Transmission-Compensated Migration of Vertically-Stacked (Plane-Wave) Sections

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As the previous article has indicated, one might reasonably expect transmission diffractions from a deep sea floor with severe topography. This may impair meaningful interpretation of the data. This article will describe a scheme for the migration of this type of data in such a way that recovery of the original geometry of the reflectors is possible. We will treat only plane waves here; we hope in the near future to be able to generalize this scheme to NMO corrected gathers so that the analysis can be done before stacking.

Propagation through Regions of Inhomogeneity

Any migration scheme to compensate for transmission diffractions using the wave equation, must necessarily be concerned with wave propagation through localized regions of inhomogeneity. The following development is contained in more detail in "2-D Inhomogeneous Media Wave Calculations" by Jon F. Claerbout in the March 1974 SEP report.

Let us begin with the two-dimensional wave equation,

$$P_{xx} + P_{zz} = v(x,z)^{-2} P_{tt} \tag{1}$$

Recognizing that we are here concerned with only the plane wave problem, our transformations are familiar ones. We have, for example, for the downgoing wave transformation,

$$\begin{aligned} x' &= x \\ z' &= z \\ t' &= t - z/\bar{v} \end{aligned} \tag{2}$$

Chain rule differentiation allows us to rewrite equation (1) as

$$P_{x'x'} + P_{z'z'} - (2/\bar{v})P_{z't'} + (\bar{v}^{-2} - v(x,z)^{-2})P_{t't'} = 0 \qquad (3)$$

Defining a slowness $s(x,z) = (\bar{v}^{-2} - v(x,z)^{-2})\bar{v}/2$ and dropping primes we have

$$P_{xx} + P_{zz} - (2/\bar{v})(P_{zt} - s(x,z)P_{tt}) = 0 \qquad (4)$$

As before we drop the P_{zz} term to obtain

$$P_{zt} = s(x,z)P_{tt} + (\bar{v}/2)\,P_{xx} \qquad (5)$$

Here we make the assumption that the increment Δz is taken small enough so that we may separate (5) into two equations. Namely,

$$P_z = s(x,z)\,P_t \qquad (6a)$$

$$P_{zt} = (\bar{v}/2)\,P_{xx} \qquad (6b)$$

where (6a) has been integrated once with respect to time. Equation (6b) is the familiar form of the wave equation used in propagation through regions of homogeneity. Equation (6a) involves the term which allows us to propagate through inhomogeneous regions and is essentially a time-shifting of the wave field, P .

Separating equations (6a) and (6b) as we have done assumes that we may treat their respective operations alternately. This is physically equivalent to the thin lens approximation, where shifting of the waveform is done between propagation steps.

We like to make decisions on the size of Δz from sampling considerations, and this criterion will still apply when equation (6a) is used alternately. In other words, we shall presume that if

Δz is small enough so that we are sampled adequately in the z-dimension both before and after shifting, the separation of equations (6a) and (6b) will be valid.

Turning our attention to equation (6a), we are aware that its essential feature is time-shifting. The amount of shifting is given by the slowness factor $s(x,z)$. Since $s(x,z)$ is a function of only x and z , for a given depth in our downward continuation (z constant) and for a single trace (x constant), the shift implied by (6a) will be uniform over the entire trace. Indeed, over many traces we find that it is a function of only the x-coordinate.

There is really no advantage in doing this time-shifting by means of a differential equation (i.e., equation (6a)). If we know the velocity structure through which the waveform is propagating, we can then do this time-shifting by any means we choose, but hopefully by one which is more economical than a differential equation. (We have been using a linear shifting routine, SHIFT4 , described in another section of this report.)

The Sea Floor as a Region of Inhomogeneity

The specific localized inhomogeneity to which we shall turn our attention is the sea floor. As implied in the previous paragraph, the velocity structure of the inhomogeneous region must be known before we can make an attempt at a deterministic propagation through the region. Usually, a good guess of time-shifting at the sea floor can be made with some knowledge of the sea floor topography and the velocity contrast. As will be shown later, an exact knowledge of the shifting is not necessary. The data enhancement after processing will, however, be better for a more accurate estimate of the shift.

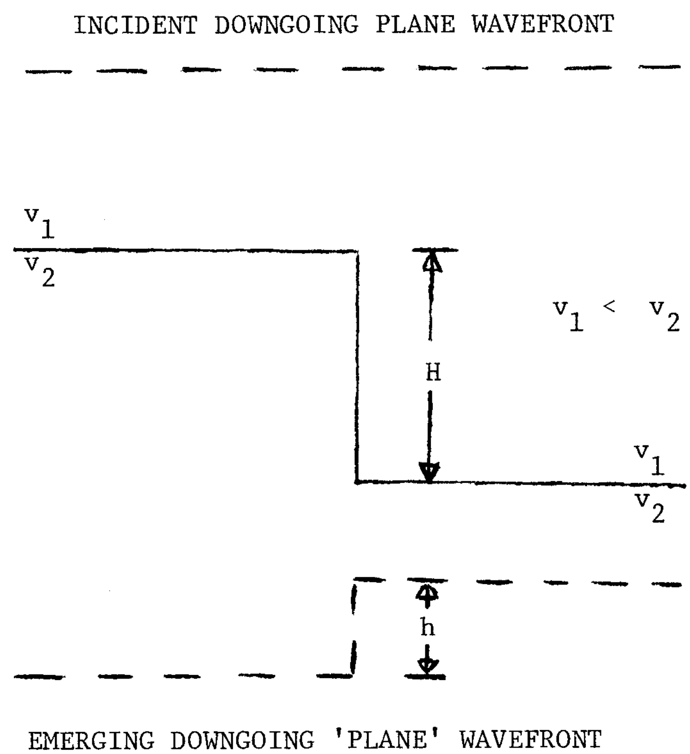

Figure 1.

Figure 1 shows a cross-section of a velocity interface whose two-dimensional topography is a step. A downgoing vertical plane wavefront is distorted in proportion to the topography. The relief, h , in the wavefront is simply related to a time-shifting, τ , by

$$h = v_2 \tau \tag{7}$$

This time-shifting can, in turn, be related to the velocities and topography in Figure 1 by

$$\tau = H \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \qquad (8)$$

The generalization of (8) to an arbitrary two-dimensional topography, $H(x)$, is simply,

$$\tau = H(x) \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \qquad (9)$$

Let us calculate the sensitivity of the time-shift, τ , to errors in topography and velocity estimates. We shall assume that we have perfect knowledge of v_1 , the water velocity.

$$d\tau = \left(\frac{1}{v_1} - \frac{1}{v_2} \right) dH + \frac{H}{v_2^2} \, dv_2$$

and, dividing through by (8),

$$\frac{d\tau}{\tau} = \frac{dH}{H} + \left(\frac{v_1}{v_2 - v_1} \right) \frac{dv_2}{v_2} \qquad (10)$$

So, the fractional error in time-shift is equal to the fractional error in topography estimate plus the fractional error in velocity estimate times a velocity dependent factor greater than unity for v_2 less than twice water velocity. It appears that the time shift estimate is more sensitive to velocity errors than to errors in the topography estimate by a factor of $\left(\frac{v_1}{v_2 - v_1} \right)$. Note that this factor reduces in magnitude as v_2 increases, a desirable situation since we expect transmission effects to be strong when the velocity contrast is high. Figure 2 shows this factor plotted against v_2 .

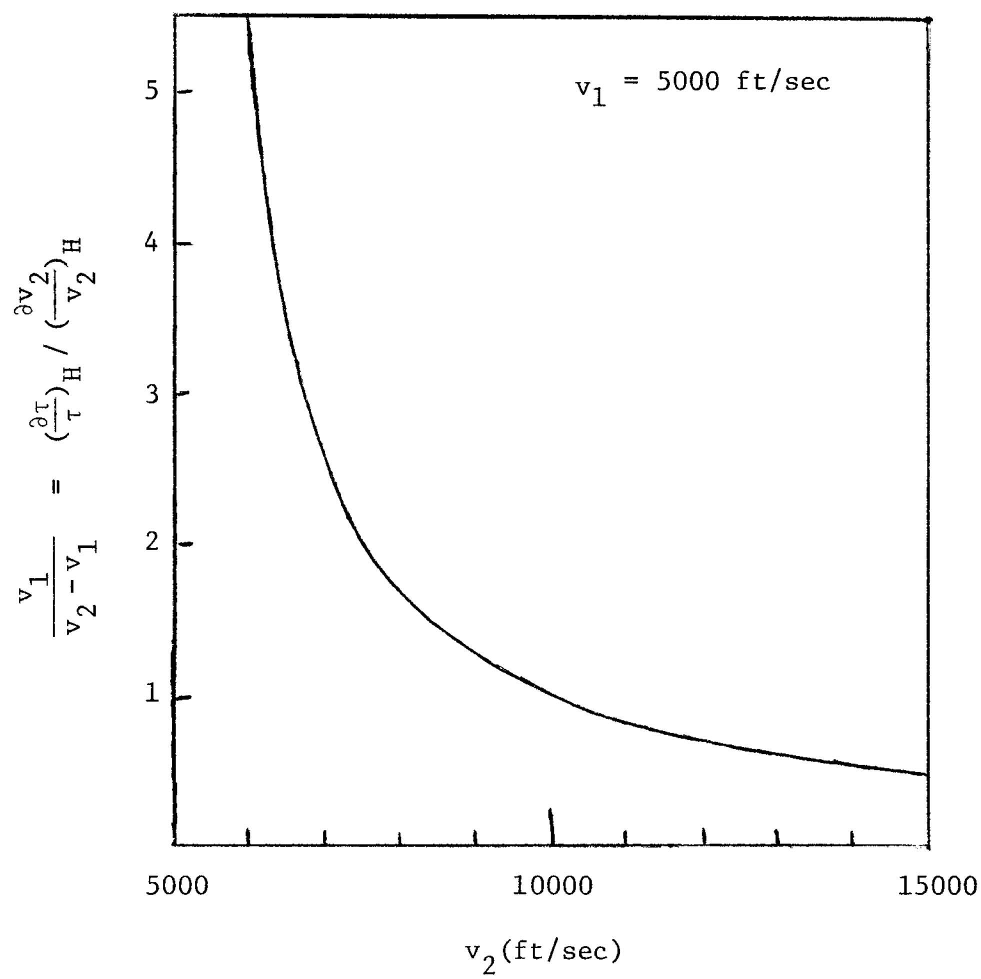

Figure 2.

The factor, $v_1/(v_2-v_1)$, multiplying dv_2/v_2 in equation (10).

This factor shows the relative strength of the contribution to the total fractional error in time shift estimate (i.e., $d\tau/\tau$) by fractional errors in v_2 estimate, as compared to that contributed by fractional errors in topography estimate, whose factor is always unity. Notice the factor becomes unity at $v_2 = 2\,v_1$.

Migrating Plane-Wave Seismic Sections

In this section we will treat the problem of migrating a vertically-stacked section (obtained by summing over the shot coordinate in common receiver gathers without prior normal moveout correction) with a localized-in-depth strong velocity inhomogeneity causing transmission diffractions. Since the operations involved in this section are more easily developed considering time-sections rather than depth-sections, we will define all wave fields and reflection coefficients in the (x,t) domain, rather than (x,z). Recall that since we are considering here only plane waves, the horizontal coordinate, x, is defined unambiguously as a position in the horizontal dimension.

The figure below shows the geometry and some definitions for the wave fields. Ray paths are shown with a horizontal component merely to separate them in the figure. Ray paths are vertical (actually, we allow a beam of $\pm$ 15 degrees from the vertical).

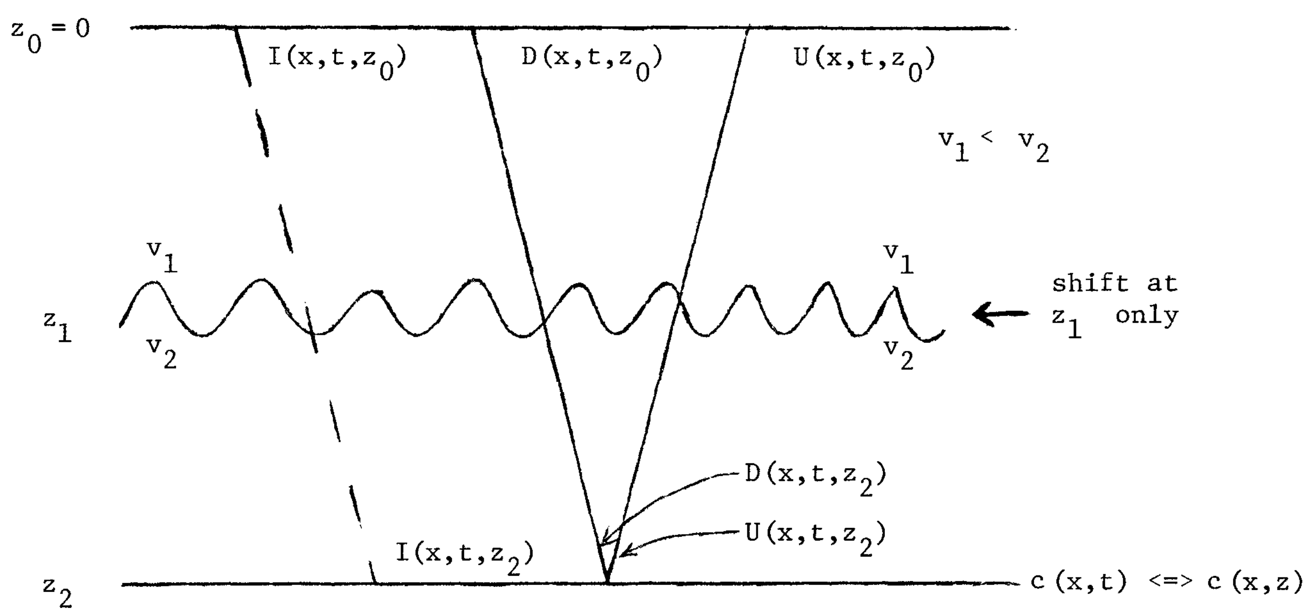

Figure 3

The figure shows only one reflector for simplicity, and we will restrict ourselves here to only one, although generalization to many will not be difficult and will be discussed later. We consider the field of reflection coefficients to be $c(x,t)$, where t refers to the two-way vertical travel time to the reflector.

$U(x,t,z_0)$ refers to the upcoming wave field at the surface (i.e., $z=z_0$) . The third coordinate, z , of the wave field allows us to keep a physical interpretation on the downward continuation process. For example, if we take the surface upcoming wave field, $U(x,t,z_0)$, that is, the data at the surface, and downward continue that data one Δz step, we would then have in our notation, $U(x,t,z_0+\Delta z)$.

Now $D(x,t,z_0)$ refers to the original downgoing plane-wave field, again as would have been recorded at the surface, $z=z_0$. The wave field $D(x,t,z_0)$ includes a downgoing plane delta function wavefront with a source waveform, $w(t)$, convolved on it in the t-dimension. The third wave field, $I(x,t,z_0)$, is defined to be precisely that downgoing plane delta function wave field. So,

$$D(x,t,z_0) \;=\; I(x,t,z_0) *_t w(t) \qquad (11a)$$

$$D(x,t,z_2) \;=\; I(x,t,z_2) *_t w(t) \qquad (11b)$$

where the subscript, t , on the convolution symbol refers to the fact that the operation is single-channel, and in the time-domain.

The argument, t , in the wave fields is not simply time, but rather the transformed time coordinate that we commonly use in our plane-wave downward continuation schemes. It is given by

$$t \;=\; t'' \pm z/\bar{v} \qquad (12)$$

where t" is clock time, the plus referring to an upcoming wave frame, and the minus to a downgoing frame. The unprimed time coordinate, t , then, refers either to $t'' - z/\bar{v}$ in D(x,t,z) and I(x,t,z) , or to $t'' + z/\bar{v}$ in U(x,t,z) . Notice that the two transformed time coordinates are equal at $z=z_0=0$, and at the surface we can attach the normal meaning to the time coordinate. While c(x,t) is described in these transformed coordinates, no confusion should result, since when viewed at the surface, c(x,t) is our migrated and deconvolved time-section.

The reflector in Figure 3 is at z_2 , and at z_1 there is a region, presumed to be the sea floor, where transmission will involve distortion to such an extent that the wave field will be subsequently diffracted. In the previous article some real parameters were calculated showing the situations where transmission distortions and subsequent diffractions might be important.

We will here define a notation for forward and backward propagation of wave fields between initial and final depths z_i and z_j . Recall that this propagation will include our usual downward continuation (equation (6b)) plus a shifting (equation (6a)) <u>at the appropriate z-step</u> (here, $z=z_1$). There will be two unique operations involved, forward and backward propagation. Forward will include propagation in the normal direction (i.e., down for D and up for U) plus shifting at z_1 ; backward will include propagation in the reverse direction (i.e., up for D and down for U) plus "unshifting" at z_1 . We will define operator notation so that M_+ will signify the forward propagation, while M_- will signify the backward propagation. The numerical subscript will be i in the initial depth, z_i , the superscript will be j in z_j, while the midscript will be the index of the depth where shifting occurs (if any). For example,

$$U(x,t,z_2) = \underset{0}{\overset{2}{1}} M_- \, U(x,t,z_0) \tag{13a}$$

$$U(x,t,z_0) = \underset{2}{\overset{0}{1}} M_+ \, U(x,t,z_2) \tag{13b}$$

$$D(x,t,z_2) = \underset{0}{\overset{2}{1}} M_+ \, D(x,t,z_0) \tag{13c}$$

$$D(x,t,z_0) = \underset{2}{\overset{0}{1}} M_- \, D(x,t,z_2) \tag{13d}$$

$$I(x,t,z_2) = \underset{0}{\overset{2}{1}} M_+ \, I(x,t,z_0) \tag{13e}$$

$$I(x,t,z_0) = \underset{2}{\overset{0}{1}} M_- \, I(x,t,z_2) \tag{13f}$$

From equations (13) it is obvious that

$$\underset{0}{\overset{2}{1}} M_- \underset{2}{\overset{0}{1}} M_+ = \underset{2}{\overset{0}{1}} M_+ \underset{0}{\overset{2}{1}} M_- = 1 \tag{14a}$$

$$\underset{0}{\overset{2}{1}} M_+ \underset{2}{\overset{0}{1}} M_- = \underset{2}{\overset{0}{1}} M_- \underset{0}{\overset{2}{1}} M_+ = 1 \tag{14b}$$

Note that equations (14) specify no relation between $\underset{0}{\overset{2}{1}} M_-$ and $\underset{0}{\overset{2}{1}} M_+$ or between $\underset{2}{\overset{0}{1}} M_-$ and $\underset{2}{\overset{0}{1}} M_+$. This becomes reasonable when one considers physically what these operations actually do. For example, $\underset{0}{\overset{2}{1}} M_+$ is a natural forward operation on the downgoing wave field, D, at z_0. It involves forward propagation between z_0 and z_1, a shift at z_1 , and a forward propagation from z_1 to z_2 . Note that the natural inverse operation to $\underset{0}{\overset{2}{1}}M_+$ begins with a backward propagation from z_2 to z_1 . This is found in the operation $\underset{2}{\overset{0}{1}}M_-$, not $\underset{0}{\overset{2}{1}}M_-$.

Our notation formalism allows other obvious identities to be written.

$$\underset{0}{\overset{2}{1}}M_+ = {}^{2}_{1}M_+ \, \underset{0}{\overset{1}{1}}M_+ \quad \text{or} \quad \underset{0}{\overset{2}{1}}M_- = {}^{2}_{1}M_- \, \underset{0}{\overset{1}{1}}M_- \tag{15a}$$

$$\underset{2}{\overset{0}{1}}M_+ = {}^{0}_{1}M_+ \, \underset{2}{\overset{1}{1}}M_+ \quad \text{or} \quad \underset{2}{\overset{0}{1}}M_- = {}^{0}_{1}M_- \, \underset{2}{\overset{1}{1}}M_- \tag{15b}$$

Now let us develop our migration scheme. Our aim will be to find a relation for the reflection coefficient field, $c(x,t)$, with the source waveform, $w(t)$, convolved on it. We call this quantity $c'(x,t)$. So,

$$c'(x,t) = c(x,t) *_t w(t) \tag{16}$$

The convolution is obviously in the time domain. (Deconvolution to eliminate the source waveform can be done when the quantity c' is obtained, but deconvolution is not of concern to us here.)

We will consider as known quantities the surface data, $U(z_0)$ (note $U(z_0) = U(x,t,z_0)$), the depth, z_1 , at which shifting is to occur, and the amount of shifting (i.e., we have good knowledge of the two-dimensional sea floor topography and the velocity contrast). We also shall assume that we have a downgoing vertical plane wave as a source wave field (this can be simulated by a vertical stack), but we will not need to know the source waveform, $w(t)$. We will therefore, be able to construct $I(z_0)$, being a vertical plane wavefront with a delta function waveform.

The conventional wave equation migration to recover the reflector at z_2 involves the operation

$$\hat{U}(z_2) = {}^{2}_{0} M_- \, U(z_0) \tag{17}$$

where no shifting has been done at the sea floor. $\hat{U}(z_2)$ is not equal to $U(z_2)$, but is an approximation because there is no compensation for any transmission effects. Since we know what this shifting will be, we can write a more exact expression for $U(z_2)$,

$$U(z_2) = \begin{matrix} 2 \\ 1 \\ 0 \end{matrix} M_- \, U(z_0) \tag{18}$$

The only difference between $\hat{U}(z_2)$ of (17) and $U(z_2)$ of (18) is the inclusion of the "unshifting" operation at z_1 , done because we know that the upcoming wave was shifted on passing upward through z_1 on its way to the surface, z_0 . We must not do the shifting before we downward continue to z_1 because some diffraction will have occured by the time U has reached the surface (mathematically, we can say that equations (6a) and (6b) commute only for ranges z to $z \pm \Delta z$ where Δz is given by sampling criteria).

Well, equation (18) is already an improvement on conventional wave equation migration (17) because it includes the shifting in the upcoming wave. It does not, however, include the shifting and diffraction that the downgoing wave has suffered on its path to the reflector. Let us instead of dealing with D , deal with I because from our original premises we know I precisely, but do not know D because of our poor knowledge of the waveform, $w(t)$.

We have, for the reflector at z_2 ,

$$U(z_2) = c(x,t) *_t D(x,t,z_2)$$

(19)

$$= c(x,t) *_t I(x,t,z_2) *_t w(t) \qquad (19)$$

where all convolutions are single channel and in the time domain. Equation (19) is a statement of a single-channel coupling between the upcoming and downgoing waves. The physical interpretation of (19) is specular reflection at z_2 , that is, immediately before and after reflection, the upcoming wave is merely a differentially time-shifted version of the downgoing wave, with the shifting given by the reflector topography. The reason, of course, that this relation is true only at z_2 , is that upward and downward continuation operators separate each wave field at successive depths, and these operators are multi-channel.

Now we can rewrite (19) as

$$U(z_2) = w(t) *_t c(x,t) *_t I(z_2) \qquad (20)$$

We would like to bring $I(z_2)$ to the left side of equation (20) and as we convolve I in the time domain, so also will we invert I in the time domain.

$$U(z_2) *_t [\ I(z_2)\ ]_t^{-1} = w(t) *_t c(x,t) \qquad (21)$$

Recalling equation (16) and invoking our downward continuation operators, equation (21) (transposed) becomes

$$c'(x,t) = [\prod_0^2 M_- U(x,t,z_0)] *_t [\prod_0^2 M_+ I(x,t,z_0)]_t^{-1} \qquad (22)$$

Now equation (22) is a final statement for our migration scheme. The final migration involves the single-channel time convolution of two fields: the factor on the left is the downward continued data to the reflector at z_2 with "unshifting" done at depth z_1 , while the quantity on the right is the time-inverse of the downgoing wave field downward propagated to the reflector at z_2 with shifting done at z_1 . Equation (22) is to be compared with equation (17) which is the way simple wave equation migration of plane-wave (vertically stacked) sections was done previously.

The added expense involved in equation (22) is not substantial. The operation $\overset{2}{\underset{0}{1}}\, M_-$ as compared to ${}^{2}_{0} M_-$ is very small because it shifts only once (add 3 MAD's per array point with SHIFT4, q.v.). The convolution operation adds N multiplies per array point where N is the number of points in the time-dimension of I . We have been getting satisfactory results with N=20 . The operation $\overset{2}{\underset{0}{1}}\, M_+$ and the inversion adds computation time, but they are operations on the downgoing wave field only, and this will be in a significantly smaller array than the data.

The generalization of equation (22) to a data field with many reflectors is not difficult. The convolution factor on the left will be merely the downward continuation of the data to the reflectors. (With the exception of the shift at z_1 , this has been the stopping point of our previous migration algorithms.) The factor on the right will be the inverse of the downward continued and shifted downgoing wave, but will only be valid for z to $z+\Delta z$, where z is the current depth in the downward continuation. The time-convolution, then will only be valid for reflectors between z and $z+\Delta z$. For reflectors between $z+\Delta z$ and $z+2\Delta z$, I must be downward continued an additional Δz step and then inverted and convolved onto the data in the proper range.

We have been able to devise a scheme for obtaining transmission-compensated migration without having to know the source waveform, $w(t)$. We must, however, have a reasonably good estimate of the shift at the sea floor. (Examples will be done for this type of migration with imperfect knowledge of the shift.)