A new Jacobian for dip-moveout

Lin Zhang

ABSTRACT

It is well known that the dip-moveout correction removes reflector point dispersal from prestack data. A new derivation of the DMO operator shows this fact explicitly. The impulse response of the new operator has an amplitude response that is qualitatively consistent with wave theory. Synthetic results show that after stacking, the new DMO-corrected data tends to have more uniform amplitudes for events of varying dips.

INTRODUCTION

Dip-moveout is an effective post-NMO process when dipping events are present. Deregowski and Rocca (1981) describe the DMO operation as common offset migration followed by zero offset diffraction. This definition does not have the plane reflector assumption. They mention that, according to wave theory, the amplitude of the operator should increase along the impulse response at large offsets and small travel times. Deregowski (1982) states in his paper that DMO removes reflector point dispersal. He shows that, for a plane dipping reflector under a constant velocity overburden, the DMO operator moves dipping energy on common offset gathers in such a manner that common midpoint gathers become true common depth point gathers. Hale's work (1983) provides an efficient way to do dip correction under the constant velocity assumption. He shows that under these conditions his DMO operator is schematically correct. Several important features of DMO, however, are not obvious from his derivation.

To derive the DMO operator in a way that shows the removal of reflector point dispersal explicitly, I worked out a pair of equations that relate the constant offset travel time and midpoint to the zero offset travel time and midpoint corresponding to a common depth point. By following Hale's approach I obtained a new operator. As expected, the new operator has the same phase response as Hale's operator but different amplitude response because of the different Jacobian involved. I noticed

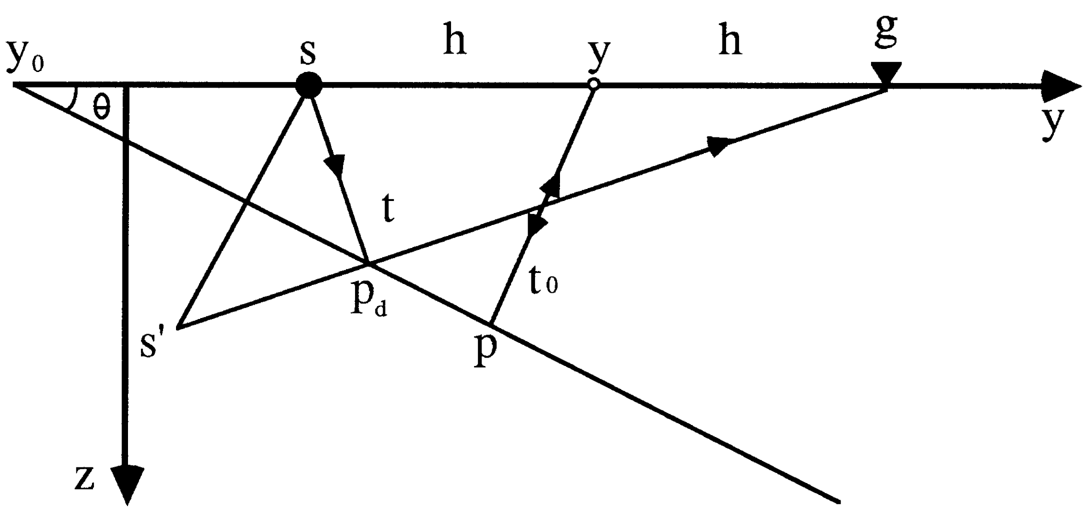

FIG. 1. Common midpoint geometry.

that the impulse response of the new operator has properties that agree qualitatively with wave theory. I used synthetic results to show that the new operator tends to give a uniform amplitude response for varying dips while Hale's operator has a lowpass dip filtering effect.

In the following sections, I compare the derivations of these two operators and then use synthetic examples to confirm my results.

THEORY

The derivation of the new operator is so similar to what Hale did that I will review his results first.

Hale's DMO

Let's begin with some notation. Let $P(t, y, h)$ represent recorded data, $P_n(t_n, y, h)$ represent the NMO corrected data, and $P_0(t_0, y, h)$ represent the NMO and DMO corrected data.

From the geometry shown in Figure 1, the relation between the travel time for zero offset and the travel time for a half offset h at a common midpoint is found to be:

$$t_0 = \left(t^2 - \frac{4h^2}{v^2} + \frac{4h^2 \sin^2\theta}{v^2}\right)^{\frac{1}{2}}, \tag{1}$$

$$t_n = \left(t^2 - \frac{4h^2}{v^2}\right)^{\frac{1}{2}}, \tag{2}$$

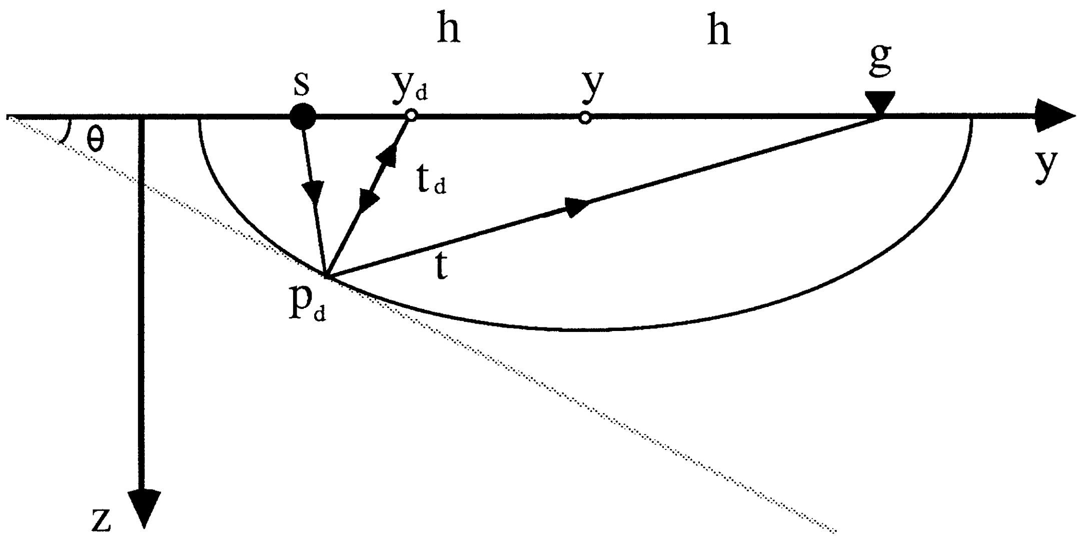

FIG. 2. Common depth point geometry.

where θ is the dip angle of the subsurface. By substitution, Hale derives his DMO to be:

$$P_0(\omega_0, k, h) = \int \int P_n(t_n, y, h)|J|e^{j\Phi} dt_n dy, \tag{3}$$

$$P_0(t_0, y, h) = FT^{-1}\left[P_0(\omega_0, k, h)\right], \tag{4}$$

where $|J|$ is the Jacobian and Φ is the phase shift,

$$|J| = \frac{1}{\sqrt{1 + \frac{h^2k^2}{\omega_0^2 t_n^2}}},$$

$$\Phi = \omega_0 t_n \sqrt{1 + \frac{h^2k^2}{\omega_0^2 t_n^2}} - ky.$$

Equations (1) and (2) relate two travel times at a common midpoint, the zero offset travel time t_0 and the constant offset travel time t. From Figure 1 it is clear that the events associated with these two travel times are not from the same reflection point. This is why the removal of reflector point dispersal is not obvious.

A new derivation

To have true common depth point gathers we need to make both time and midpoint corrections to the dipping events on common offset gathers. From Figure 2 we can find the equations relating the travel time t and midpoint y of half offset h to the zero offset travel time t_d and midpoint y_d, with the dip angle θ as a parameter.

$$t_d = \frac{t_n}{\sqrt{1 + \frac{4h^2 \sin^2 \theta}{v^2 t_n^2}}}, \tag{5}$$

$$y_d = y - \frac{\frac{2h\sin\theta}{vt_n}}{\sqrt{1+\frac{4h^2\sin^2\theta}{v^2t_n^2}}}h, \tag{6}$$

where t_n is defined in equation (2). Now the two travel times are associated with a common reflector point and θ is the dip angle at this common reflector point.

Using the formula

$$\frac{2\sin\theta}{v} = \frac{dt_d}{dy_d} = \frac{k_d}{\omega_d}, \tag{7}$$

where ω_d is the frequency corresponding to t_d and k_d is the wavenumber corresponding to y_d, we have

$$t_d = \frac{t_n}{\sqrt{1+\frac{h^2k_d^2}{\omega_d^2t_n^2}}}, \tag{8}$$

$$y_d = y - \frac{\frac{hk_d}{\omega_d t_n}}{\sqrt{1+\frac{h^2k_d^2}{\omega_d^2kt_n^2}}}h. \tag{9}$$

Now we can derive the new DMO integral. Let $P_d(t_d, y_d, h)$ be the new dip-moveout corrected data,

$$\begin{aligned} P_d(\omega_d, k_d, h) &= \int\int P_d(t_d, y_d, h)e^{j\omega_d t_d - jk_d y_d}dt_d dy_d \\ &= \int\int P_n(t_n, y, h)|J|e^{j\Phi}dt_n dy \end{aligned} \tag{10}$$

and

$$P_d(t_d, y_d, h) = FT^{-1}\left[P_d(\omega_d, k_d, h)\right], \tag{11}$$

where $|J|$ is the Jacobian and Φ is the phase shift

$$|J| = \frac{1+2\frac{h^2k_d^2}{\omega_d^2t_n^2}}{(1+\frac{h^2k_d^2}{\omega_d^2t_n^2})^{\frac{3}{2}}},$$

$$\Phi = \omega_d t_n\sqrt{1+\frac{h^2k_d^2}{\omega_d^2t_n^2}} - ky.$$

Comparing these results with Hale's DMO we see that the phase shifts are identical but the Jacobians differ by a factor of

$$\frac{1+2\frac{h^2k_d^2}{\omega_d^2t_n^2}}{1+\frac{h^2k_d^2}{\omega_d^2t_n^2}},$$

which can be recognized as a time and offset dependent highpass dip filter.

The above derivation also serves as another way to show that:

1. DMO removes reflector point dispersal.
2. DMO works for any curved beds that have reflections consistent with ray theory.

Impulse responses

We can obtain the impulse response of the DMO operator by setting $P_n(t_n, y, h)$ to be an impulse; the results are shown in Figures 3 and 4. At large offsets the amplitude of the operator increases along the impulse response at small travel time.

SYNTHETIC RESULTS

I generated a synthetic example to confirm the theory. The synthetic section consists of a horizontal reflector and a dipping reflector. The velocity of the medium is assumed to be constant. The results are shown in Figure 5. For the horizontal reflector the results of two operators are the same but for the dipping reflector the new operator gives larger amplitude than does Hale's operator. The new operator, therefore, tends to give uniform amplitudes for varying dips.

CONCLUSION

This paper presented a new derivation of the DMO operator. Theoretically the new operator is more consistent with wave theory, but in practice, it would not do significantly better than Hale's DMO. Nevertheless, the derivation itself provided another way to show several properties of the DMO operation.

ACKNOWLEDGEMENTS

I acknowledge the support of the sponsors of the Stanford Exploration Project (SEP) and its director, Jon Claerbout.

REFERENCES

Claerbout, J.F., 1985, Imaging the Earth's Interior: Blackwell Scientific Publications.

Deregowski, S.M., and Rocca, F., 1981, Geometrical optics and wave theory of constant offset sections in layered media: Geophysical Prospecting, **29**, 384-406.

Deregowski, S.M., 1982, Dip-moveout and reflector point dispersal: Geophysical Prospecting, **30**, 318-322.

Hale Ira David, 1983, Dip-moveout by Fourier Transform: Ph.D. thesis, Stanford University.

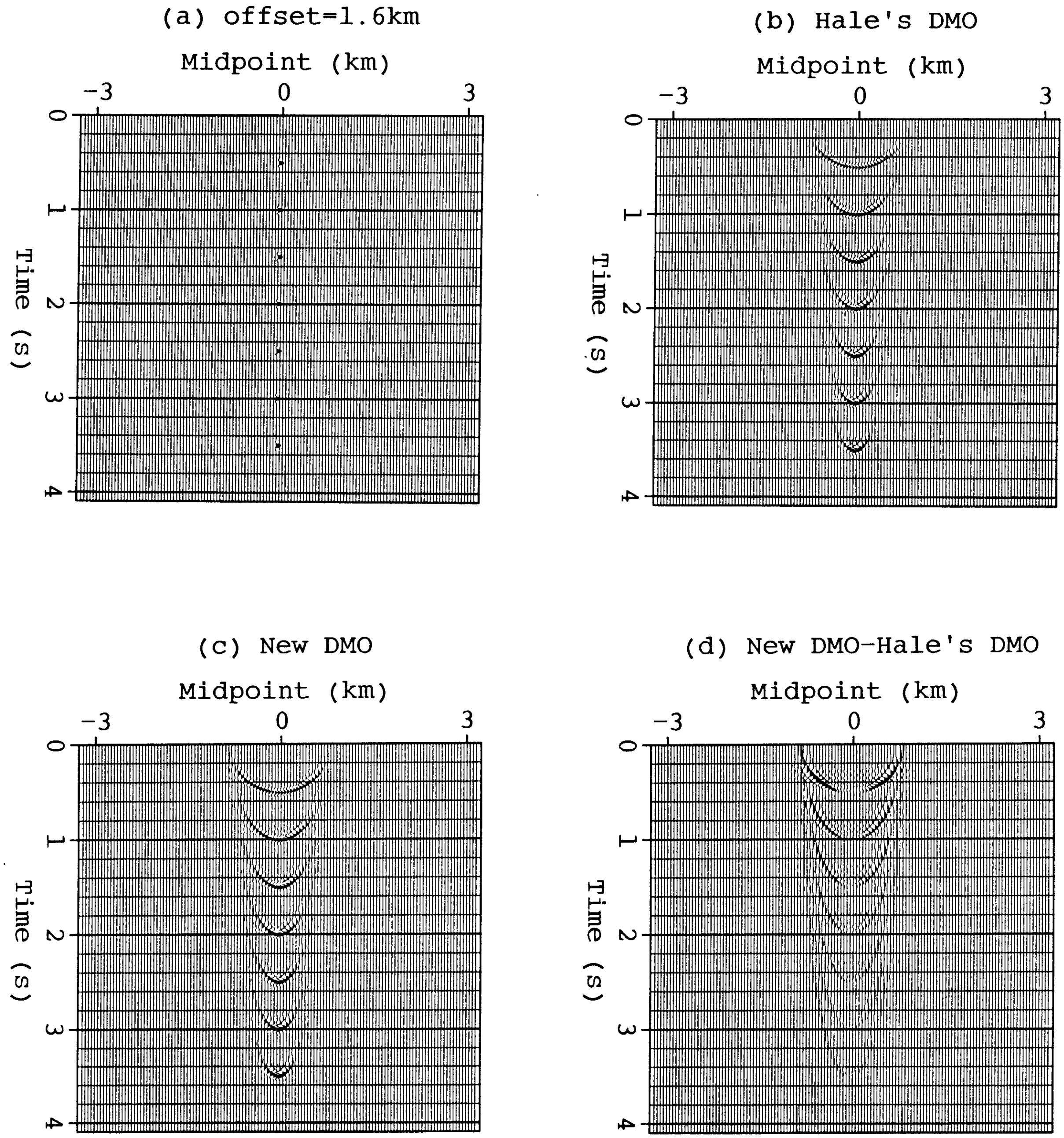

FIG. 3. The impulse responses of DMO operator at offset h=1.6km. (a) Impulses in the NMO corrected common offset section. (b) The impulse responses of Hale's operator. (c) The impulse responses of the new operator. (d) (c)-(b).

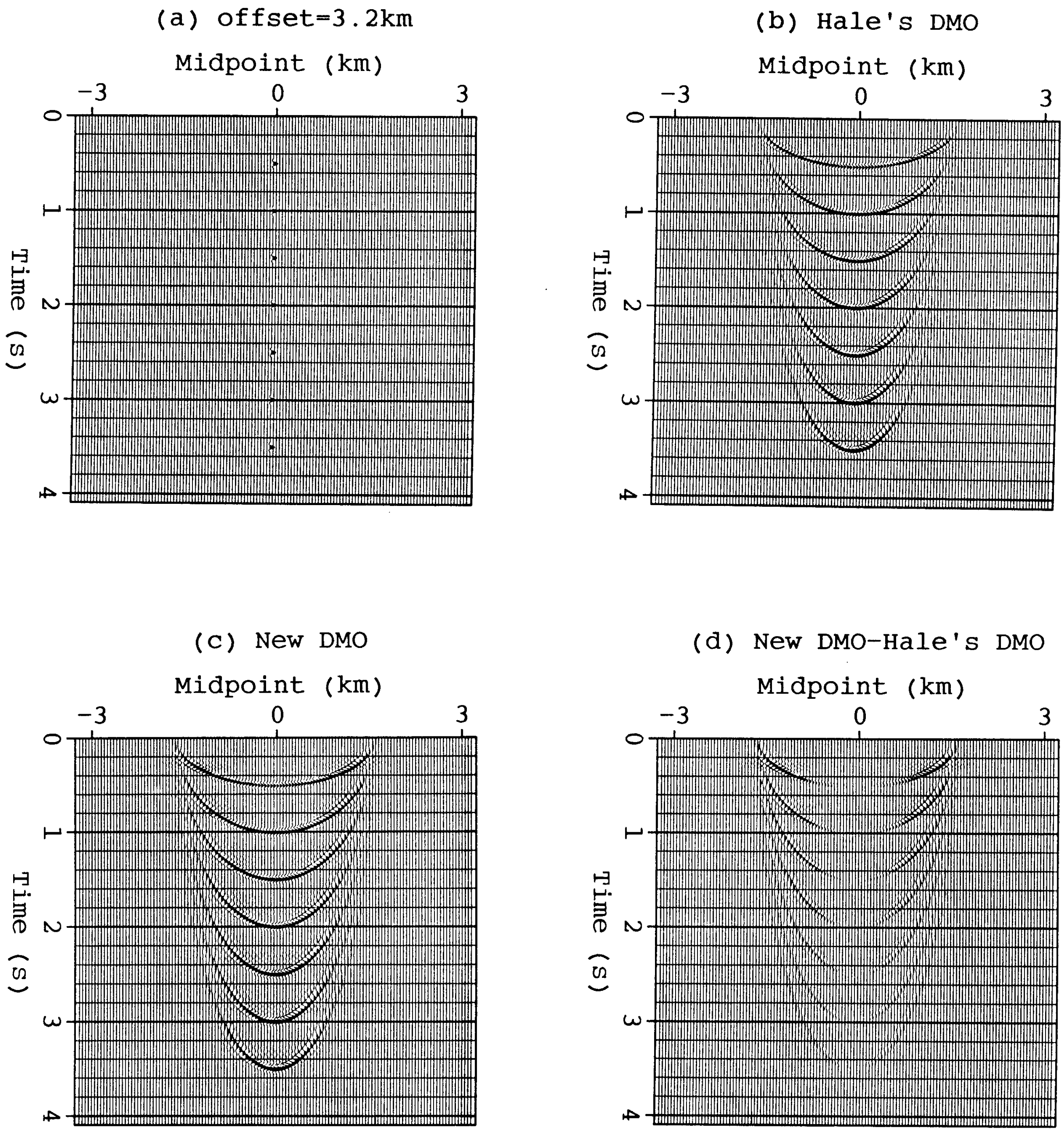

FIG. 4. The impulse responses of DMO operator at offset h=3.2km. (a) Impulses in the NMO corrected common offset section. (b) The impulse responses of Hale's operator. (c) The impulse responses of the new operator. (d) (c)-(b).

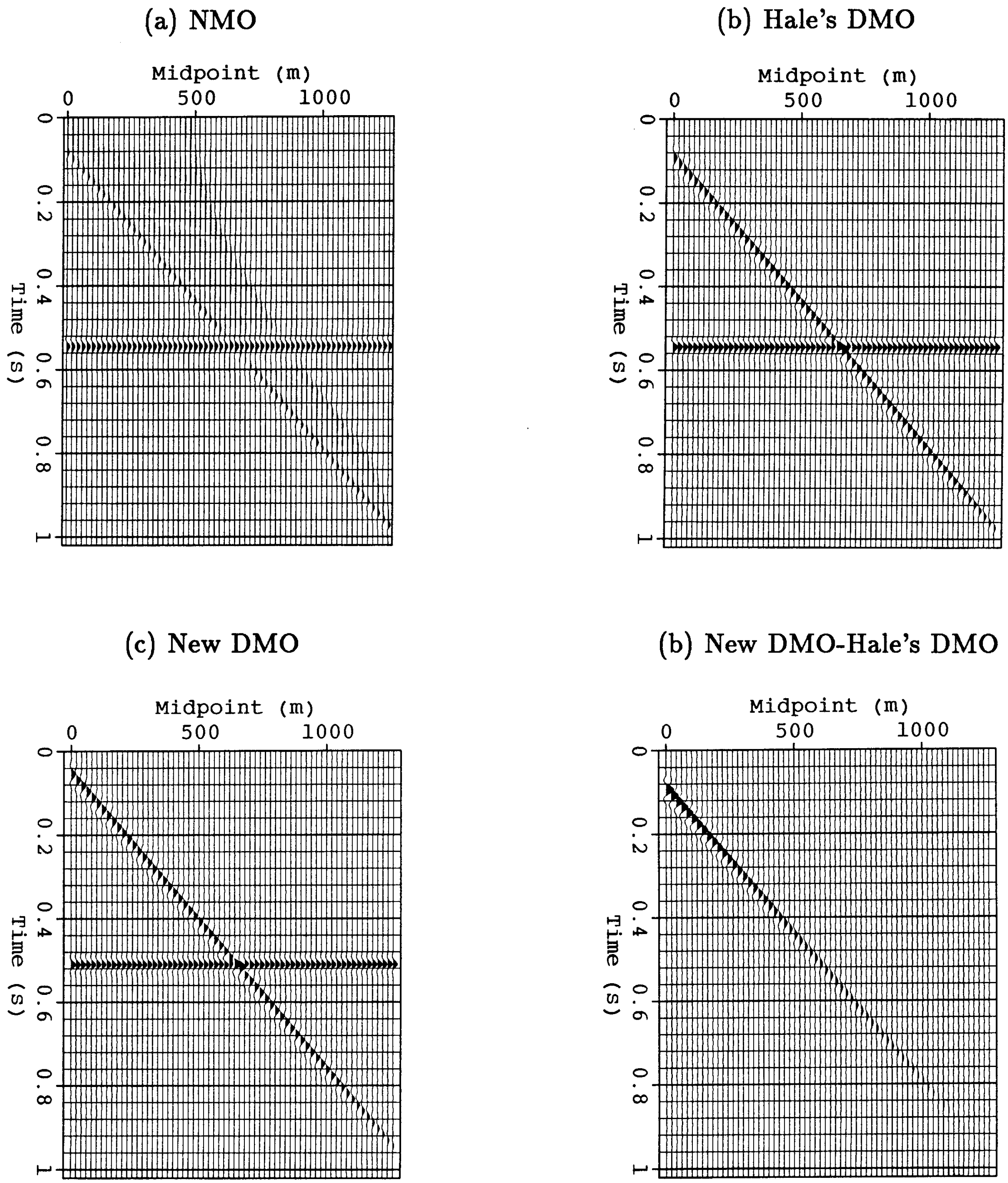

FIG. 5. The stacked results after the DMO operation. The synthetic section contains one horizontal reflector and one dipping reflector. For the dipping reflector, the new operator gives larger amplitude response than Hale's operator. (a) Stacking after the NMO correction. (b) Stacking after Hale's DMO correction. (c) Stacking after the new DMO correction. (d) (c)-(b).