An Expanding Time Scale

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Because seismic data is higher frequency at earlier times than at later times there will be occasional advantages to resampling the time axis. A time invariant spectrum is an asset in filtering and partial difference equation applications. A sensible expansion function is

$$t' = 2 \, (t_0 \, t)^{1/2} \tag{1}$$

with the inverse relation

$$t = (t')^2 \, / \, 4 \, t_0 \tag{2}$$

We have

$$\frac{dt'}{dt} = (t_0 / t)^{1/2} = 2 \, t_0 / t' \tag{3}$$

The meaning of t_0 is that when $t = t_0$ then $dt' / dt = 1$. Thus, we may choose $\Delta t = \Delta t' = 4$ millisec at $t_0 = 2$ sec for a typical example. We get

t	$t' = (8t)^{1/2}$ sec $\Delta t = (t/t_0)^{1/2} \, \Delta t'$ $= (8t)^{1/2}$ millisec	Points in Interval
		353
1/4	1.4	
		146
1/2	2.0	
		207
1	2.8	
		292
2	4.0	
		414
4	5.6	
	total	1412

These numbers seem to be reasonable although some of the 353 points in the early interval 0 - 1/4 might often be omitted. The last column was calculated by

$$\text{No. Pts} = \int_{t_1}^{t_2} \frac{dt}{\Delta t(t)} = \int_{t_1}^{t_2} (t_0/t)^{1/2}\, dt \,/\, \Delta t'$$

$$\frac{2}{\Delta t'} \left[(t\ t_0)^{1/2} \right]_{t_1}^{t_2} = 707\,(t_2^{1/2} - t_1^{1/2})$$

In differential equation applications we have say

$$P(t) = Q(t') \quad \text{and} \quad P_t = Q_{t'} t'_t = Q_{t'}\, 2t_0 / t' \quad .$$

Thus, an equation like $P_{zt} = P_{xx}$ becomes

$$Q_{zt'} = (t' / 2t_0)\, Q_{xx} \quad .$$