

ELASTIC MODELING AND MIGRATION IN EARTH MODELS

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By  
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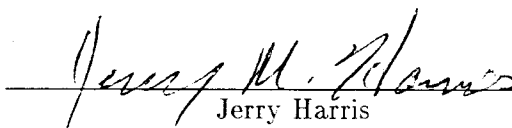
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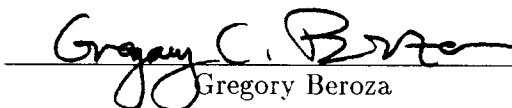
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# Elastic modeling and migration in earth models

*Carlos A. Cunha Filho, Ph.D.  
Stanford University, 1992*

## ABSTRACT

Migration and inversion of marine seismic data using the elastic wave equation requires the transformation of the recorded pressure data into a vector particle-displacement field. This can be done easily when the recording geometry samples the wavefield both horizontally and vertically. However, only experimental surveys have cables located at different depths. Using a few assumptions, I derive a method for performing this transformation, which is applicable to standard surveys. The assumptions are: smooth water surface, cable nearly parallel to water surface, and perfect seismic-reflection at the water surface. Results in a realistic example, where these assumptions are only partially fulfilled, demonstrate that the method is robust.

Elastic, reverse-time migration/inversion schemes in the space-time domain are usually implemented by finite-difference or finite-element methods. When imaging beyond structures, a dynamically accurate scheme must be used. For models characterized by layers with sharp boundaries traditional finite-difference methods fail to correctly describe the dynamics of the propagation process. Failure comes from the lack of distinction between model and field variables; the same difference operator is applied to discontinuous (model) and continuous (wavefield) components. The problem is solved with a modified finite-difference scheme (dual-operator), which uses long operators for wavefields, short operators for elastic parameters, Shoenberg-Muir (1989) equivalence relations and a modified Virieux (1984) staggered grid scheme. Tests show that the the dual-operator is dynamically more accurate than traditional finite-difference schemes and comparable to Haskell-Thomson schemes.

In structurally complex media, accurate recovery of angle-dependent reflectivities requires elastic prestack migration. Mode separation can be done before or after depth extrapolation. Though more complex, the latter is more complete because it images mode-converted waves. Standard depth-extrapolation and imaging approaches are unsuitable

for true-reflectivity recovery. I introduce an extrapolation method which properly compensates for transmission/conversion losses. This method is combined with an imaging condition that performs the plane-wave decomposition of the downward extrapolated data to define the plane-wave-response (PWD) migration. The four image-cubes generated by the PWD migration correspond to the plane-wave angle-dependent reflectivities for PP, PS, SP, and SS modes, and directly relate to the Zoeppritz equations because they represent the in-depth plane-wave response of the medium.

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