A Wide Offset Migration Equation

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Previously we migrated by taking an upcoming wave and marching it back down the z-axis. Data at later times (variously called late t' or late d) was moved to greater depths z . Here we introduce a diffraction coordinate r by

$$r = z / d \tag{1a}$$

$$z = r d \tag{1b}$$

Instead of integrating down the z-axis from the surface $z=0$ to $z= vd$ we now accomplish the same goal by integrating down the r-axis from $r=0$ to $r = 1/v$. For simplicity we set velocity $v=1$. The idea is depicted in figure 1.

By means of the r coordinate we can now do a rather more general treatment of offset than was done in the Claerbout-Doherty 1972 paper. The earlier treatment which retained only first order in offset terms in equation (49) is now generalized to retain all orders. Although the present analysis is restricted to a constant velocity coordinate frame it is possible that the diffraction coordinate r being introduced will be generalizable to facilitate treatment of all offsets in arbitrary stratified frames. We define the coordinate transform

$$x = k (2 - r) \tag{2a}$$

$$z = r d \tag{2b}$$

$$t = (2 - r) (d^2 + k^2)^{1/2} \tag{2c}$$

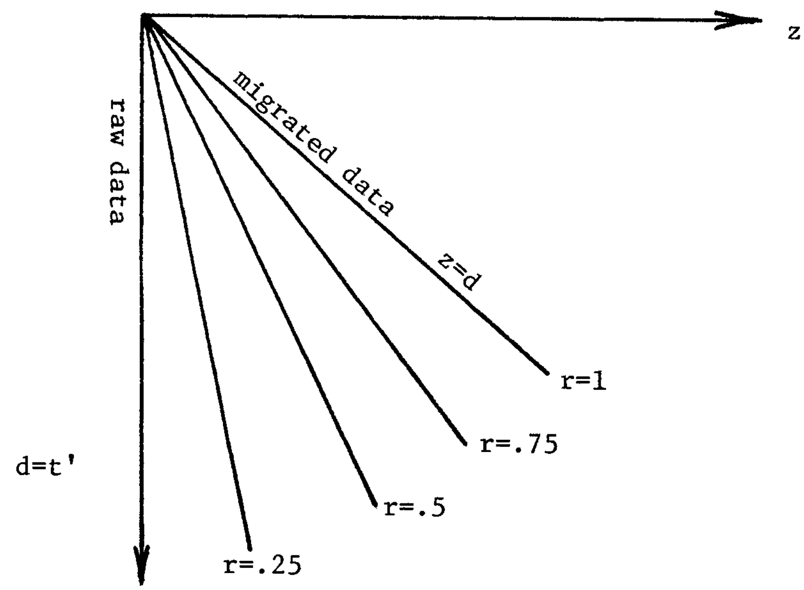

Figure 1. Migration using a diffraction coordinate r in place of a depth coordinate z . Instead of continuing the data from z=0 to z=d in equal intervals of Δz we march along in equal intervals Δr of $r = z / d$.

Reference to figure 2 shows a geometrical interpretation.

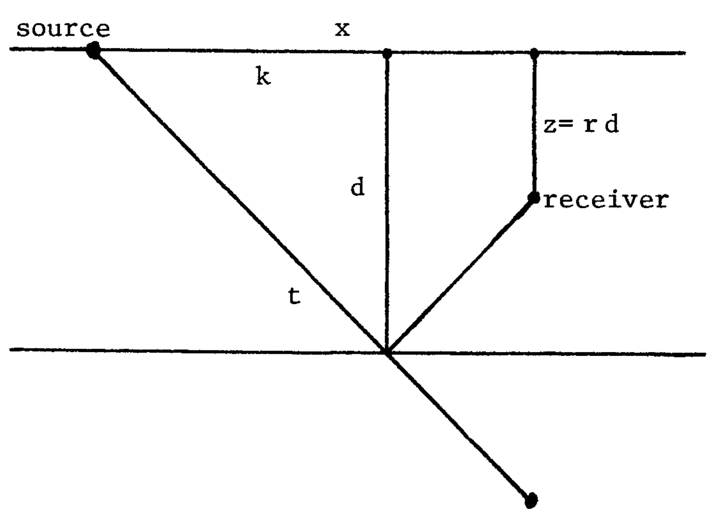

Figure 2. Geometry for continuing receiver down from surface to a reflector.

The inverse transformation can be easily worked out from geometrical considerations and the expressions below can be readily checked by substitution from (2).

$$k = (1 + z / (t^2 - x^2)^{1/2})\, x / 2 \tag{3a}$$

$$r = 2z / ((t^2 - x^2)^{1/2} + z) \tag{3b}$$

$$d = ((t^2 - x^2)^{1/2} + z) / 2 \tag{3c}$$

Naturally we will require the Jacobian matrices of the transformations. Using the letter s as an abbreviation for $(d^2 + k^2)^{1/2}$ we calculate the Jacobian of (2)

$$\begin{bmatrix} x_k & x_r & x_d \\ z_k & z_r & z_d \\ t_k & t_r & t_d \end{bmatrix} = \begin{bmatrix} 2-r & -k & 0 \\ 0 & d & r \\ (2-r)k/s & -s & (2-r)d/s \end{bmatrix} \tag{4}$$

The inverse to (4) is tedious to compute but not hard to check. It is

$$\begin{bmatrix} k_x & k_z & k_t \\ r_x & r_z & r_t \\ d_x & d_z & d_t \end{bmatrix} = \frac{1}{2d^2} \begin{bmatrix} \frac{2d^2+rk^2}{2-r} & kd & -\frac{rks}{2-r} \\ rk & (2-r)d & -rs \\ -kd & d^2 & ds \end{bmatrix} \tag{5}$$

Given that

$$P(x, z, t) = Q(k, r, d) \tag{6}$$

The wave equation in a homogeneous medium is

$$0 = (k_x \partial_k + r_x \partial_r + d_x \partial_d)^2 Q + + (k_z \partial_k + r_z \partial_r + d_z \partial_d)^2 Q - (k_t \partial_k + r_t \partial_r + d_t \partial_d)^2 Q \tag{7}$$

Making the usual assumption that the wave's gradients exceed the coordinate system gradients, we square the operators in (7) as if the coefficients were constants. In (7) we find the Q_{dd} term. This time shifting term is

$$Q_{dd} (d_x^2 + d_z^2 - d_t^2) =$$

$$= Q_{dd} (k^2 d^2 + d^4 - d^2 (k^2 + d^2)) / 4 d^4 = 0$$

The lateral shifting Q_{kd} term is

$$Q_{kd} \; 2 (k_x d_x + k_z d_z - k_t d_t) = 0$$

The last two terms vanished because of our selection of the ray coordinate system.

The main extrapolation term Q_{dr} is

$$Q_{dr} \; 2 (r_x d_x + r_z d_z - r_t d_t) = Q_{dr} / d \tag{8}$$

The unfamiliar Q_{kr} term is

$$Q_{kr} \; 2 (k_x r_x + k_z r_z - k_t r_t) = Q_{kr} \; k / d^2 \tag{9}$$

The diffraction term Q_{kk} is

$$Q_{kk}(k_x^2 + k_z^2 - k_t^2) = Q_{kk}(1 + k^2 / d^2) / (2 - r)^2 \qquad (10)$$

The parabolic approximation term Q_{rr} is

$$Q_{rr}(r_x^2 + r_z^2 - r_t^2) = Q_{rr}(1 - r) / d^2 \qquad (11)$$

These are all the terms there are in (7), namely, three cross terms and three second derivative terms. Thus, with the parabolic approximation we have the grand result that

$$(\frac{k}{d} \partial_k + \partial_d) \partial_r Q = - d \frac{(1 + k^2 / d^2)}{(2 - r)^2} \partial_{kk} Q \qquad (12)$$

Notice that no poles are encountered in the region $r = 0$ to 1 .

The unfamiliar term Q_{kr} has given us a directional derivative in the data plane $Q(k,d)$ along a line of constant k/d instead of the usual time derivative.

Now a valid question to ask is "Why so much effort just to achieve only an approximate equation for propagating in a homogeneous material, even if it does turn out to be generalizable to stratified media?" The answer involves data interpretation. It is harder to make inferences about the 2 (or 3) dimensionally complex earth from surface data than it is from downward continued data <u>even though the downward continuation may incorrectly assume a homogeneous velocity</u>. The reason is based on the fact that structural complexity of the earth usually exceeds its velocity complexity. Equation (12), although it assumes velocity homogeneity, makes no assumption about the complexity of reflector shapes other than the improvable Fresnel-like approximation.

Downward continuation with the wrong velocity at least will reduce the data complexity due to the earth's structural complexity.

In summary, before we interpret data we have no knowledge of velocity or structural variations within the earth. In order to get some knowledge we downward continue the data with (12) which assumes constant velocity. This reduces structural complexity. From the downward continued data we get our best estimates of inhomogeneity. Then we can propose the task of obtaining a more inhomogeneous version of (12) and starting all over again with our improved knowledge.