Appendix D

B-spline parametrization of the velocity model

The medium interval velocity is described as the slowness function $S(x, z)$. I use slowness instead of velocity because ray tracing is more linear with respect to slowness than with respect to velocity. The model is defined as the sum of B-spline basis functions. The parameter $m_{i,j}$ of the slowness model $\mathbf{m}$ is the amplitude of the B-spline function centered at the knot $(\bar{x}_i, \bar{z}_j)$. The distances Δx and Δz between the knots determine the smoothness of the model.

The slowness at each point (x, z), with $\bar{x}_I \leq x < \bar{x}_{I+1}$ and $\bar{z}_J \leq z < \bar{z}_{J+1}$, can be evaluated by use of the following expression:

$$S(x,z) = \sum_{i=I-1}^{I+2} \sum_{j=J-1}^{J+2} m_{i,j}\, Fx(x - \bar{x}_i) Fz(z - \bar{z}_j), \tag{D.1}$$

where $Fx(x)$ and $Fz(z)$ are B-splines.

The horizontal B-splines $Fx(x)$ are piece-wise defined as follows:

$$Fx(x) = \begin{cases} 0 & if\ x < -2\Delta x \\ \frac{(2\Delta x + x)^3}{6\Delta x^3} & if\ -2\Delta x \leq x < -\Delta x \\ \frac{(-x)^3}{2\Delta x^3} - \frac{(-x)^2}{\Delta x^2} + \frac{2}{3} & if\ -\Delta x \leq x < 0 \\ \frac{(x)^3}{2\Delta x^3} - \frac{(x)^2}{\Delta x^2} + \frac{2}{3} & if\ 0 \leq x < \Delta x \\ \frac{(2\Delta x - x)^3}{6\Delta x^3} & if\ \Delta x \leq x < 2\Delta x \\ 0 & if\ 2\Delta x \leq x; \end{cases} \tag{D.2}$$

the vertical basis functions $Fz(z)$ are similarly defined.

The spatial derivatives of the model are easily computed because of the linearity of equation (D.1). The first derivative of slowness with respect to the horizontal coordinate is, for example,

$$S(x,z)_x = \sum_{i=I-1}^{I+2} \sum_{j=J-1}^{J+2} m_{i,j}\ Fx_x(x-\bar{x}_i)Fz(z-\bar{z}_j). \tag{D.3}$$

The slowness function and its derivatives are efficiently evaluated by use of tables for the values of the B-spline functions and their derivatives. More particulars on the properties of B-splines can be found in Inoue (1986).