A Shot-Offset Frame for Velocity Estimation

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It is well known that in a flat layered earth the material velocity in each layer may be determined from the normal moveout correction which flatens the events on the common reflection point gathers. We will establish that the same method can be applied to a dipping and faulted earth provided that surface observations are not only moveout corrected but also downward continued. We begin with a coordinate transformation based on the geometry in figure 1.

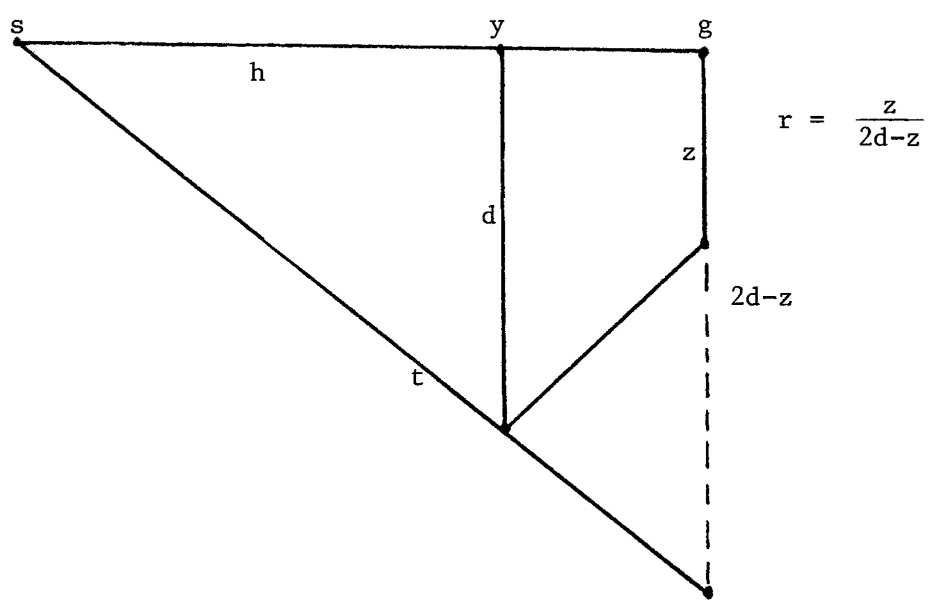

Figure 1. Geometry for shot-offset coordinate transformation. The shot-offset h is the lateral offset between the shot s and the midpoint y .

In figure 1 we have the forward problem variables (s, g, z, t) with the surface shot coordinate s , the lateral coordinate of the geophone g , its depth z and the travel time for a hypothetical echo t . We also have the inverse problem variables (h, y, d, r) where y is the lateral coordinate of the reflection point, d is the depth coordinate of the reflection point, h is the lateral offset between the shot point and the reflection point, and r is a parameter which is zero when the receivers are at the surface and increases to +1 when the receivers have been continued downward to the reflector. The easiest transformation to prescribe is the one from the inverse problem variables to the forward problem variables (because that is like solving a forward problem). For a unit velocity homogeneous medium we have

$$z = d\, 2r / (1+r) \tag{1a}$$

$$s = y - h \tag{1b}$$

$$g = y + h - h\, 2r / (1+r) \tag{1c}$$

$$t = (d^2 + h^2)^{1/2}\, 2 / (1+r) \tag{1d}$$

For a homogeneous medium the inversion of the transformation works out to be

$$h = (g-s) \left(1 + z / (t^2 - (g-s)^2)^{1/2} \right) / 2 \tag{2a}$$

$$y = (g+s)/2 + z(g-s) / 2(t^2 - (g-s)^2)^{1/2} \tag{2b}$$

$$d = ((t^2 - (g-s)^2)^{1/2} - z) / 2 \tag{2c}$$

$$r = z / (t^2 - (g-s)^2)^{1/2} \tag{2d}$$

Rather than repeat the chore of transforming the wave equation we can pick up from where Steve Doherty stopped on his 17 June work. He used the relationship $z = d\,r'$ but we are now using $z = d\ 2r\,/\,1{+}r$. Thus we find that

$$r \;=\; 2\,/\,(2-r') \;-\; 1$$

and

$$r_{r'} \;=\; 2\,/\,(2-r')^2$$

and

$$\partial_{r'} \;=\; r_{r'}\,\partial_r \;=\; \frac{2\,\partial_r}{(2-r')^2}$$

So Steve's equation (13) becomes

$$2\left(\frac{p\,\partial_y}{(1-p^2)^{1/2}} + \partial_d\right)\partial_r\,Q \;=\; -\,\frac{d}{2}\,\frac{1}{(1-p^2)}\,Q_{yy} \;-\; \frac{(1-p^2)^2}{2\,d}\,Q_{pp} \;-\; (1-p^2)^{1/2}\,Q_{yp} \qquad (3)$$

Obviously the reason I made the new definition of r was to eliminate it from the coefficients of the differential equation (3). Next I got the idea of cleaning up the coefficients still more by using a tangent coordinate instead of a sine, that is,

$$q \;=\; p\;/\;(\,1-p^2\,)^{1/2} \;=\; \tan\theta$$

and

$$q_p \;=\; (\,1-p^2\,)^{-3/2}$$

so that

$$\partial_q \;=\; q_p\,\partial_p \;=\; (\,1-p^2\,)^{-3/2}\;\partial_p$$

reducing (3) to

$$2\left(\frac{p\,\partial_y}{(1-p^2)^{1/2}} + \partial_d\right)\partial_r Q = \frac{-1}{2(1-p^2)}\left[\, d\,Q_{yy} + 2\,Q_{yq} + \frac{Q_{qq}}{d}\,\right]$$

or (4)

$$(\,q\,\partial_y + \partial_d\,)\,\partial_r Q = -\frac{(1+q^2)}{4}\left[\, d\,Q_{yy} + 2\,Q_{yq} + Q_{qq}/d\,\right]$$

On inspection of (4) my next objective was to eliminate the presence of $1/d$ as a coefficient because a wide range of numerical values of coefficients is an annoyance in numerical integration. This suggested the definition

$$h = d\,q = d\tan\theta$$

which was incorporated in (1) and (2). Thinking of

$Q(q, d) = Q'(h, d)$ we get

$$Q_q = Q'_h\, h_q = d\, Q'_h$$

and

$$Q_d = Q'_d + Q'_h\, h_d$$

$$= Q'_d + q\, Q'_h$$

Bringing these into (4) and dropping the primes we get

$$(\,h/d\,(\,\partial_y + \partial_h\,) + \partial_d\,)\,\partial_r Q = -\frac{(1+h^2/d^2)}{4}\, d(\,\partial_y + \partial_h\,)^2\, Q \qquad (5)$$

The disadvantage of the new Q_{hr} term is mitigated by a common geophysical situation in which $Q_d >> Q_y >> Q_h$. In fact we might often, for simplicity, approximate (5) by

$$Q_{dr} = - (1 + h^2/d^2) \, d/4 \; Q_{yy} \tag{6}$$

which amounts to migrating each offset separately.

Finally we return to the question of velocity analysis. The statement is that $Q(h, y, d, r=1)$ is independent of h if the earth has the same velocity as the transformation velocity (which was unity) regardless of whether the earth is layered $(Q_y = 0)$ or not. To see this we will check the case of a point scatterer. Excluding steeply dipping waves we have for the upcoming wave an infinitesimal distance above a scatterer at (x_0 , z_0) .

$$U(s,g,z,t) = \delta(g-x_0) \, \delta(t^2 - (s-x_0)^2 - z_0^2)$$

Converting this to inverse coordinates at $r = 1$ we get

$$U'(h,y,d,r=1) = \delta(y-x_0) \, \delta(d^2 + h^2 - (y-h-x_0)^2 - z_0^2)$$

The existence of $\delta(y - x_0)$ allows us to set $y = x_0$ in the other delta function getting

$$= \delta(y - x_0) \, \delta(d^2 - z_0^2)$$

We see that the wave in moveout corrected coordinates is indeed independent of the shot to reflection point lateral offset h . Thus, the migrated common reflection point gathers can now be analyzed for velocity without regard for whether reflection has taken place from point scatterers or plane layers.