Gallery of the double-square-root equation

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ABSTRACT

The program *Mathematica* is useful for making tutorials of mathematical relations. Here are pictures of solutions to the double-square-root equation (DSRE): the pyramid, radial traces, and Snell's bat.

INTRODUCTION

We bought the symbolic mathematics system called *Mathematica* (Wolfram, 1988) because it passed an essential test— it could automatically solve for variables inside the roots of the double-square-root equation. It's competitor, *MACSYMA* could not. The DSRE gives the travel time between separated sources and receivers (Claerbout, 1985). It is an essential function of much past and current SEP research (Etgen, 1988).

Though the main use of *Mathematica* is as an interactive algebraic calculator, it is also a programming language (Nichols and Ottolini, 1988) and facilitates the writing of mathematical tutorials. This article demonstrates the latter. Useful tutorial tools include (1) a built-in outline utility called *notebooks*, and (2) powerful (though slow[1]) three dimensional graphics.

Each of the following sections explores an aspect of the double-square-root equation of interest to me. It contains an interesting figure generated by *Mathematica*, the mathematical expression that generated it, and the mathematical rules that derived the expression from the DSRE.

One final note—this document could have been written entirely within the *Mathematica* system. *Mathematica* notebooks have *styles* for formatting titles, texts, equations, and pictures. These could be customized to SEP report format rules,

[1]We currently operate *Mathematica* on 4MB Macintosh IIs, and hope to have a faster NeXT workstation shortly.

but I didn't have the time to do this. This has been partly done in Nichols and Ottolini (1988).

THE TRAVEL TIME PYRAMID

We begin with the double-square-root equation in *Mathematica* notation

```
DSRE = v t == Sqrt [ z^2 + (x+h)^2 ] + Sqrt [ z^2 + (x-h)^2 ] .   (1)
```

where v is velocity, z is depth of a point scatterer, x is the distance of the shot-receiver midpoint from the point scatterer, and $2h$ is the shot-receiver separation. [In *Mathematica* '==' means an equation anf '=' means a definition.] Then we solve for t (trivial), set constants, and plot the time surface sampled in offset and midpoint.

```
Solve [ DSRE, t ] ,
v = 1. ,
z = 100. ,
xmin = -800 ,
xmax = 800 ,
hmin = -800 ,
hmax = 800 ,
Plot3D [ t, {x,xmin,xmax}, {h,hmin,hmax} ] .
```

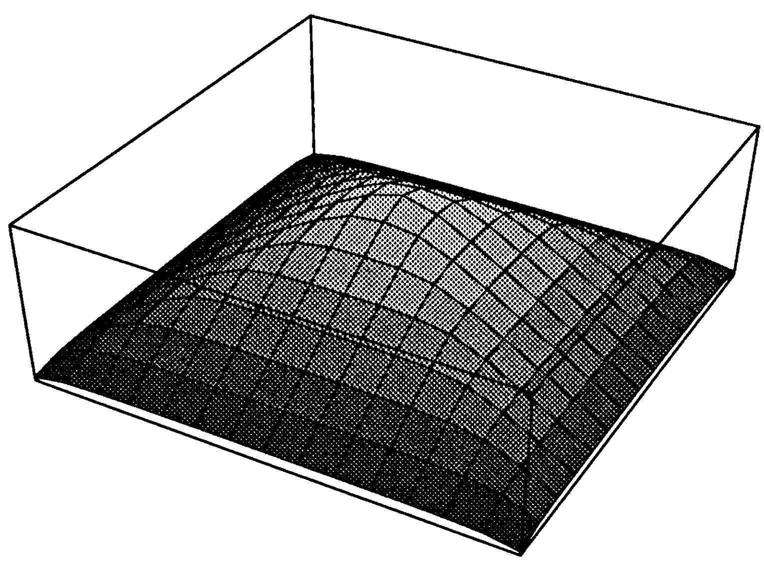

FIG 1. A plot of the DSRE travel time pyramid.

RADIAL TRACE HYPERBOLAS

Ottolini, 1983, discovered diagonal cross sections through the DSRE pyramid were hyperbolas. This suggested new methods of seismic data processing. The radial curve is the intersection of the DSRE with the equation, $h = rt$, where r is the *radial parameter*. This is computed in *Mathematica* by

```
Solve [ Eliminate [ { DSRE, h==r*t }, h ] , t ] .
```

The result is

```
t -> 2 / v * Sqrt [ x^2 + z^2 / (1 - 4 r^2 / v^2 ) ] .      (2)
```

This is the equation of a hyperbola. [In *Mathematica* '$->$' means a rule.]

Plotting a set of radial hyperbolas shows that they are indeed the cross section of diagonal planes and the DSRE pyramid. First, equation (2) is tabulated along x to obtain a set of coordinates. These coordinates are converted into the $(x, h = rt, t)$ coordinates of Figure 1.

```
curve [r_] := Table[ {x, r*t, t}, {x, xmin, xmax, nx} ] .
```

[In *Mathematica* ':=' means a function definition and the underscore means internal variables.] Next render the curve as a shaded polygon to improve visibility:

```
radpoly [r_] := Polygon [ curve [r] ] .
```

Then plot a set of curves for a range of radial parameters

```
rmin = -.45 ,
rmax = .45 ,
rdelta = .09 ,
Show [ Graphics3D [ Table [ radpoly [r], {r,rmin,rmax,rdelta} ]]]
.
```

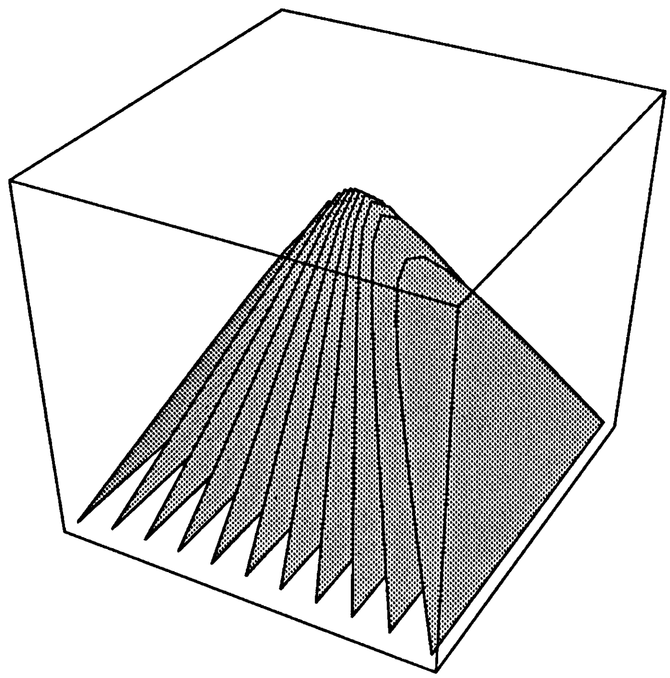

FIG 2. A plot of radial trace hyperbolas.

SNELL'S BAT

Snell's bat is a plot of the DSRE in depth-variable velocity media. This object is used to study the depth-variable velocity radial traces, called *Snell traces* by Ottolini (1983). Chuck Sword thought this function looked like a bat.

Slotnick (1959) derived the travel time for a linear increase of velocity with depth, $v = v_0 + az$.

```
t[x_,z_] := Arccosh [ 1 + a^2(x^2 + z^2) / (2 v0(v0 + a z)) ] / a    (3)
```

The total travel time is the sum of the source leg contribution at $x - h$ and the receiver at $x + h$.

```
total_time = t[x-h,z] + t[x+h,z]                                     (4)
```

A plot of this function is given by

```
v0 = .5 ,
a = .1 ,
Plot3D [ total_time, {x,xmin,xmax}, {h,hmin,hmax} ] .
```

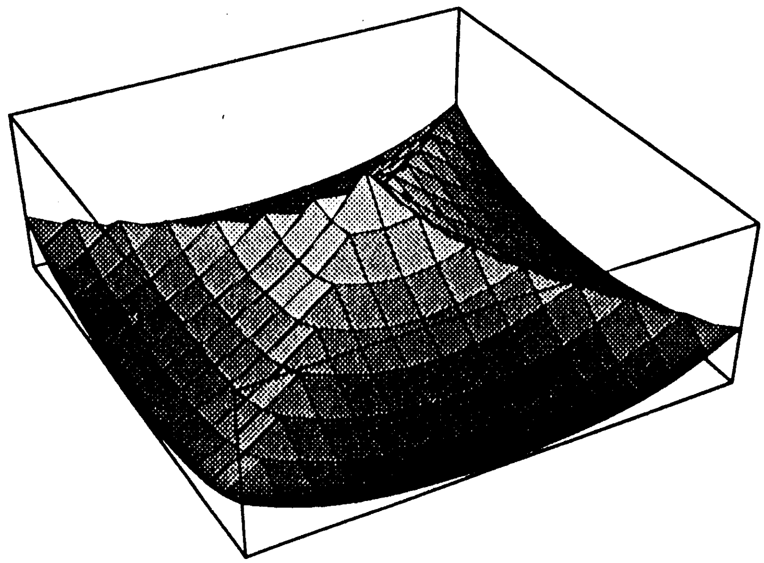

FIG 3. A plot of Snell's bat.

REFERENCES

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Ocean Listen Hexagon Array