WAVE-EQUATION TOMOGRAPHY

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ABSTRACT

Seismic tomography reconstructs velocity fields from integrals over paths through the fields. Tomography is most familiar in its ray-theoretic form as traveltime inversion, where the integrals are traveltimes over raypaths. In this formulation, traveltimes are measured for each source-receiver experiment, and expected traveltimes are calculated by ray tracing through an assumed background velocity field. An updated velocity field is produced by projecting the differences between the measured and expected traveltimes (the traveltime delays) back through the medium over the traced raypaths.

Ray-trace tomography works well when two requirements are met. First, because the method relies on the high-frequency assumption of ray theory, the velocity field being examined must vary slowly on the scale of the source wavelengths. In this case, there is no scattering: phase delay is linear with frequency and the source wavelet is not distorted. Second, because without scattering rays sample very narrow regions in space, the source-receiver geometry must provide many view angles through the medium. When these requirements are not met, wave-theoretic tomography provides a better image.

Wave-theoretic tomography accommodates scattering by replacing traveltime delays with scattered wavefields. The wavefields are backpropagated through the medium, using a Born or Rytov linearization of the scalar wave equation. Wave-theoretic tomography is usually formulated in the frequency-wavenumber domain, under the title of diffraction tomography. This thesis reformulates the method in the frequency-space domain, under the title of wave-equation tomography. Wave-equation tomography is shown to project monochromatic, scattered wavefields back over source-receiver *wavepaths*, just as ray-trace tomography projects traveltime delays back over source-receiver *raypaths*.

I have two purposes in reformulating diffraction tomography in the space domain. First, the method becomes more flexible in dealing with irregularly sampled surveys and inhomogeneous background media. Second, and more importantly, a comparison of single source-receiver wavepath and raypath backprojection patterns clarifies the relation between wave-theoretic and ray-theoretic techniques. The comparison shows that wave-theoretic tomography is closely related to migration under the Born approximation, and to ray-theoretic transmission tomography under the Rytov approximation. Examined under the linear-phase-delay assumption of ray theory, Rytov wavepaths are identified as monochromatic raypaths, and traditional raypaths as wavepaths averaged over an infinite bandwidth. ***Bandlimited raypaths*** are introduced as wavepaths averaged over a finite bandwidth, graphically linking rays and waves through the uncertainty relation.

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