Appendix A

Resolution of local stacks

Beam stacks and local slant stacks select the data in a gather according to the ray parameter of the recorded wavefronts. Ideally the stacked gather is different from zero only at the traveltimes and surface locations in which the recorded reflection has the desired ray parameter. In practice the stacked data are different from zero also in a region around the semblance peaks; the temporal and spatial extensions of this region are related to the time and spatial resolutions of the stacking operators. The time resolution depends on the duration of the wavelet, and the spatial resolution depends on the frequency of the signal, the length of the stacking trajectory, and on the local curvature of the wavefront.

In this appendix I derive the relations for estimating the spatial resolution of local stacking operators. The results are approximate, but they give a good idea of how resolution is related to the data quality and to the processing parameters.

In the first section the resolution in ray parameter is derived as a function of the frequency of the data and the length of the stacking array. In the second section the spatial resolution is related to the ray parameter resolution as a function of the curvature of the wavefront. Finally in the third section I derive an upper bound on the resolution of local slant stacks, caused by the curvature of the wavefront.

A.1 RAY-PARAMETER RESOLUTION OF LOCAL STACKS

When the ray parameters of plane waves are estimated with a stacking operator, the resolution depends on the frequency of the data and the length of the stacking array. The following derivation of resolution as a function of these parameters is a classical result of beamforming theory (Dudgeon and Merserau, 1984).

The data recorded from a monochromatic plane wave with ray parameter p_0, and frequency f_0 are

$$\text{Data}(t,x) = \cos\left[2\pi f_0(t - p_0 x)\right], \tag{A.1}$$

where t is the traveltime and x the surface location.

The value of the stack as a function of ray parameter computed at time $\bar{t} = 0$ for an array centered at surface location $\bar{x} = 0$ is

$$\begin{aligned} &\text{Stack}(p) = \\ &\frac{1}{L}\int_{\bar{x}-\frac{L}{2}}^{\bar{x}+\frac{L}{2}} \text{Data}\left[\bar{t} + p(x-\bar{x}), x\right] dx = \frac{1}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \cos\left[2\pi f_0(p-p_0)x\right] dx = \\ &\frac{\sin\left[2\pi f_0(p-p_0)L\right]}{2\pi f_0(p-p_0)L} \end{aligned} \tag{A.2}$$

The stacking spectrum is equal to

$$S_{\text{Stack}}(p) = |\text{Stack}(p)|^2 = \frac{\sin^2\left[2\pi f_0(p-p_0)L\right]}{(2\pi f_0(p-p_0)L)^2} \tag{A.3}$$

The maximum of the stacking spectrum is at $p = p_0$ and the zeros are at $p = p_0 \pm j(1/f_0 L)$. Figure A.1 shows an example of a stacking spectrum for a monochromatic plane wave with frequency $f_0 = 25$ Hz, and ray parameter $p_0 = .25$ s/km; the plane wave was recorded by an array 400 m long.

The stacking spectrum resolves two plane waves if their ray parameters differ by at least the half width of the main lobe of the stacking spectrum. Therefore the resolution of the stacking spectrum is

$$\gamma = \frac{1}{\Delta p} = f_0 L. \tag{A.4}$$

This relation is approximately valid also for wide-band data when f_0 is the central frequency in the data and when semblance is used instead of simple stacking.

In the next section I analyze the resolution of local stacks applied to wavefronts not having linear moveout. The expression in equation (A.4) is still approximately valid in this case, but only under the assumption that the stacking trajectories well approximate the real moveouts in the data. In the last section I consider the case in which this assumption

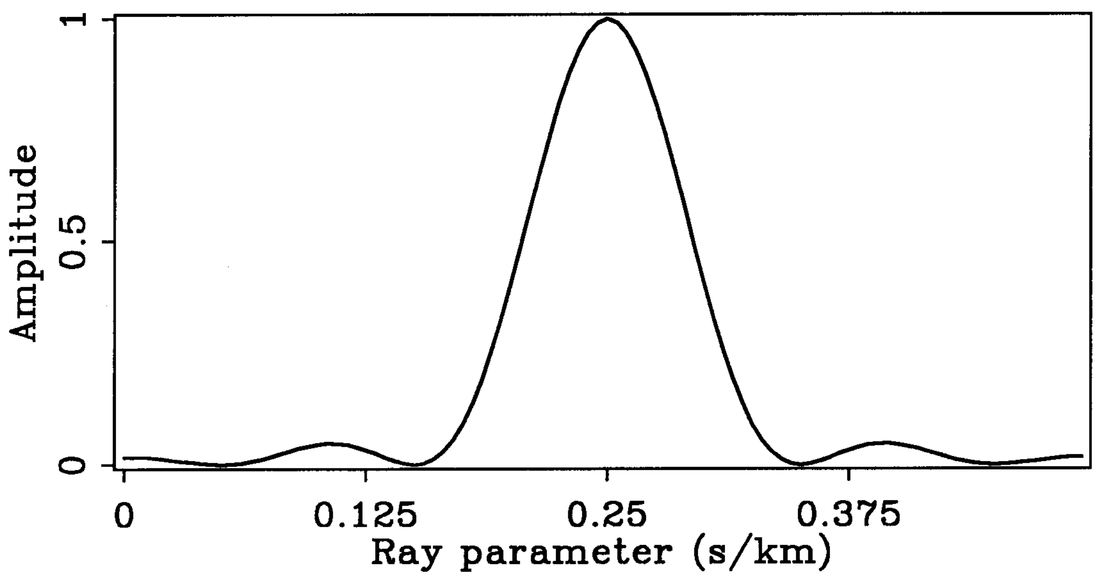

FIG. A.1. The stacking spectrum for a monochromatic plane wave with frequency $f_0 =$ 25 Hz, and ray parameter $p_0 = .25$ s/km; the plane wave was recorded by an array 400 m long. The maximum of the spectrum is at $p = p_0$ and the zeros are at $p = p_0 \pm j(1/f_0 L)$.

does not hold.

A.2 SPATIAL RESOLUTION OF LOCAL STACKS

The spatial resolution of local stacks is given by the width of the semblance peaks in a stacked gather. The width of the semblance peaks depends not only on the ray-parameter resolution of the stacking operator expressed in equation (A.4), but also on the rate of variation of the ray parameter with the surface coordinate x. In first approximation the spatial width of the semblance peaks Δx is related to Δp by

$$\Delta p = \Delta x \frac{dp}{dx} = \Delta x \frac{d^2 \mathrm{Time}(x)}{dx^2} = \Delta x \rho, \tag{A.5}$$

where Time(x) is the moveout of a reflection and ρ is its second derivative with respect to x. The parameter ρ is related to the curvature of the wavefront.

The spatial resolution of a stacking operator with ray-parameter resolution γ can then be expressed as

$$\phi = \frac{1}{\Delta x} \approx \rho \frac{1}{\Delta p} = \rho\gamma = \rho f_0 L. \tag{A.6}$$

The last equation shows that the spatial resolution of stacking operators is directly proportional to the curvature of the wavefront, the frequency of the data, and the length of the stacking trajectory.

If the traveltime curves are well approximated by Dix's hyperbolas the parameter ρ can be analytically derived. From equation (2.10) in the main text, we find that

$$\rho_h = \frac{dp}{dx} = \frac{1}{V^2 \bar{t}}, \tag{A.7}$$

where V is the velocity of the unique Dix hyperbola passing through the point $(\bar{t}, \bar{x})$ with ray parameter p. Substituting equation (A.7) in equation (A.6) I obtain the following expression for the resolution of local stacks,

$$\phi_h \approx \gamma\rho = \frac{f_0 L}{V^2 \bar{t}}. \tag{A.8}$$

As expected, equation (A.8) shows that the resolution of the local stack operator decreases with time and with the velocity of the hyperbola.

Figure A.2 shows the semblance functions computed by applying beam stacks to two hyperbolas with three different lengths for the stacking trajectories. The velocities of the hyperbolas are $V_1 = 2.2$ km/s and $V_2 = 2.5$ km/s. The central frequency of the wavelet is $f_0 = 50$ Hz. The widths of the semblance peaks are approximately predicted by equation (A.8).

A.3 THE UPPER BOUND IN THE SPATIAL RESOLUTION OF LOCAL SLANT STACKS

In the previous section I derived a relation with which the resolution of stacking operators could be estimated, under the assumption that the stacking trajectories well approximate the moveout of the reflection. For local slant stacks, the assumption is not valid when the stacking trajectories are too long, because the "effective" length is limited by the curvature of the wavefronts. In this section I derive an upper bound on the "effective" length of the

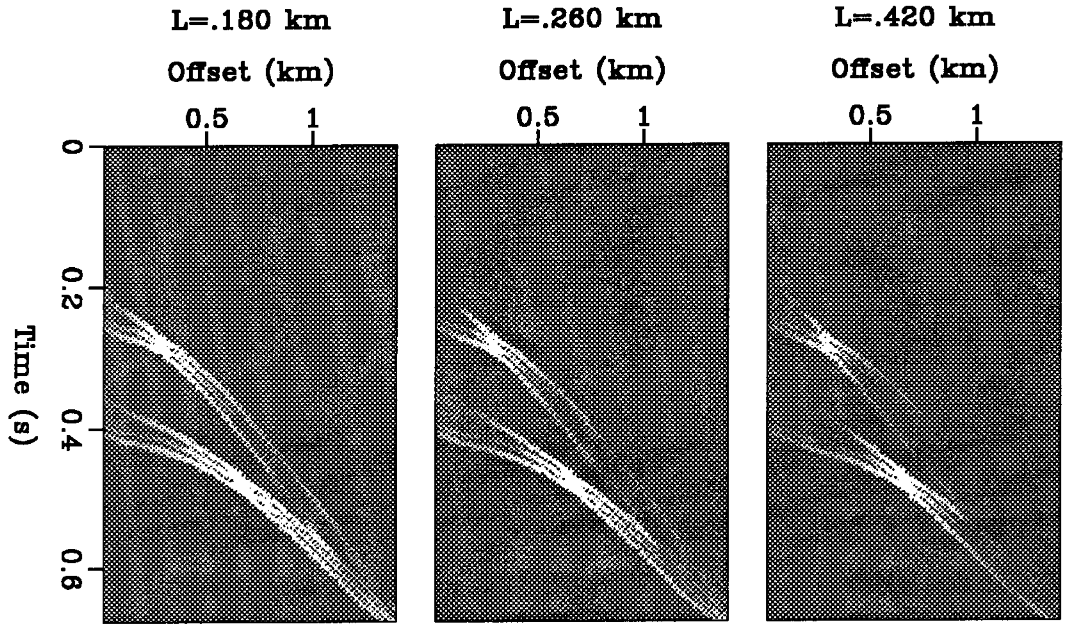

FIG. A.2. Semblance panels computed by applying beam stacks to two hyperbolas with three different lengths L for the stacking trajectories. The velocities of the hyperbolas are $V_1 = 2.2$ km/s and $V_2 = 2.5$ km/s. The central frequency of the wavelet is $f_0 = 50$ Hz.

stacking trajectories of local slant stacks, and consequently I derive an upper bound on the spatial resolution of local slant stacks.

The limit in the length of the stacking trajectories depends on the admissible time difference between the data moveouts and the stacking trajectory. The choice of the maximum time error is quite arbitrary; I constrain the time error $t_{\rm err}$ to be such that the correspondent phase errors at the extremes of the trajectory turn out to be lower than π, that is

$$t_{\rm err}\left(x = \bar{x} \pm \frac{L}{2}\right) \leq \frac{1}{2f_0}. \tag{A.9}$$

Because data moveouts can be well approximated by parabolas, the time error is

$$t_{\rm err}(x) \approx \left[\bar{t} + p(x - \bar{x}) + \frac{\rho(x - \bar{x})^2}{2}\right] - [\bar{t} + p(x - \bar{x})] = \frac{\rho(x - \bar{x})^2}{2}. \tag{A.10}$$

Consequently the upper bound on the length of the stacking trajectories is

$$L_{\mathrm{lss}} \leq \sqrt{\frac{4}{\rho f_0}}. \tag{A.11}$$

Substituting this value of the length of the stacking trajectory in equation (A.6), I get the following upper bound on the resolution achievable by local slant stacks:

$$\phi_{\mathrm{lss}} \leq 2\sqrt{\rho f_0}. \tag{A.12}$$

Comparing this relation with the theoretical limit for the resolution of local stacking operators [equation (A.6)], we appreciate the advantage of beam stacks over local slant stacks.

When the hyperbolic assumption is valid, the parameter ρ_h given in equation (A.7) can be substituted into the last equation, and the bound on the resolution becomes

$$\phi_{\mathrm{lss-h}} \leq \frac{2}{V}\sqrt{\frac{f_0}{t_0}}. \tag{A.13}$$

Figure A.3 shows the semblance panels computed by slant stacking the same hyperbolas used for the beam stacks shown in Figure A.2; three different lengths for the stacking trajectories were used. The resolution of the slant stacks does not increase monotonically with the stacking trajectory length because of the limit derived in equation (A.13).

A.4 CONCLUSIONS

In this appendix I derived approximate expressions that relate the resolution of local stacking operator to the frequency of the data, the curvature of the wavefronts, and the effective length of the stacking trajectories.

The effective length of the stacking trajectories of local slant stacks is limited by the wavefront curvature, and thus there is an upper bound on the resolution that can be achieved by slant stacks. On the contrary, beam stacks do not suffer this limitation in resolution.

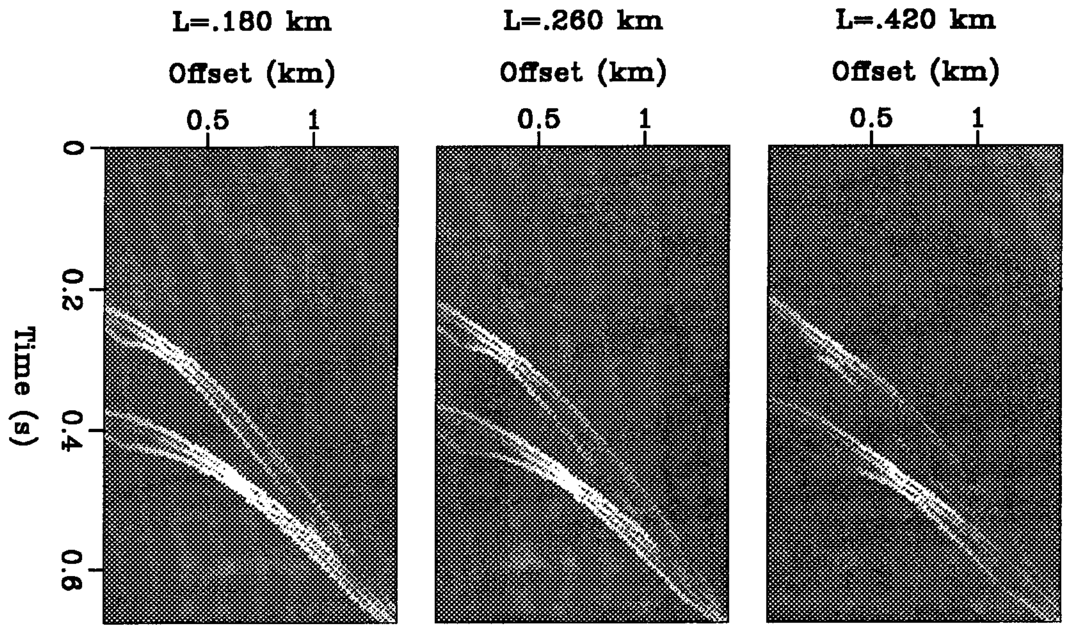

FIG. A.3. The semblance panels computed by slant stacking the same hyperbolas used for the beam stacks shown in Figure A.2; three different lengths L for the stacking trajectories were used. The resolution of the slant stacks does not increase monotonically with the stacking trajectory length because of the limit derived in equation (A.13).