Chapter 1

Introduction

1.1 Imaging seismic reflection data

The goal of processing seismic reflection data is to convert a recorded wave field into an image of the subsurface. Although seismic processing consists of many steps, the imaging step *migration* positions reflector images. Migration needs a model of the wave-propagation velocities of the subsurface, but obtaining this model is often the most difficult processing step in areas of complex structure. This thesis describes how to use prestack depth migration to obtain velocity estimates and images in areas of complex structure and lateral velocity variation.

1.1.1 Migration

Migration converts traveltime to distance and time dip to geological dip (Claerbout, 1985; Stolt and Benson, 1986) by modeling wave propagation, and to perform this task migration needs a model of the wave propagation velocity of the subsurface. The interval velocities of the rocks in the subsurface vary on many scales, and waves propagating through the Earth are affected differently by different wavenumbers of the velocity field. The reflected waves of interest to exploration geophysicists are generated by discontinuities or the high wavenumbers of the velocity field, but the arrival times of these reflected waves are primarily affected by the low wavenumbers of the velocity field between the reflector and the surface. Since the most important action of migration is focusing and positioning scattered energy, migration primarily needs a model of the low wavenumbers of the Earth's velocity. Although this separation of wave effects into transmission and scattering is imperfect (Claerbout, 1985; Stolt and Benson, 1986; Woodward, 1989), it usually works well in practice.

Time migration vs. depth migration

Migration algorithms can be divided into two categories, time migration and depth migration. Time migration uses the kinematics of constant-velocity wave propagation to focus and position reflected waves. Although velocity is almost never constant, time migration focuses and positions an event with a single velocity, the "migration velocity" v_m, that represents some average of the true interval velocities of the overburden. Different parts of the data can be migrated with different migration velocities to account for variation of the velocity field. As long as there are no rapid interval-velocity variations, replacing wave propagation in the true velocity model with wave propagation in an average constant-velocity model adequately focuses and positions seismic reflections.

Depth migration, which uses wave propagation in an interval-velocity model $v(x, z)$, gives a more accurate image than time migration in a variable velocity medium because it honors ray bending. The inclusion of depth conversion with migration is another important difference between time migration and depth migration. Time migration only focuses the extant time structure; it does not perform depth conversion. Depth migration alters time structure when the velocity model varies laterally by both focusing and depth converting the image. It is often said that time migration is "safe" because is doesn't alter the time structure as much as depth migration. Although it is true that when used improperly, depth migration is "dangerous" because it can highly distort migrated images; depth migration *is needed* when the subsurface velocity varies significantly laterally.

Figure 1.1 shows an unmigrated, stacked data set that displays a significant time structure (on the bottom-of-salt reflector) caused in part or entirely by lateral velocity variation. We need to use migration to properly image the structure. After prestack time migration (Figure 1.2), the events are more properly focused, but the false time structure remains. After prestack depth migration (Figure 1.3) with a crude, but geologically reasonable velocity model, the shape of the structure is altered significantly; much of the pull-up is removed. At this point however, it is unknown if the velocity model and the depth-migrated image are correct because conventional velocity-estimation methods don't give adequate interval-velocity estimates when lateral velocity variation is strong.

Depth migration leans more heavily on the velocity analysis step of seismic processing than time migration. To position events correctly and to stack the data coherently, the velocity model used for depth migration must closely resemble the true Earth velocity; CMP stacking and time migration only need approximate or average velocity information. This illustrates the central problem of this thesis; when prestack depth migration is needed to image data correctly, it can't be used because conventional velocity-analysis tools do not give adequate interval-velocity estimates.

Migration before stack, after stack, or with stack

Migration can be applied to stacked data or to unstacked (prestack) data. Also some migration algorithms that work on unstacked data output only stacked images; others can output unstacked images. Stacking looses all information about traveltime versus offset; but if a satisfactory stack has been made, and the velocity model that explains the observed moveout is reasonable, post-stack migration is an effective and economical method of imaging seismic data. The velocity information that remains after stacking, the positioning of the image and diffraction focusing, can sometimes be used to correct a migration-velocity model (Whitmore, 1984).

Prestack migration is often applied when a satisfactory stack cannot be made by simpler methods because of complex structure and/or lateral velocity variation. Some prestack migration algorithms such as $f-k$ (Stolt, 1978) and S–G downward continuation (Claerbout, 1985) combine the information from all offsets at once. They perform prestack migration and stacking simultaneously. Prestack time-migration algorithms such as the $f-k$ method of Stolt are often applied to data for a range of migration velocities. Then, velocity analysis can take place after migration (Shurtleff, 1984; Fowler, 1984; Li et al., 1987).

Other prestack migration algorithms such as constant-offset (Deregowski, 1985) or common-shot migration (Reshef and Kosloff, 1986) postpone stacking until after migration. The unstacked output after prestack migration can be examined for residual moveout and thus for errors in the velocity model. Figure 1.4 shows a set of common-midpoint (CMP) gathers after constant-offset prestack depth migration from the data set of Figures 1.1-1.3. If the velocity model used to migrate the constant-offset sections is correct, events should be images at the same depth regardless of offset. If the velocity model is incorrect, events will have residual moveout. Since residual moveout is seen on many of

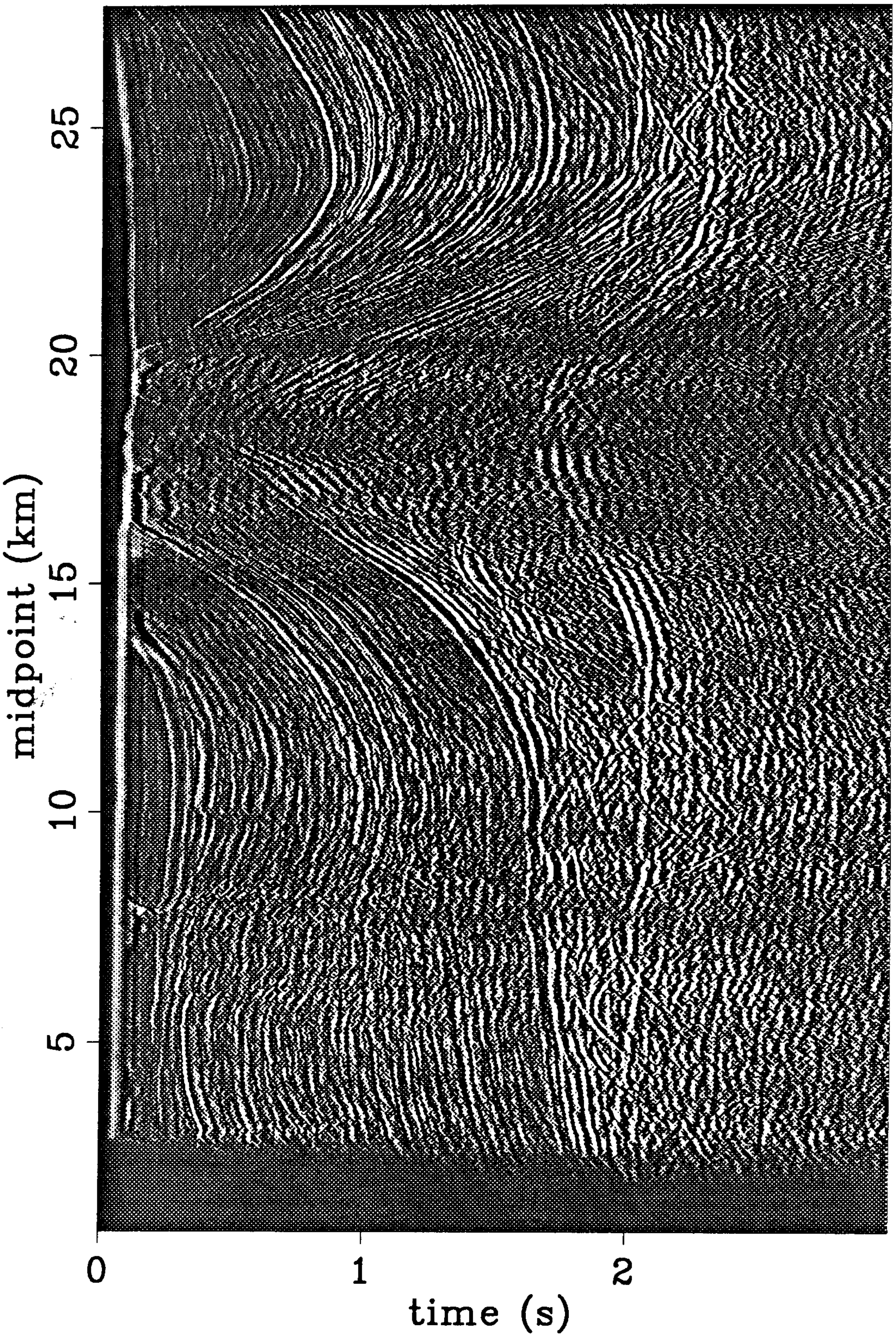

FIG. 1.1. CMP stack of a salt-dome data set from the North Sea. The difference between the interval velocities of the salt and the interval velocities of the neighboring sediments distort the image of the bottom-of-salt reflector (pull-up around midpoint 18 km, 1.8 s.).

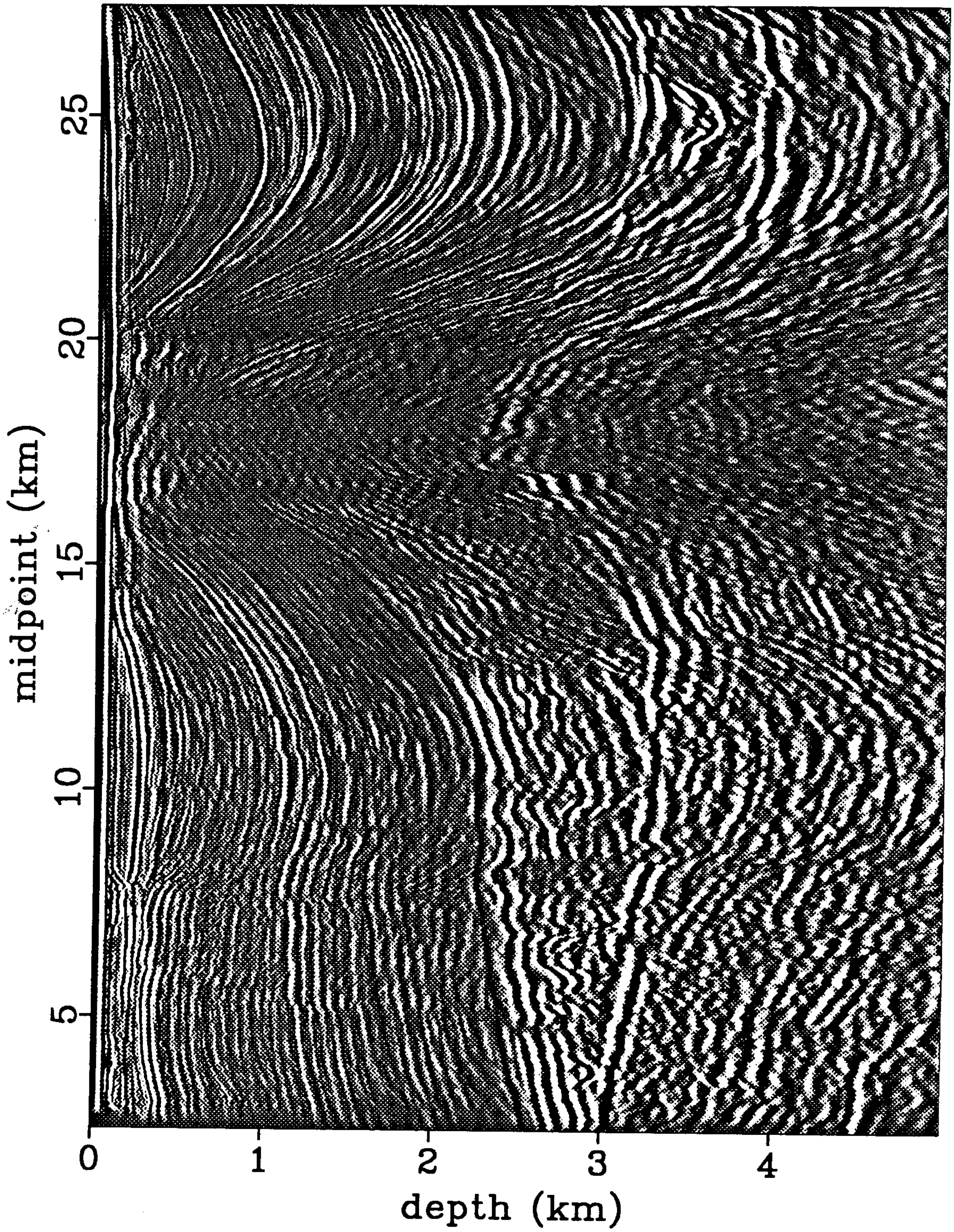

FIG. 1.2. Prestack time-migrated image of the salt-dome data set. Although the image is more correctly positioned than in the CMP stack, significant and probably false time structure remains on the bottom-of-salt reflector.

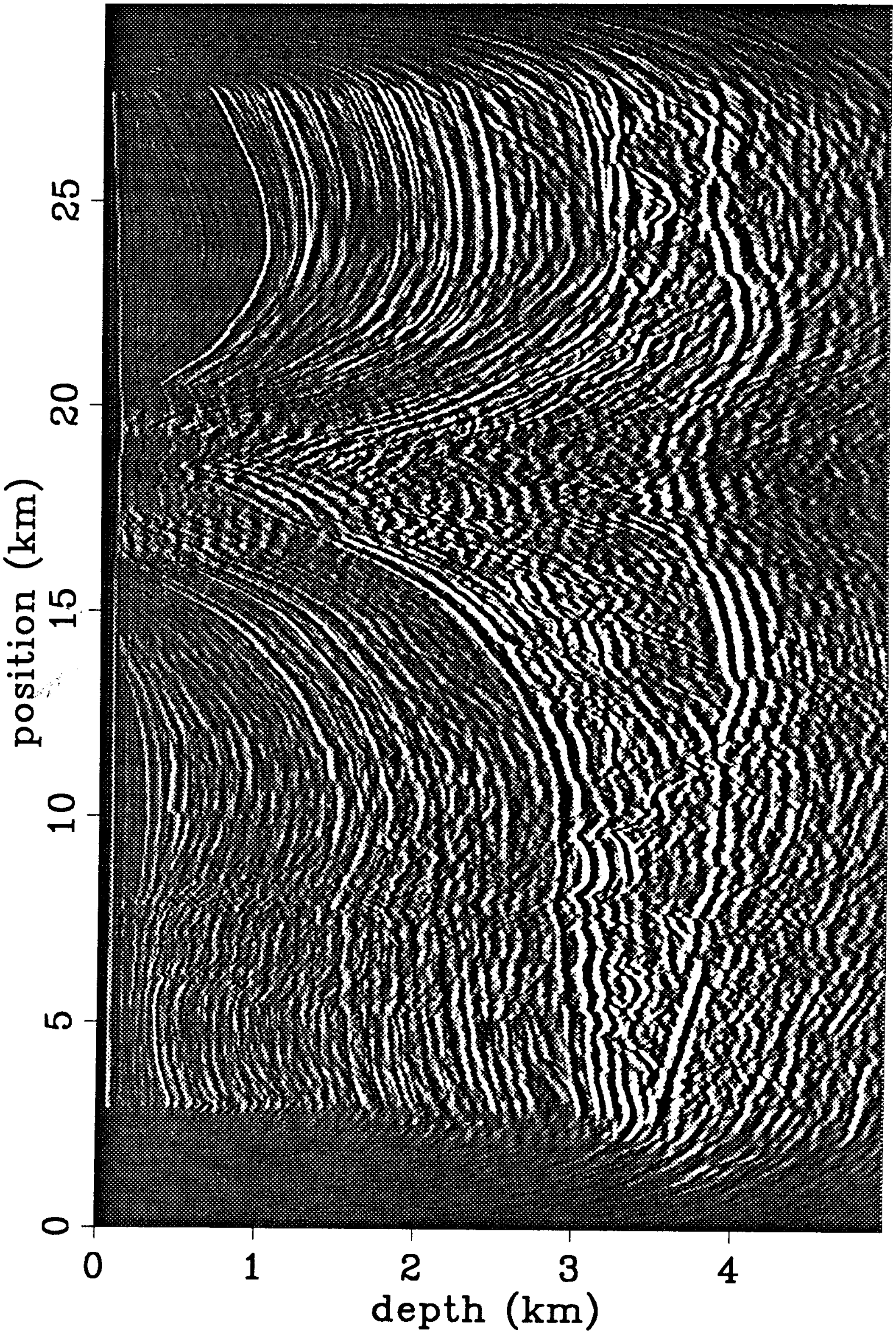

FIG. 1.3. Prestack depth-migrated image of the salt-dome data set. The image of the bottom-of-salt reflector is altered, but without a velocity-analysis tool that works with prestack depth migration it is still unknown if the image is correct.

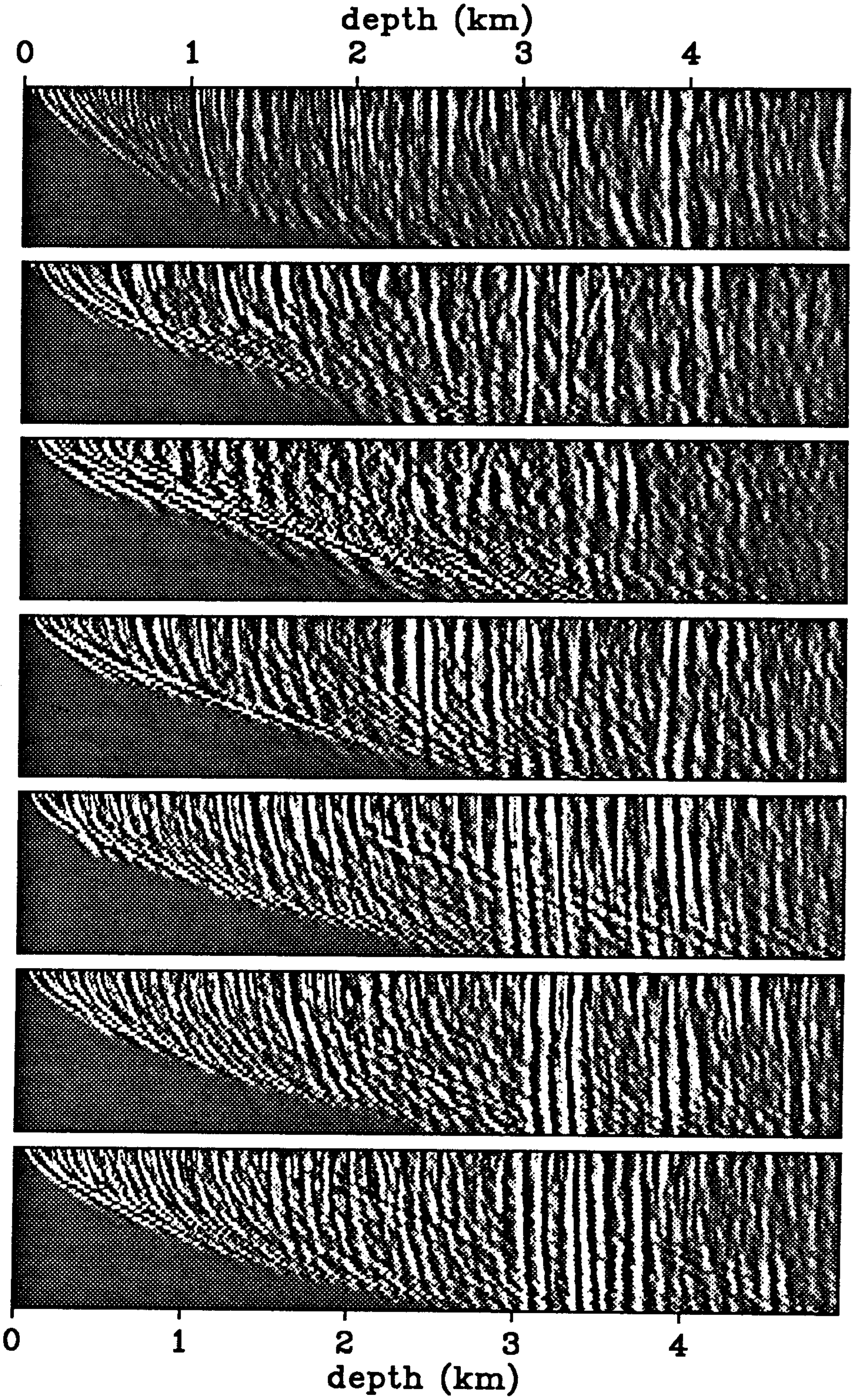

FIG. 1.4. CMP gathers after prestack depth migration. Each panel is a different midpoint, the bottom panel at 6 km surface location, the rest spaced at 3 km increments. The vertical axis of each panel is offset; inner offsets at top; outer offsets at bottom. The events aren't flat vs. offset; the interval-velocity model used for depth migration is incorrect.

the post-migration CMP gathers, the velocity model used to produce Figure 1.3 and 1.4 is incorrect.

Migration algorithms that output unstacked images are particularly useful for finding velocity errors after prestack depth migration. Velocity-dependent residual corrections can be applied to the migrated constant-offset sections to remove residual moveout found in the initial migrated images; this provides the basis for a velocity-estimation method that uses prestack depth migration.

1.1.2 Velocity analysis

The velocities of the subsurface are unknown or known only approximately, and we must estimate the velocity of the subsurface before or concurrent with imaging. There are several different attributes of seismic data that can be used for velocity analysis. Moveout over offset of the traveltimes of reflected waves can be measured with stack-coherence methods (Taner and Koehler, 1969; Neidell and Taner, 1971). Diffraction focusing can be measured by minimum-entropy methods (Harlan et al., 1983; DeVries and Berkhout, 1984). The migrated location of events can be checked for geological reasonability (Whitmore, 1984). Moveout-correction stack-coherence methods like NMO and stack are most commonly used because they increase signal relative to noise through stacking, are easy to automate, and are inexpensive to apply. These conventional velocity-estimation methods do not obtain interval velocity directly but estimate some average or function of the interval velocity. Since the interval velocity is needed for depth migration, it must be derived from the "average" velocities obtained with these methods.

stacking velocities

The NMO equation used to apply moveout corrections is given by

$$t_h^2 = t_0^2 + 4h^2/v_s^2 \ . \tag{1.1}$$

The two-way traveltime of a reflected wave at offset h is denoted by t_h; t_0 is the two-way zero-offset traveltime. Stacking velocities are found by applying NMO to CMP gathers with several values of the stacking velocity v_s and finding the one that best corrects the moveout of each event. Measuring coherence over offset after NMO correction with stack power or stack semblance quantifies how well a given stacking velocity corrects the observed moveout. In a horizontally layered medium, v_s is approximately equal to the vertical root-mean-square average (rms) of the interval velocity (Dix, 1955). As long as the horizontally layered approximation is valid, rms velocity can be converted to interval velocity by layer stripping. Although the form of equation (1.1) is simple, it is by a wide margin the most common velocity estimator applied to seismic data and has enjoyed great success.

DMO-corrected stacking velocities and prestack time-migration velocities

When seismic waves reflect from dipping horizons, common-midpoint gathers are not common-reflection-point (CRP) gathers. Stacking velocities for dipping reflectors are increased relative to the stacking velocities of horizontal reflectors by a factor equal to the secant of the dip angle of the reflector (Levin, 1971). DMO (Deregowski and Rocca, 1981; Hale, 1984) corrects for the geometrical effect of CMP smearing observed for dipping reflectors and converts common-midpoint gathers to common-reflection-point gathers. DMO can be used between NMO and stacking to obtain dip-independent stacking velocities for dipping reflectors.

Hale (1983) showed that prestack time migration was equivalent to NMO followed by DMO, stacking, and zero-offset migration when both use the same constant velocity. Although the two methods can produce equivalent images, they treat velocity information differently. Prestack time migration uses the same velocity for event positioning and moveout correction, that is, $v_s = v_m$. Applying NMO, DMO, and stacking followed by zero-offset migration allows one to specify a different velocity model for moveout correction than for migration. When the assumptions behind prestack time migration are violated, that is, when velocity varies laterally or strongly vertically, better images can sometimes be obtained by using a different velocity field for stacking and than for migration.

DMO-corrected stacking and prestack time migration also differ in how they position velocity information. Prestack time migration positions the velocity information at time-migrated locations while DMO positions velocity information in unmigrated time. For example, the velocity information from the tails of a diffraction is collapsed by prestack time migration to nearly a point. DMO does not apply zero-offset migration to reposition events, so the velocity information from diffraction tails stays at unmigrated locations. When velocity varies laterally or strongly with depth, DMO-corrected stacking velocities can suffer from multivaluedness; the velocity information from the diffraction is spread out along the unmigrated diffraction hyperbola and possibly conflicts with the information from other underlying events. Prestack time migration may position velocity information more accurately by moving events much closer to their true positions and unscrambling crossing events (Black et al., 1986).

Unless there is strong vertical or lateral velocity variation, these two methods usually give equivalent results. Like stacking velocities, the DMO-corrected stacking velocity or prestack time-migration velocity is an average velocity that can often be converted to interval velocity.

1.2 Handling lateral velocity variation

The presence of *any* lateral velocity variation in the subsurface technically violates the assumptions made in routine processing and velocity analysis. Nonetheless, standard processing methods are almost always applied and are often successful. The effect of lateral velocity variation is a function of two things: its magnitude and its wavelength. When the lateral velocity variation is gradual compared to the length of the recording spread, standard methods of imaging and velocity analysis can still be applied successfully even though there may be substantial velocity change from one end of a survey to the other. Rapid changes in velocity such as near-surface effects, can be handled as well, as long as the magnitude of the velocity changes are small. Standard processing methods are less successful when there are significant lateral velocity variations at scales from a few cable lengths to less than a cable length.

1.2.1 Imaging

If a coherent stacked section can be formed, time-to-depth conversion with a geologically derived velocity model is often sufficient to image horizontal or nearly horizontal structure when lateral velocity variation exists. Static or dynamic traveltime corrections before stack can correct for mild short-wavelength velocity variations. If more complex structure is apparent, depth conversion after post-stack time migration or post-stack depth migration may be necessary. When structure is complicated and has crossing events, DMO or prestack time migration are often needed to stack all events coherently. One successful imaging method in areas of moderate lateral velocity variation and complex structure uses DMO to stack the data coherently followed by post-stack depth migration with a geologically derived interval-velocity model to position events correctly.

What happens when there is strong lateral velocity variation? Prestack time migration gives only approximate common-reflection-point gathers and reflector positions. DMO followed by post-stack depth migration gives correct reflector positions, but only approximate common-reflection-point gathers; so it can be difficult to get a coherent stack or reasonable stacking-velocity estimates. Prestack depth migration performs true common-reflection-point gathering and correctly positions reflectors. Therefore, when subsurface structure is complicated and there are strong lateral velocity variations, prestack depth migration should be the preferred imaging tool. I say ***should be*** rather than ***is*** because prestack depth migration requires detailed knowledge of the interval velocity of the subsurface that we don't often have (Larner et al., 1989). Most of the burden of imaging data affected by strong lateral velocity variation falls on the velocity analysis step and not on the migration algorithm itself.

1.2.2 Velocity analysis

Velocity-analysis methods based on NMO or prestack time migration should work so long as the traveltimes of reflected waves from a reflection point are nearly hyperbolic over offset, or can at least be fit with hyperbolas, and so long as the supposed common-reflection-point gathers are approximately true common-reflection-point gathers. This requirement can be boiled down to requiring that prestack time migration produce a somewhat coherent, if mispositioned and distorted image. Careful smoothing and interpretation of stacking velocities or prestack time-migration velocities still give adequate interval-velocity estimates when velocity variation is gradual. However, when the velocity varies laterally on scales similar to or smaller than the recording spread, conventional velocity analysis can give confusing results (Lynn, 1979). Stacking velocities, DMO-corrected stacking velocities, or prestack time-migration velocities are no longer simple averages of the interval velocity.

If the magnitude of the lateral velocity variation is not large, there are methods that interpret the stacking velocity (for horizontal reflectors) or the DMO-corrected stacking velocity (for dipping reflectors) as a complicated function of overlying interval velocities. Loinger (1983) and Toldi (1985) derived linear operators that relate a laterally varying interval-velocity model to the stacking velocity that best NMO corrects reflections from horizontal reflectors. These linear operators can be inverted to estimate an interval-velocity model from the measured stacking velocities. Traveltime corrections or depth conversion based on this interval-velocity model improve the image of the data.

Fowler (1988) extended Toldi's work by deriving operators that relate a laterally varying interval-velocity model to the DMO-corrected stacking velocity or prestack time-migration velocity that best stacks or migrates horizontal and dipping reflectors. The interval-velocity model derived with this method can be used for post-stack or prestack depth migration.

When long-wavelength lateral velocity variation is large as in Figures 1.1–1.3 or when there are significant local lateral velocity variations, the velocity-analysis methods based on NMO or time migration fail for interval-velocity estimation. Strong lateral velocity variations cause moveout to be non-hyperbolic; DMO or prestack time migration do not perform true common-reflection-point gathering, and time migration does not adequately position events. Stacking or migration-velocity estimates, if interpreted as simple averages, can imply rapidly changing and possibly unphysical interval velocities. Moreover, since the positions of events are uncertain, it is difficult to unravel the relation between the observed moveout and interval velocity. Since only prestack depth migration performs adequate imaging when lateral velocity variation is significant, a velocity-analysis method that takes advantage of the properties of prestack depth migration is needed.

1.3 Tenets of velocity analysis

Before describing a method to combine prestack depth migration with velocity estimation, it is important to describe the foundation that underlies successful conventional velocity-estimation methods. Applying these tenets imparts robustness to velocity-estimation techniques.

1.3.1 Common-reflection-point gathering

Moveout or traveltimes should be analyzed in common-reflection-point gathers. Data converted to common-reflection-point format have traveltime differences over offset because the reflected waves travel along different paths from source to a single reflector point to receiver and encounter different velocities along those paths. They do not have traveltime differences because they travel to different reflection points in the subsurface. Stacking data reflected from the same point in the subsurface enhances the reflected signals relative to noise since the information contained by different offsets are presumably the same (ignoring AVO). Whether or not the images of a common reflection point formed from different offsets are the same allows us to tell if the velocity model is correct (Gardner et al., 1974; Al-Yahya, 1987). Reflected waves from different reflection points do not necessarily stack together coherently, so stacking without common-reflection-point gathering can degrade velocity analysis.

As an example, prestack time migration performs common-reflection-point gathering for arbitrary structure in a constant-velocity background model. Prestack depth migration performs common-reflection-point gathering for an arbitrary background model. NMO and CMP stacking only performs common-reflection-point gathering for horizontal reflectors beneath a laterally invariant overburden.

1.3.2 Velocity space

An effective way of performing velocity analysis with NMO, NMO and DMO, or prestack time migration is to apply the chosen imaging operator to the data for a range of velocities and organize the output data as a "cube," computing a coherence or stack-power spectrum versus velocity for all midpoints (Taner and Koehler, 1969). The output can be called a cube because it is often displayed a function of three variables, time, midpoint, and velocity. This organization of the output data space allows easy identification of desired and undesired events and allows spatial correlation of velocity information.

If a velocity-analysis cube is built using NMO or prestack time migration for a range of constant velocities, the stack power for any arbitrary NMO or prestack time-migration velocity field can be obtained by "slicing" the velocity cube. As demonstrated by Fowler (1984) and Shurtleff (1984), NMO or prestack time migration can be powerful interactive velocity-estimation and imaging methods. More advanced velocity-estimation methods like Toldi's and Folwer's that convert stacking or migration velocity to interval velocity using tomography need the velocity cube to evaluate rapidly the stack power or semblance implied by trial interval-velocity models.

Unfortunately prestack depth migration uses interval velocities that must be specified at every point in the subsurface. It would be computationally infeasible to build a multidimensional velocity spectrum consisting of all possible prestack depth migrations by allowing the interval velocity to cover a range of values at every point in the subsurface.

As suggested by Figure 1.4, it is possible to build a velocity-analysis cube *after* prestack depth migration.

1.3.3 Fixed events versus fixed positions

Moveout corrections applied by an imaging operator should be applied for fixed events rather than for fixed points in the subsurface. This results in a velocity-analysis cube where fixed events are stationary in space as velocity changes. For example, stacking-velocity analysis for a horizontal reflector is performed for fixed zero-offset reflection times, not for fixed depth points. The difference between fixed-event analysis and fixed depth-point analysis is most easily demonstrated using NMO. The NMO equation (equation (1.1)), is derived using the Pythagorean theorem given in equation (1.2).

$$t_h^2 = \frac{4}{v_s^2}(z^2 + h^2) \ , \tag{1.2}$$

followed by replacing $2z/v$ with t_0. It is entirely possible to perform a moveout correction using equation (1.2). This equation includes both the NMO of equation (1.1) and depth conversion with velocity v_s. If equation (1.2) is used for NMO correction, the maximum of stack power for a given event is more difficult to determine than using equation (1.1) since the event moves to a different depth for each velocity. More importantly, picking the maximum of stack power at fixed depths is incorrect because there might be several maxima in stack power for a given depth as different prominent reflectors map to that depth for different trial velocities.

Both NMO and NMO+DMO are performed for fixed zero-offset times and have the property of keeping fixed events at fixed output locations. Unfortunately, a velocity cube built with prestack time migration does not have this property. Although the position and pseudo-depth of horizontal events are unchanged by prestack time migration, the position, dip, and pseudo-depth of dipping events change as the velocity varies; this movement confuses velocity analysis. Fowler (1988) noticed this effect and recommended using DMO-corrected stacking velocities rather than prestack time-migration velocities for velocity analysis.

Prestack depth migration works directly with interval velocities in depth and positions events in depth. It is not clear how to keep the positions of events fixed as interval velocity changes. One possibility discussed by Popovici and Biondi (1989) builds a "depth-migration DMO" that performs the moveout correction and common-reflection-point gathering of prestack depth migration given some interval-velocity model but omits the positioning of reflectors and keeps the output at fixed zero-offset times.

1.3.4 Model-driven velocity analysis

Even though stacking velocities can often be determined robustly, the interval-velocity model implied by a set of picked stacking velocities may be infeasible if the picks are fit exactly. As many have noted, the inversion of stacking velocity to interval velocity is unstable. Toldi (1985) applied optimization theory to the problem of interval-velocity estimation from stacking velocities. Rather than pick stacking velocities and find the implied interval velocities, he used a model-driven approach that found an interval-velocity model that when used to apply NMO corrections, maximized the stack power of the events in the data. This approach can avoid undue influence of bad or inconsistent picks and produces a "reasonable" interval-velocity model that explains the observed stacking velocities.

1.4 Velocity analysis for prestack depth migration

Most interval-velocity estimation methods are based explicitly or implicitly on traveltime tomography. Conventional traveltime tomography (Bishop, et al., 1985) uses the traveltimes of picked events and ray tracing to find a velocity model and reflector positions that agree with the picked traveltimes. The velocity—reflector-depth ambiguity problem (Stork and Clayton, 1987) makes solving for both velocities and reflector depths difficult. To get more coherent images of reflectors, Stork (1988) used traveltime tomography combined with depth migration to estimate velocity models. Using depth migration makes reflector-depth location easier and more stable, but doesn't solve the greatest drawback of these approaches, the extensive traveltime picking required. Picking is labor intensive and prone to errors and is biased by the interpreter. It is my personal preference (influenced by Toldi and Fowler) to avoid directly picking moveout in the data or even picking peaks of semblance. Sometimes the extra knowledge or bias injected by picking is of great benefit; but then it is hard to tell how much the velocity model is constrained by the data and how much it is just invented by the interpreter.

Sword (1987) described another tomographic technique that picked traveltimes and step-outs automatically and used a tomographic inversion to estimate the velocity model. In an important improvement, he made reflector positions a function of the fixed data and the variable velocity model, avoiding many of the ambiguity problems with both picked reflectors and picked traveltimes. Biondi (1990) introduced an improved method for estimating step-outs with beam stacks and eliminated the need for picking altogether by using semblance as an objective function. Biondi's beam stack transform resembles a prestack time migration. A single transformation is applied to the data, and the results are interpreted much the same way Toldi's method interprets stacking velocity spectra. Sword's and Biondi's methods were more automatic than the method of Stork, but the stabilizing effects of migration were excluded from velocity analysis; migration was only applied after velocity estimation. Migration should be a part of velocity estimation since an accurately migrated image is what we hope to obtain from detailed interval-velocity analysis.

Faye and Jeannot (1986) combined depth-focusing analysis with prestack depth migration and introduced a powerful velocity-analysis tool. Based on the remarks of Doherty and Claerbout (1974) that a velocity error during downward continuation is similar to an error in imaging depth and imaging time, the method detects average velocity errors by imaging downward continued data not just at imaging time $t_i = 0$ but in a range $t_- < t_i < t_+$. If the velocity model is in error, events focus best at imaging times other than $t_i = 0$ and at depths other than their true depth $z = z_r$. The depth error measured from the focusing analysis can be related to an error in the average velocity. Missing from this approach is the tomographic connection between the error in the average velocity and the interval-velocity error. Because of this, the interval velocity is not updated automatically but by an interpreter, guided by the depth-focusing analysis. It would probably not be difficult to find a relation between interval velocity and depth-focusing error to make the method more automatic.

Van Trier (1990) developed a velocity-estimation method that used constant-offset residual event migration. The advantage of his method is its speed; event migration is much more economical than wave-field migration. Although picking is required, migrated events are picked rather than raw traveltimes. His method is highly interpretive and its advantages and disadvantages come from the large amount of human interaction involved.

The velocity-analysis method of this thesis combines elements of conventional velocity analysis with the work of Toldi, Fowler, and Biondi and uses prestack depth migration to estimate interval velocities in structurally complex areas when lateral velocity variation is strong. Since prestack depth migration cannot be used directly as a velocity-analysis tool like NMO or prestack time migration, velocity analysis must be performed after an initial prestack depth migration and if necessary, after subsequent prestack depth migrations.

1.4.1 Residual prestack migration

Chapter 2 derives a residual prestack-migration operator that makes it possible to evaluate rapidly the effect of a change in interval velocity on migrated images. Like conventional migration, residual migration can be a residual time migration or a residual depth migration. If the method is a residual depth migration, it will depend directly on the change in interval velocity; so it will still be infeasible to compute a velocity-analysis cube of all possible residual migrations. I derive the residual migration as a time migration, and like conventional prestack time migration it depends only one parameter called residual velocity.

Residual NMO+DMO

Residual prestack time migration can be separated into a sequence of three processes. The first parts are residual NMO and residual DMO. They perform the required moveout correction and common-reflection-point gathering. The last part of residual prestack time migration is exactly residual zero-offset migration (Rothman et al., 1985). After residual NMO and DMO, the data can be stacked since the residual zero-offset migration is the same for all offsets.

According to section 1.3, it is useful to ignore the movement of reflectors caused by residual zero-offset migration when building a velocity-analysis cube. Thus, like DMO-corrected stacking velocities, residual velocities are better measured for fixed events and than for fixed locations in the subsurface. Residual-velocity analysis is analogous to conventional velocity analysis, but it detects residual moveout over offset of events in common-reflection-point gathers after prestack depth migration. The true positions of reflectors given by the residual zero-offset migration are essential for converting residual velocities to interval velocities; so although I don't apply zero-offset migration during velocity analysis, it is necessary to keep track of the implied kinematic repositioning of reflectors.

1.4.2 Relating residual velocity to interval velocity

The relation between interval velocity and residual velocity described in Chapter 3 makes residual prestack migration useful for interval-velocity analysis. The relation is a filtered traveltime-tomography operator like those of Toldi, Fowler, and Biondi. The operator converts changes in the interval-velocity model used for prestack depth migration to changes in the residual velocity measured by residual prestack migration. The operator has two parts; the first part relates changes in interval velocity to changes in residual velocity for fixed depth points. The second part converts the changes in residual velocity for fixed depth points to the desired changes in residual velocity for fixed reflection events. The second part of the operator compensates for the movement of reflector images as the interval-velocity model changes.

1.4.3 Velocity analysis as an optimization problem

The operator of Chapter 3 is a forward-modeling or prediction operator; it predicts the residual velocity that describes the movement of reflector images after a change in interval velocity. Interval-velocity estimation is the reverse of this procedure; we wish to find a change to the interval-velocity model that explains the residual velocity measured after prestack depth migration. In Chapter 4, I pose interval-velocity estimation as an optimization problem. I find the change to the interval-velocity model that maximizes the sum over all reflectors of the stack semblance of each reflector at the residual velocity predicted for that reflector using the operator of Chapter 3. I solve this optimization problem with a conjugate-gradient method that needs the operator of Chapter 3 as well as a linearized version of the operator and its adjoint. The linearized operator is used for rapid computation of the effect of a small perturbation to the interval-velocity model. The adjoint of the linearized operator is used to compute the gradient of the objective function and is a tomographic back-projection operator.

When the velocity-analysis method is applied to data distorted by large lateral velocity variation, the interval-velocity model found by solving the optimization problem just described may not correctly image the data. However, as long as prestack depth migration using the new interval-velocity model improves the image; the velocity-analysis procedure can be restarted.

Summary of the velocity-analysis algorithm

The first step of the velocity-analysis procedure is to depth-migrate the constant-offset sections of the data with an estimate of the interval-velocity model. Then apply residual NMO+DMO for a range of residual velocities. The output velocity-analysis cube serves as the objective function for an iterative inversion of the operator of Chapter 3 to estimate a perturbation to the interval-velocity model that explains the observed residual moveout. Then depth-migrate the data with the new interval-velocity model and check for remaining residual moveout. Further iterations attempt to remove remaining residual moveout and correct the interval-velocity model.

1.5 Assumptions and terminology

Although waves that propagate through the Earth are inelastic, coupled, anisotropic, and attenuating, throughout this thesis I describe imaging methods for isotropic acoustic waves. The bulk of seismic data acquisition in structurally complex areas is designed to collect P–waves and discriminate against other wave types, and the near-vertical P–wave propagation that characterizes most seismic reflection data-acquisition geometries can be well approximated with an isotropic acoustic wave equation (Helbig, 1984) For computational reasons, I discuss only two-dimensional problems in this thesis, but the velocity-analysis concepts expounded here also apply to three-dimensional velocity analysis.

The prestack migration algorithms I use treat the wave field as if it were composed only of primary reflections; multiples should be removed before prestack migration. Moreover, the migration algorithms I use for velocity analysis treat the P–wave data using geometric optics which replaces wave calculations with simpler and more economical ray calculations.

Although this thesis deals with "velocity" analysis, it is often more convenient to use slowness (the reciprocal of velocity) in calculations rather than velocity itself. In particular, traveltime-tomography calculations are more appropriate in terms of interval slowness than interval velocity. Despite possible confusion, I treat slowness and velocity interchangeably, and at each step use whichever is most convenient for computations.

I mention several types of velocity (or slowness) throughout this thesis. I have already mentioned stacking velocity, migration velocity, and interval velocity. The distinctions are important so I reiterate them here. Interval velocity is a property of a rock that describes the speed of acoustic wave propagation in the rock. Migration or stacking velocities are not (except in special cases) the wave speed in subsurface rocks. Applying NMO and stack or time migration to data for a range of "velocities" fits a traveltime curve to the data. The velocity that best stacks or migrates an event is called its stacking velocity or time-migration velocity. These velocities really measure the shape of the traveltime versus offset or traveltime versus midpoint of reflected waves and are a function of the interval velocities of the rocks that the reflected wave transmits through.