Appendix D

Stacking in terms of the data-covariance matrix

For stacking along a time-invariant family of trajectory, Fourier Transform of the data over the time axis has several advantages. The frequencies are decoupled and inversion can be done frequency per frequency. Convolution over time, as required for a time-shift, can be applied accurately and rapidly in the frequency domain. For a data sequence of length N and a filter of length M, the ratio between floating point operations for a convolution with a Fast Fourier Transform and for a convolution in the time domain is approximately $\log(N)/M$. Thus for relatively long data sequences and filters, convolution in the frequency domain is considerably faster.

Measuring the power along hyperbolic trajectories parametrized by depth of source and velocity, provides a method for velocity analysis that is complementary to the interactive picking method presented above (Figure 3.3).

The coherency spectrum $S(z, v)$, computed by stacking along trajectories parametrized by depth z and velocity v, is given by the following expression (Equation 3.3):

$$S(z,v) = \frac{1}{M \times T} \sum_{t_0=0}^{T} \left| \sum_{x} d\left(x, t_0 + \frac{\sqrt{x^2 + z^2}}{v} - \frac{z}{v}\right) \right|^2, \tag{D.1}$$

where T is the number of samples along the time axis and M is the number of traces.

Applying Parseval's theorem (Papoulis, 1965), the above spectrum can be rewritten also in terms of the frequency components $d(x,\omega)$ of the data:

$$S(z,v) = \frac{1}{M \times T} \sum_{\omega} \left| \sum_{x} d(x,\omega) e^{j \frac{\omega\sqrt{x^2+z^2}}{v}} \right|^2 . \tag{D.2}$$

An alternative expression for the contribution of a particular frequency ω to the coherency spectrum can be given in terms of the covariance matrix of the data $R_d(\omega)$, defined as the outer product of the data vector $D(\omega)$, whose components are $d(x,\omega)$ (Biondi and Kostov, 1989):

$$R_d(\omega) = E(D(\omega) D^H(\omega)),$$

and

$$S(z,v) = \sum_{\omega} \mathbf{u}^H R_d(\omega) \mathbf{u}, \tag{D.3}$$

with the components of the steering vector $\mathbf{u}$ being equal to $\{e^{j \frac{\omega\sqrt{x^2+z^2}}{v}}\}_x$. For drill-bit data the expectation operator in the definition is replaced by the average of covariance matrices computed in windows of data.

Computation of the spectrum by means of the data-covariance matrix might (depending on the number of data and on the dimensions of the spectrum) decrease significantly the cost of the computations with respect to directly applying Equation D.2.