at 6000 ft/sec = 100 ms

NT = number of points on time axis = 1250

NZ = number of points on depth axis = 50

v = 6000 ft/sec

Δy = sampling on midpoint axis = 150 ft

a = a magic number, see text

Choice of the dimensionless number a is influenced by several considerations. First, for stability it must be less than $1/4$. Next, if it is chosen equal to $1/12$ we will have 4th order accuracy in the approximation of the differential operator $\partial^2/\partial y^2$ by the difference operator δ_{yy} . From the table on page 296 of SEP Report 2, we see that it is important to have $a \approx 1/12$ if the field data we will be processing has fewer than 8 points per wavelength on the y-axis. Inspection of Figure 1 shows that this is indeed the case. However, in equation (1) the parameters Δy and $\Delta t'$ were fixed by the recording arrangement and v was chosen by the earth. Thus, if we are to adjust the dimensionless number a for best lateral accuracy we will be simultaneously adjusting the step size $\Delta t''$ on the vertical axis. Indeed for our first trial a was chosen equal $1/12$ which by (1) implies that

$$\Delta t'' = 32 \ a \ \Delta t' \left(\frac{\Delta y}{v \Delta t'} \right)^2 = .41 \text{ sec} = 410 \text{ ms}$$

$$NZ = NT \ \Delta t'/\Delta t'' = 12 \quad .$$

It was nice to contemplate an accurate migration in only 12 steps along the depth axis, but the result was obviously not fully migrated. The reason was that the data sampling along the depth axis was too coarse. Thus, we had to drop the idea of fourth order accuracy along the horizontal axis in order to get sufficiently dense sampling on the depth axis. What was actually done for Figure 1 was to choose $\Delta t''$ about four times smaller, namely 100 ms .

We are still unsatisfied with the result. However, the project was ended because we had established the validity of the x-outer technique.

If our goal was a high quality migration what could we have done?

(1) We could have reverted to our previous implicit technique which is 2-1/2 times slower, but the choice of $\Delta t''$ is not coupled to the fourth order accuracy on the horizontal axis.

(2) We could have resampled the data about twice as densely along the midpoint axis y.

The cost of either of the above two options is about the same. Because of the nature of the x-outer algorithm the second option poses no additional memory requirement. I haven't done a detailed comparison, but I believe greater accuracy would result from the second option.