A non-linear velocity inversion of a moderately difficult synthetic model in a cross-hole geometry using ray trace tomography

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ABSTRACT

The cross-hole transmission velocity inversion problem has a significant non-linear component because horizontal geology can radically effect horizontally traveling energy. The repeated application of a travel time linear inversion to reconstruct a sample cross-hole model is generally successful at removing these non-linear effects, but not entirely. Further improvement of the model seems to require that wave form amplitude and dispersion information be included in the inversion. Analysis of the wave forms for the sample model used here indicates that although the inversions are close to the original model, they are not close enough to be within the region of validity for Born inversion schemes.

INTRODUCTION

Most two dimensional velocity inversion methods are perturbation methods based on a reference model. These methods attempt to adjust a reference model so that its synthetic data match the observed data. While the adjustments are imperfect, the resulting model is generally an improvement over the starting model. The repeated application of the perturbation method by continuously updating the reference model will hopefully produce the correct result.

This perturbation iteration procedure solves a non-linear problem by a series of linear steps. It finds the low point of an unknown non-linear objective function by linearizing about a point and moving to a new point based on the information of that point.

An analogy of this non-linear iteration procedure is trying to find the lowest point in a valley while blind but able to jump great distances. Each jump corresponds to one non-linear iteration. Lacking any other knowledge, the best direction to jump would be the direction of the steepest slope from our present position. Unfortunately, the optimal distance to jump cannot be determined except through repeated forward modeling, an expensive procedure. Instead, the distance to jump is determined from the curvature of the valley at ones present position.

If the valley is smooth, the lowest point will be located quickly. But, topography encountered along the way will cause complications by sending us off in a wrong direction. The topography may be so rough that our search is endless, or we could get fooled by a local minima. Without the ability to see the rest of the valley, we would assume that the local minimum is the low point of the valley and that the search is over.

Typical inversion problems will involve thousands of dimensions. Clearly, this non-linear iteration procedure cannot be taken for granted. It may take an unreasonably large number of jumps to find the low point, or it might not be possible to find it at all. The trick to finding the low point of the valley is to start close enough so that no intermediate topography is encountered. What distance defines close will vary greatly with each individual case.

The only procedure for analyzing non-linear behavior is by trial-and-error modeling of insightful cases. Since the Born and Rytov linear inversions are themselves relatively expensive, modeling of non-linearity is seldom performed. Ray trace tomography is a cheaper procedure where the non-linear analysis is more plausible. Some of its characteristics may be similar to Born and Rytov for broad velocity variations.

The linearization used by most velocity inversion methods is computed by back-projecting data residuals along the paths of propagation in the reference model. The difference of the paths of propagation between the reference model and the correct model causes non-linear error. Born, Rytov, and ray trace tomography are examples of velocity inversion schemes that use this linearization approach and suffer from this type of non-linearity (Tarantola, 1984; Mora, 1987; Woodward, 1988).

The cross-hole geometry has the potential for having a serious non-linear component because geologic structure is generally parallel to the direction of energy travel. Energy traveling horizontally from one bore hole to another is very susceptible to the sharp vertical velocity variations of horizontally oriented structure. Even low amplitude velocity variations can significantly effect the amplitude and propagation direction of high frequency energy and cause dispersion of low frequency energy. Figures 2a-j show how finite difference seismograms are affected by the model of Figure 1. This is the model later used for analysis of non-linear convergence behavior.

For a typical cross-hole survey, this horizontal structure is not well resolved and is the objective of the velocity inversion. The starting reference model with errors in the horizontal structure will have a significantly different energy propagation pattern than the correct model. The linearization computed using this energy propagation will correspondingly suffer. As a result, the non-linear iteration behavior for reference model errors typical of this geometry is less stable than that for other reference model errors of similar amplitude but different orientation.

The non-linear behavior may be a limiting factor in cross-hole velocity inversions. This potential problem is analyzed with inversions of synthetic data from a moderately difficult cross-hole model shown in Figure 1. Results indicate that while non-linear errors are significant, the convergence behavior is stable when ray coverage is complete. However, ray trace tomography is unable to completely resolve all of the small scale structure. This partial failure may result from either non-linear error, use of only first arrival travel times, or from ray tracing errors. In either case, the use of a wave-form method should improve upon these failures. Analysis of the wave-forms demonstrates that even though the ray trace tomography non-linear inversions are close, they are not close enough to be within the region of validity of Born inversion schemes.

METHOD

The velocity inversion method used here is a cell based, travel time inversion scheme using ray paths. It is called ray trace tomography. The method can be considered the high frequency approximation of wave equation techniques although it uses only the arrival time rather than also the amplitude or phase. Like wave equation techniques, ray trace tomography treats the velocity field as a two dimensional grid of points for both the forward modeling and inversion steps. No a-priori structure, such as layers, is assumed. A detailed description of the method can be found in Stork (1988).

INVERSIONS

Previous experience indicates that it is best to start with smooth reference models. The starting reference model used in all cases is the constant background velocity of the synthetic model.

In an attempt to analyze only the effect of the limited angular ray coverage of a cross-hole experiment rather than the non-linearity, an inversion is performed without ray path errors. The velocity variations of the original model are scaled down to be only +/- 0.5%, which have a very minor effect on the ray paths. The straight rays of the constant velocity reference model are accurate. The inversion, shown in Figure 3b, is nearly perfect. The lack of vertically traveling energy has not compromised the inversion.

However, when the velocity variations of the original model are scaled back up to +/- 10%, the ray path errors between the original and reference model increase so that the inversion is severely affected. The result, shown in Figure 4a, is dominated by artifacts with few of the original features present. An additional ray tracing and inversion is performed to attempt to improve on the ray path errors. The result, shown in Figure 4b, shows little improvement. For this case, it does not appear that the repeated application of the linear inversion will remove the non-linear errors.

One of factors contributing to the non-linear artifacts is the inhomogeneous ray coverage of the square recording geometry. The top and bottom of the model contain very few rays that travel only horizontally while the middle contains many more rays traveling at additional azimuths. The plot of ray density in Figure 5 shows how ray coverage is inhomogeneous over the inversion region. Other inversions using different damping values did not improve on the artifacts.

In order to produce a more uniform ray coverage, the model is extended above and below the zone of interest, as shown in Figure 6. Even though there is no structure of interest in these areas, data will be recorded here just to improve the ray coverage in the central section. Analysis of the ray coverage in this extended model, shown in Figure 7, shows that ray density is now uniform in the zone of interest.

The inversion using this extended ray coverage, shown in Figure 8a, does not have most of the serious artifacts of the previous inversion. While the inversion is considerably distorted, some of the original features are noticeable. An additional ray tracing and inversion, shown in Figure 8b, has improved the result. The velocity variations are larger in amplitude and are starting to take on their correct shape. The black, high velocity channel in the middle is now starting to be connected between the two boreholes.

A third ray tracing and inversion, shown in Figure 8d, improves the result only marginally. This minor improvement suggests that the repeated linear steps are converging but to a solution that is only close to the correct the result. Additional ray tracings and inversions were not performed because ray tracing through these increasingly complicated models is problematic.

In an attempt to improve the non-linear convergence behavior, the inversions are repeated with constraints that damp small scale velocity variations. The inversion of larger scale velocity variations is assumed to be more stable because they effect the ray paths less. Figures 9a-c show three successive ray tracings and inversions with these constraints. Assuming the large scale features are resolved after these inversions, a fourth inversion, in Figure 9d, is performed without constraints using the previous inversion as the reference model. Most of the structure of the original model is inverted but problems remain. The central fast velocity channel does connect through between both boreholes, but it is not quite as thick as it should be. Moreover, some of the subtleties of the slow velocity regions is missing.

While the final non-linear inversion using the initial constraints is quite different than that without constraints, the results appear to be quite comparable. In both cases, the repeated ray tracing and inversions converged to a solution that is close to the correct result, but noticeably different.

DISCUSSION

Three explanations are offered as reasons for the failure of the repeated ray tracings and travel time inversions to produce a very accurate inversion: 1) non-linear error, 2) the use of only first arrival travel times, and 3) ray tracing failures. Non-linear error is where the reference model was not close enough to the correct model that certain subtle features could be resolved. The use of first arrival travel times ignores later arriving energy and does not use the phase, amplitude, or dispersion information in the data. Ray tracing failure is the inability to trace certain rays, generally those experiencing a large amount of geometric spreading, because of numerical problems with the method used. Probably all of these factors contributed to the imperfect inversions.

Figure 10 plots the location of the ray tracing failures in black. Besides the roll-in and roll-out of the survey, all ray tracing failures occurred near zero offset, which corresponds to a horizontal ray. These missing horizontal rays can be vital for determining horizontal structure.

Travel times computed with the ray tracing code are plotted over finite difference seismograms in Figure 11a. The two methods are quite consistent with each other. However, it is apparent how many arrivals there are in addition to the first arrivals. Figure 11b plots all additional geometric arrivals over the wave forms. Energy that does not have a travel time plotted over it probably corresponds to non-geometrical, dispersive arrivals– information that can only be taken advantage of with a wave form method.

One way to improve on the inversions would be to use a wave equation method. It would incorporate the later arrivals, would include wave form information, and is generally more robust than ray tracing. It seems likely that a wave equation method would provide enough additional information to remove the non-linear errors.

To analyze the information available in the wave forms and investigate how to use that information, synthetic seismograms for one shot position are presented in Figures 12a-h for all the previous inversions performed. These synthetic seismograms may serve as a more quantitative measure for comparing the inversions than the velocity field itself. With each successive inversion we see the seismograms becoming more complicated and taking on features of the wave field through the original model. In general, the seismograms through the constrained inversions are less complex than the unconstrained ones. It is surprising how much of the original wave field has been reconstructed by using just the first arrival travel times.

In order to aid in comparing the seismograms of the two final non-linear inversions with the seismograms of the original model, they are plotted over each other in color in Figures 13 and 14. The seismograms from the original model are plotted in green while those from the inversion are plotted in purple. Where they overlap, they appear as black.

A necessary condition for the transmission component of a Born inversion scheme to produce a constructive contribution is for the wave forms to be within a phase of pi of each other. This condition is meet for most of the wave form when the corresponding arrivals in the data and synthetics overlap each other. Although the two final non-linear inversions are close to the original model, they are not close enough that most of the corresponding arrivals overlap that of the original model. This is especially true for the horizontally traveling energy, which gives us the most information about the horizontal structure. Arguments exist (Mora, 1987) that the other parts of the wave forms that overlap will contribute enough to improve the result to overlap the rest of the wave forms. However, there is question about whether this procedure is robust. If it is robust, the procedure would be slow.

One of the main criticisms of travel time inversion is that it requires the picking of the travel times. However, the wave forms of Figures 13 and 14 show that this criticism is unjustified. To use the transmitted information, the travel path of the individual arrivals must be specified, which requires the picking of the wave forms. Perturbation inversion methods improve on a reference model by comparing its synthetic data with the real data. In order to make a proper and complete comparison, arrivals in the synthetic data corresponding to those in the real data must be identified. This comparison then specifies the travel path of the arrival in the real data to be that of the corresponding arrival in the synthetic.

Born performs this comparison by assuming the arrivals overlap each other. Frequently, arrivals in both wave fields can be identified as corresponding to each other even when they do not overlap. This information that is generally apparent to the human, is lost with Born. Analysis of Figures 13 and 14 shows that there are several such cases where a human can identify arrivals that correspond to each other yet do not overlap. To make use of this additional information in with wave form technique, the technique must enable the input of picking information.

The picking of travel times is just one representation of correlating arrivals in the synthetic data with those in the real data. It is trivial to correlate first arrivals. But the correlation of later arrivals is more difficult. Comparing wave fronts between two separate plots is problematic, but much easier to do with the overlayed data of Figures 13 and 14. This method of overlaying data is proposed as the preferred method of correlating arrivals in the synthetic data with that of the real data.

CONCLUSION

Even though horizontal structure causes the most severe non-linear errors when using horizontal energy, as with cross-hole data, the repeated application of a linear inversion technique, ray trace tomography, is stable for the example used here.

While the inversion produces a result close to the correct model, errors still exist. These errors probably result from a combination of non-linear error, the use of only first arrival travel times, and ray tracing failure for horizontal rays. Improving on the last two causes with a wave equation technique would probably improve the inversion considerably.

Analysis of the wave forms demonstrates that individual arrivals in the synthetic data can be identified with the corresponding ones in the real data but they do not overlap, a criterion for transmission Born inversion schemes. To make use of the transmission information, the arrivals of the synthetic seismograms must be correlated with those of data, which is tantamount to picking. An effective way to aid in performing this picking, or correlation of arrivals, is to plot the wave forms on top of each other.

ACKNOWLEDGEMENTS

Discussions with Marta Woodward showed me how Rytov is a wave form inversion scheme analogous to Born but enables the input of picked information by the unwrapping of phases. It is probably more appropriate for transmission inversion than Born.

All synthetic wave forms were produced using a finite difference program written by John Etgen. I appreciate having such an efficient and easy-to-use tool available. The improvements to SEP's plotting package written by John Dellinger and Steve Cole were very useful. In general, the environment and facilities of SEP enabled me to concentrate on my research and be very efficient. I regret not having made a more substantial contribution in helping to improve the facilities.

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Stork, C., 1988, Ray trace tomography velocity analysis of surface reflection data: PhD Thesis, California Institute of Technology.

Tarantola, A., 1984, Linearized inversion of seismic reflection data: Geophys. Prosp., 32, 998-1015.

Woodward, M., 1988, Wave equation tomography: SEP-57.

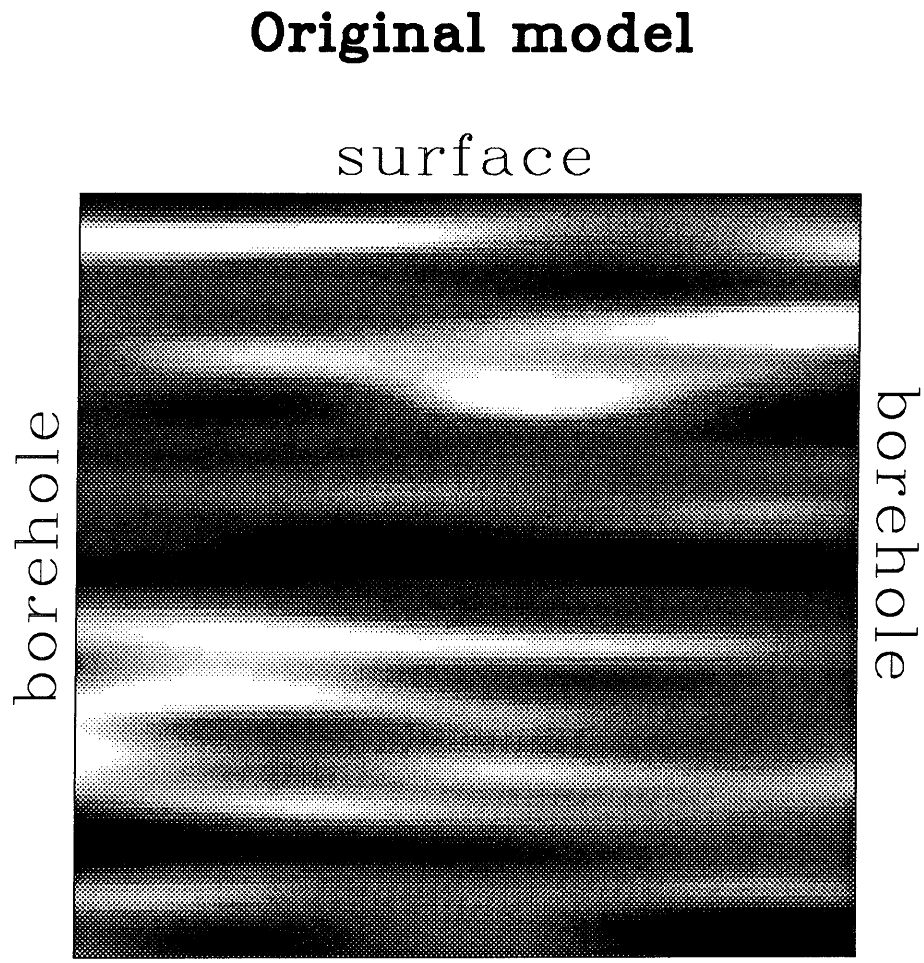


Figure 1: Sample cross-hole model which inversions will attempt to reconstruct. Data is collected by setting off numerous shots in one borehole with a dense string of receivers in the other borehole.

Time slice of wave propagation through cross–hole model

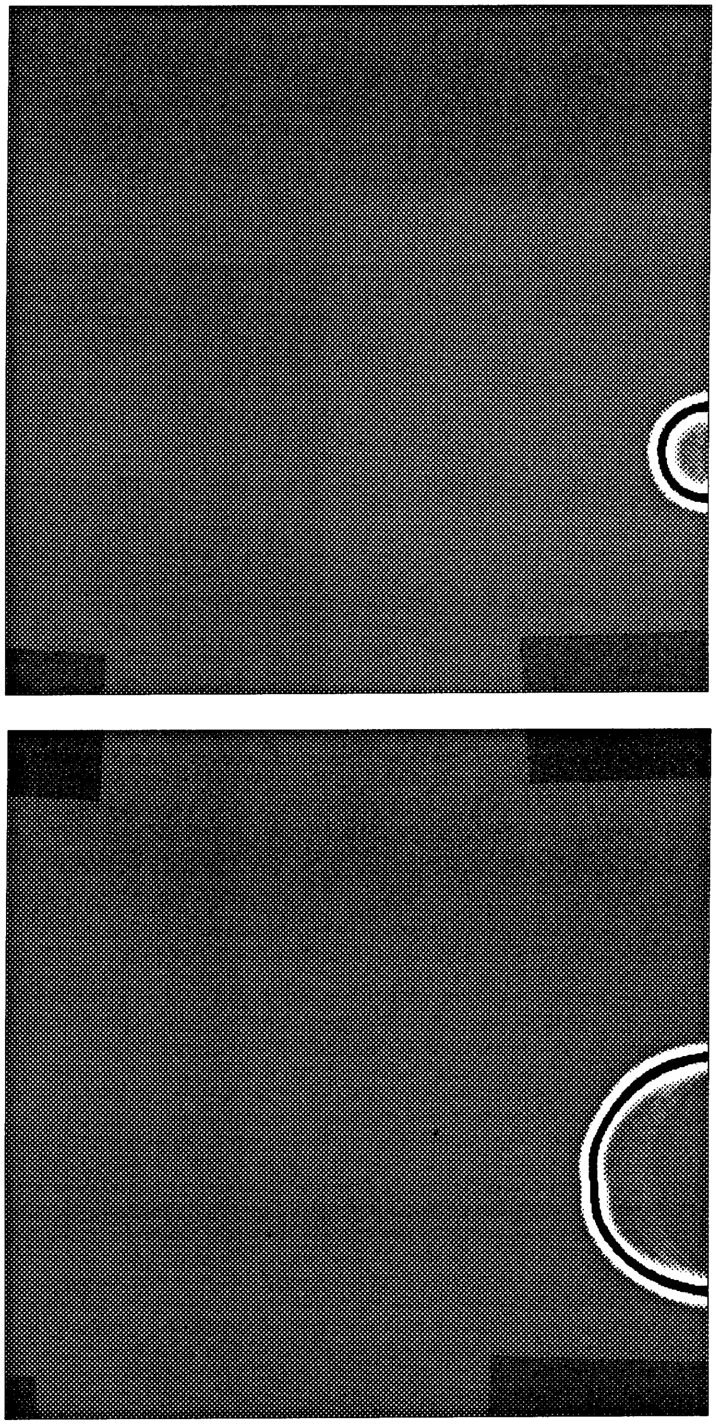


Figure 2a & b: Slices in constant time of wave propagation through sample cross-hole model.

Time slice of wave propagation through cross−hole model

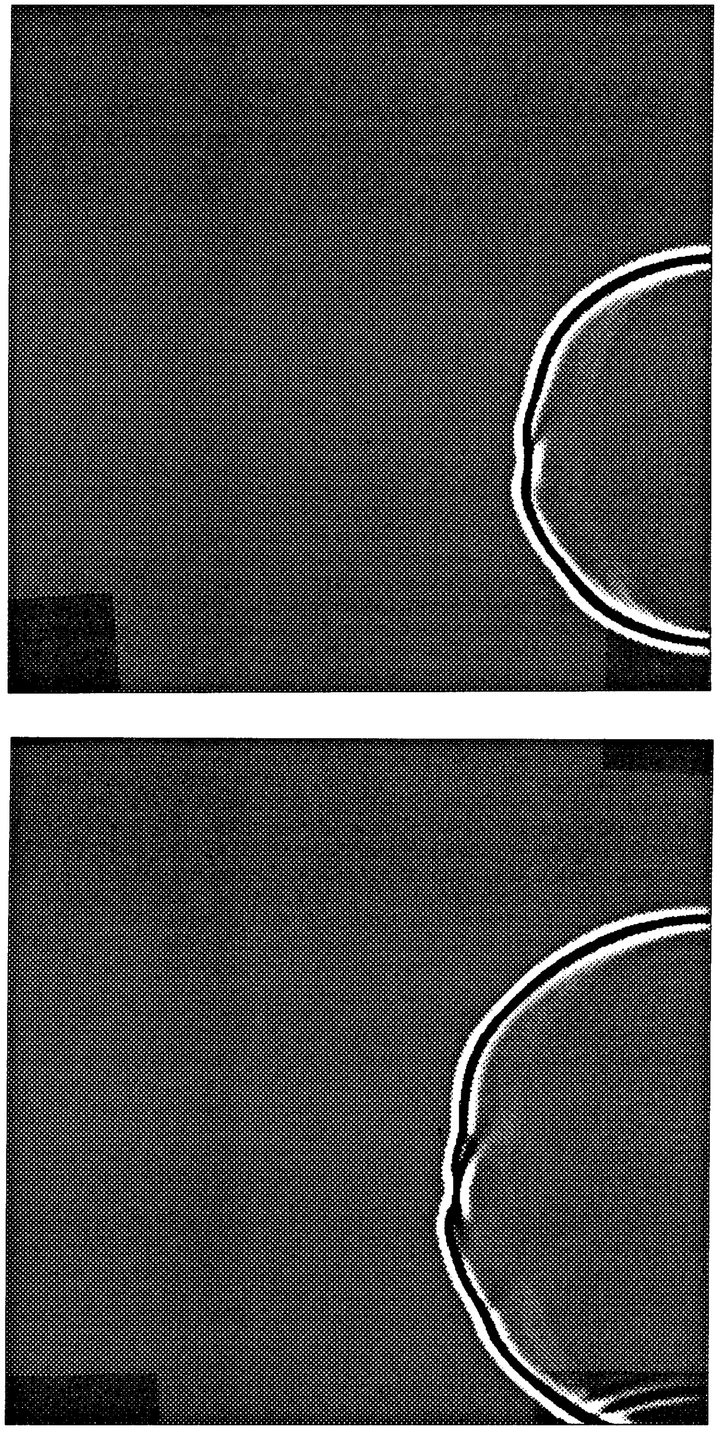
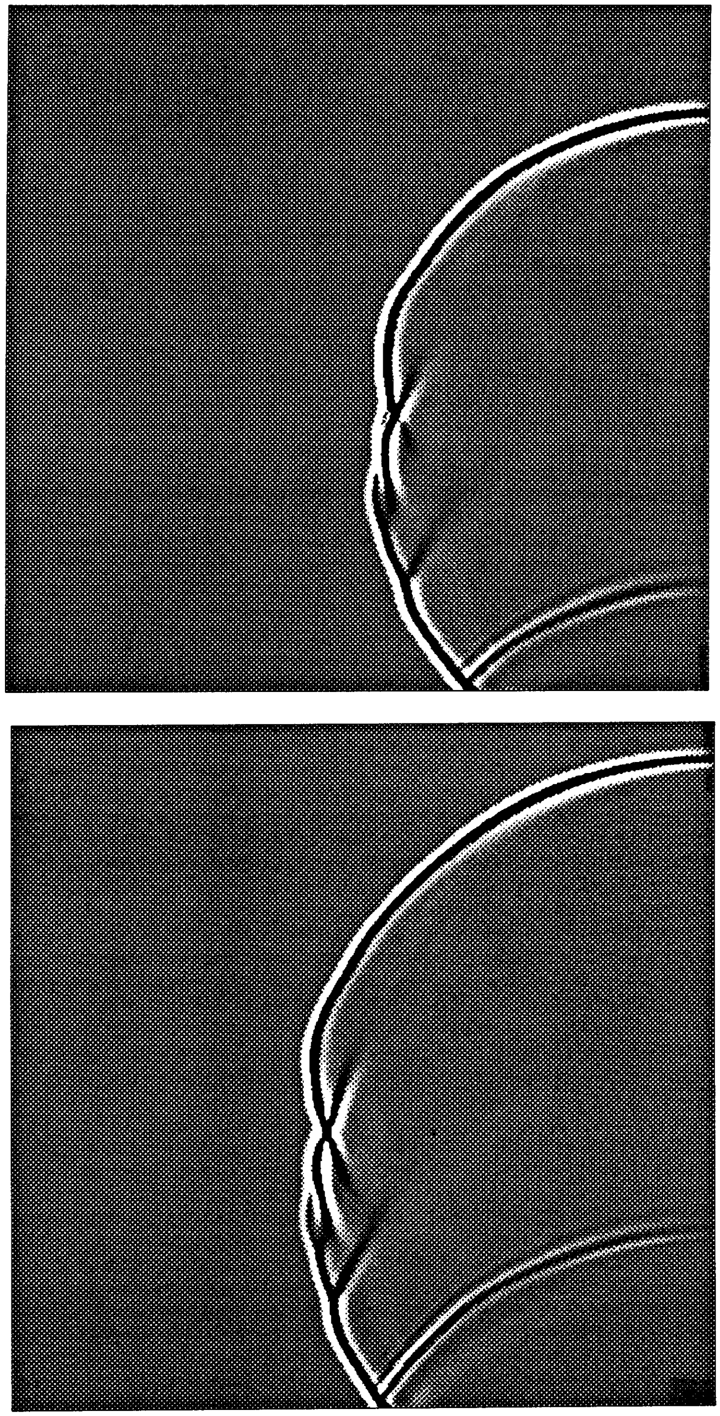


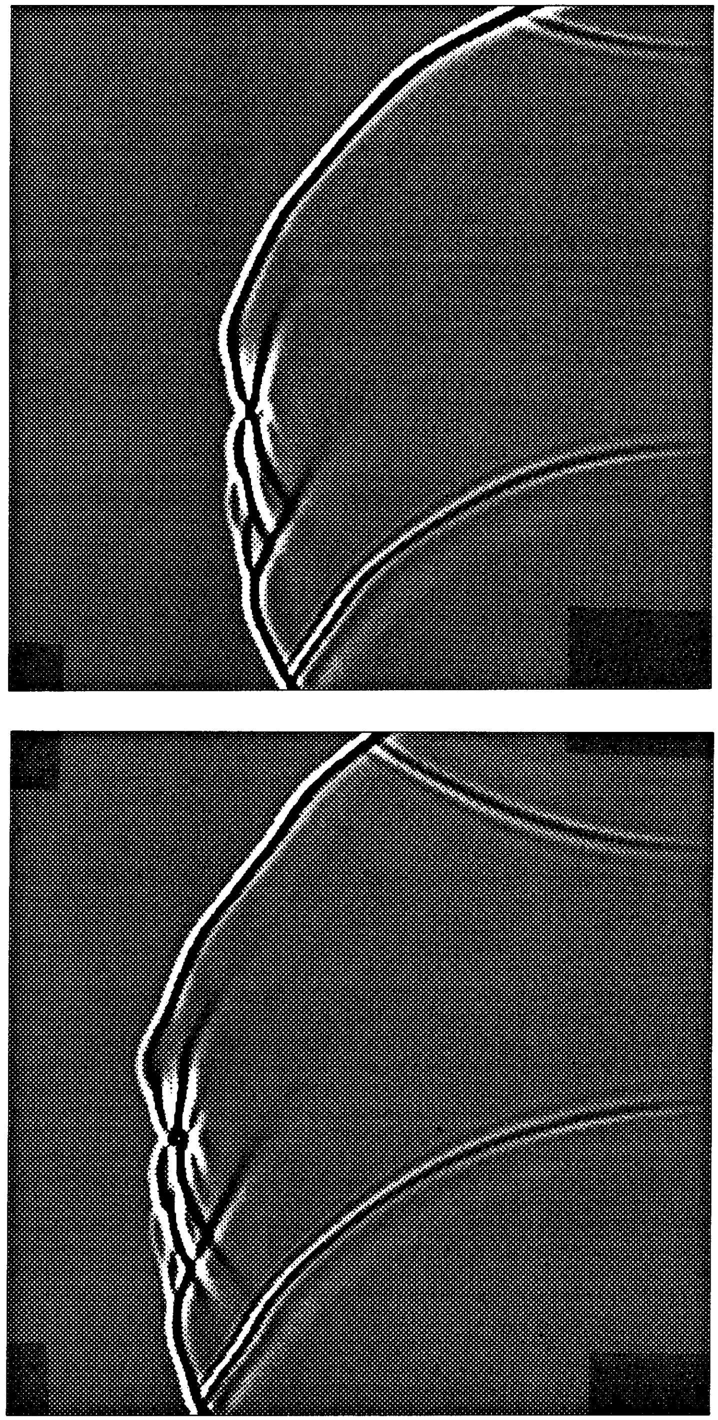
Figure 2c & d

Time slice of wave propagation through cross–hole model



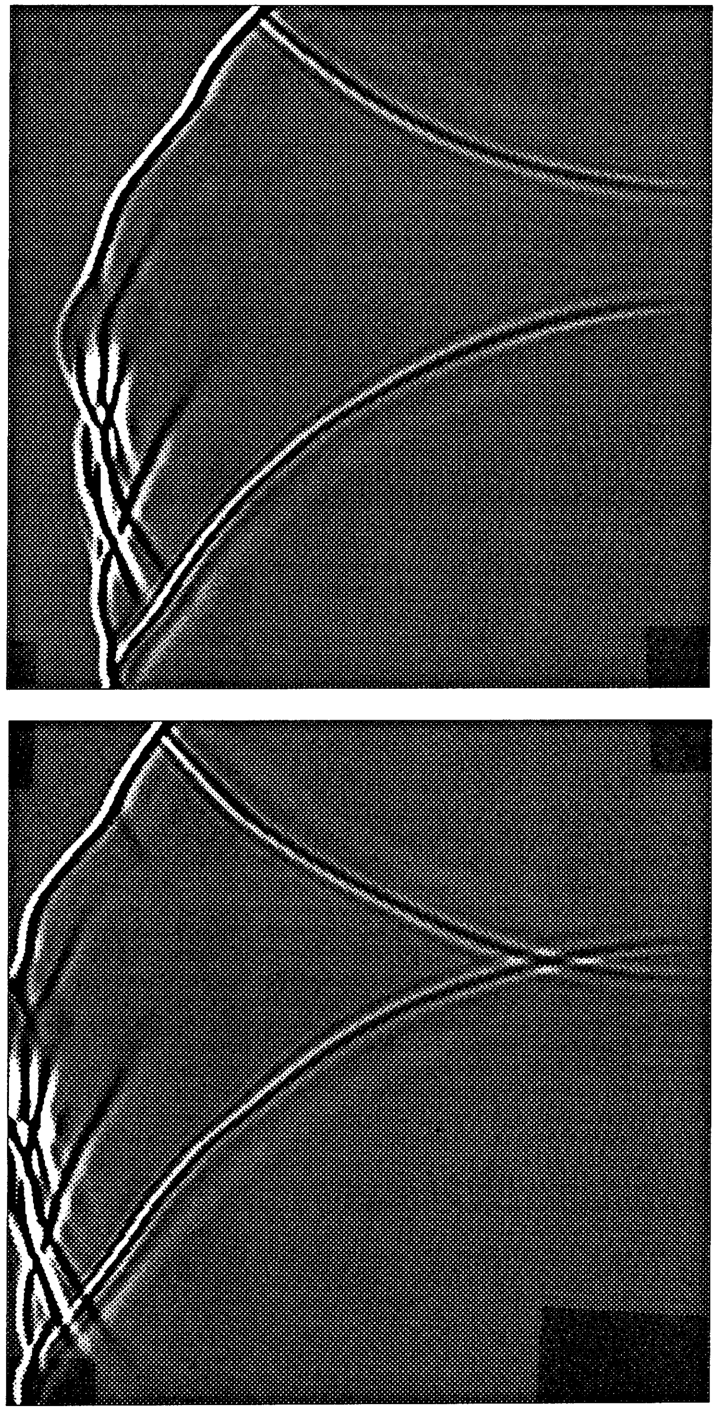
Figures 2e & f

Time slice of wave propagation through cross-hole model



Figures g & h

Time slice of wave propagation through cross-hole model



Figures 2i & j

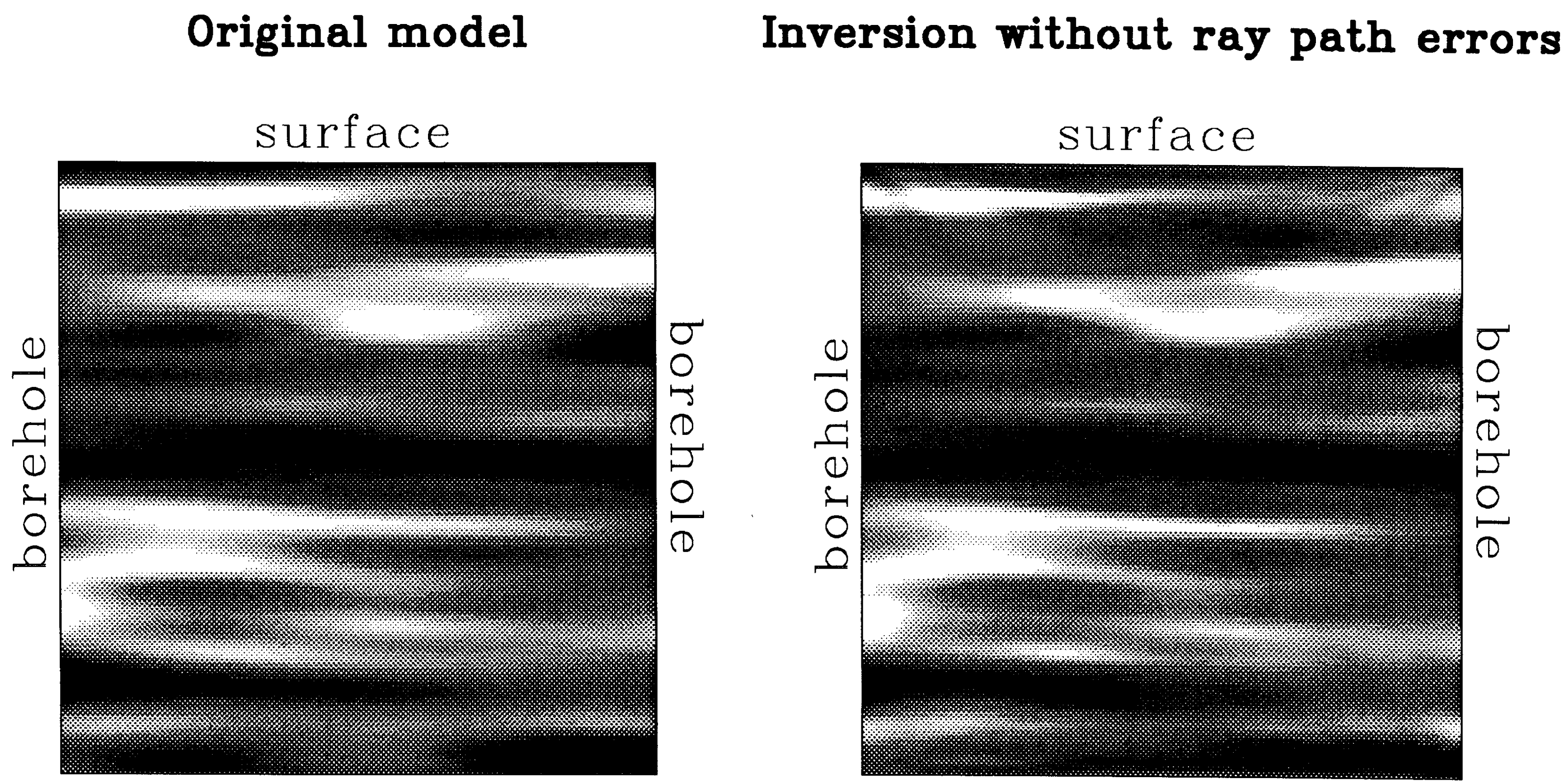
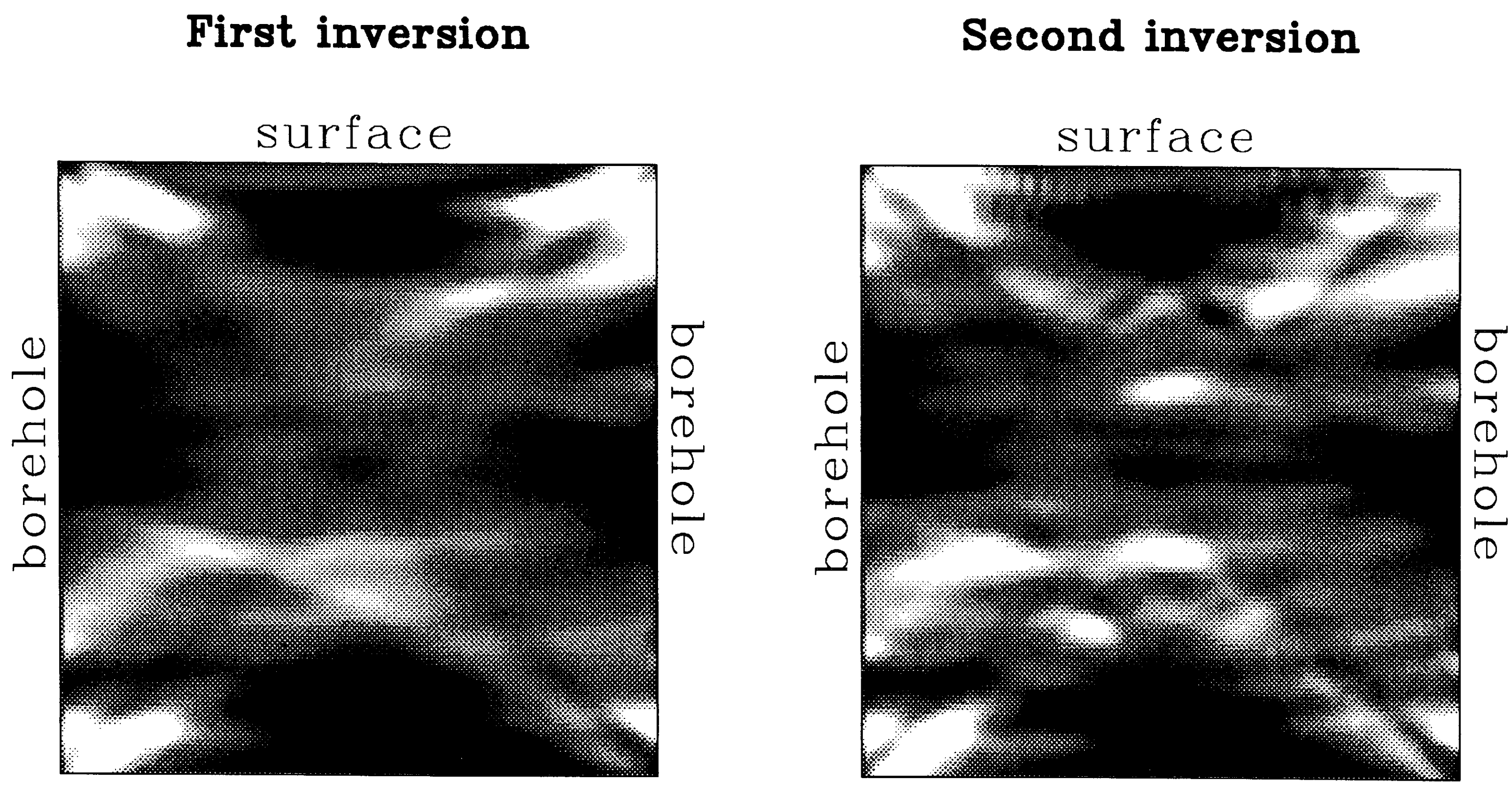


Figure 3a & b: Inversion without ray path errors. Velocity variations of the original model are small enough (0.5%) that they do not effect ray paths. The limited ray coverage of cross-hole experiment has not effected the inversion.



Figures 4a & b: Inversions with ray path errors. Velocity variations of original model are now large enough (±10%) to significantly effect ray paths. The inversion is significantly compromised. A second inversion produced by re-tracing rays through first inversion has not improved result.

Ray density in square model

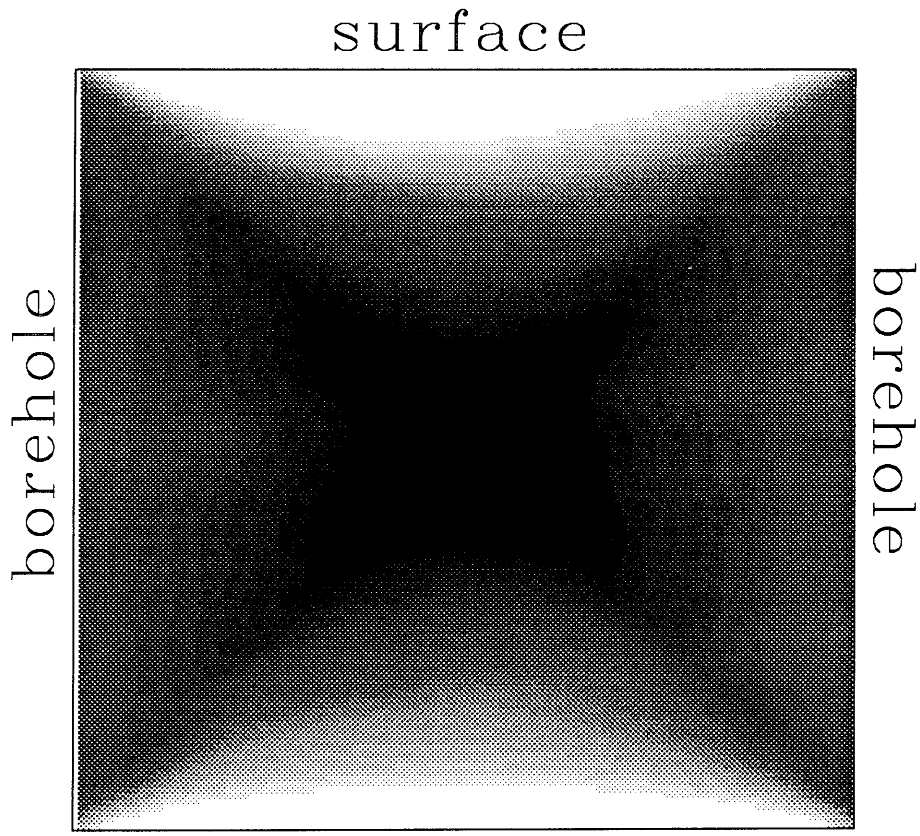


Figure 5: Ray density of square cross-hole recording geometry is very inhomogeneous.

Extended cross-hole model

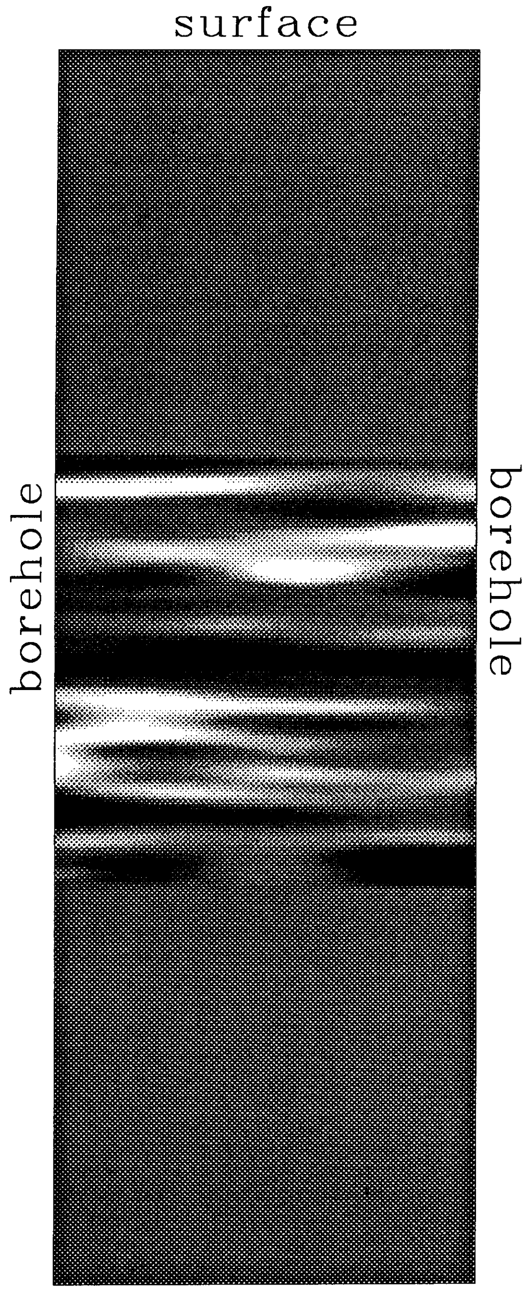


Figure 6: Extended cross-hole model used to produce a more unoform ray coverage in the zone of interest. Central part of model is identical as previous square model.

Ray density in extended model

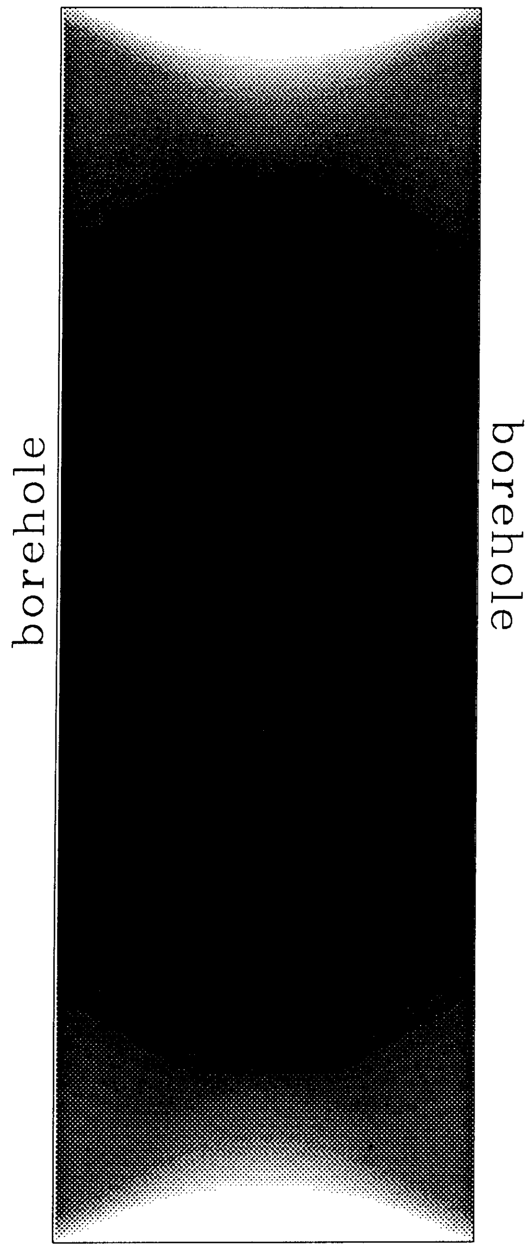
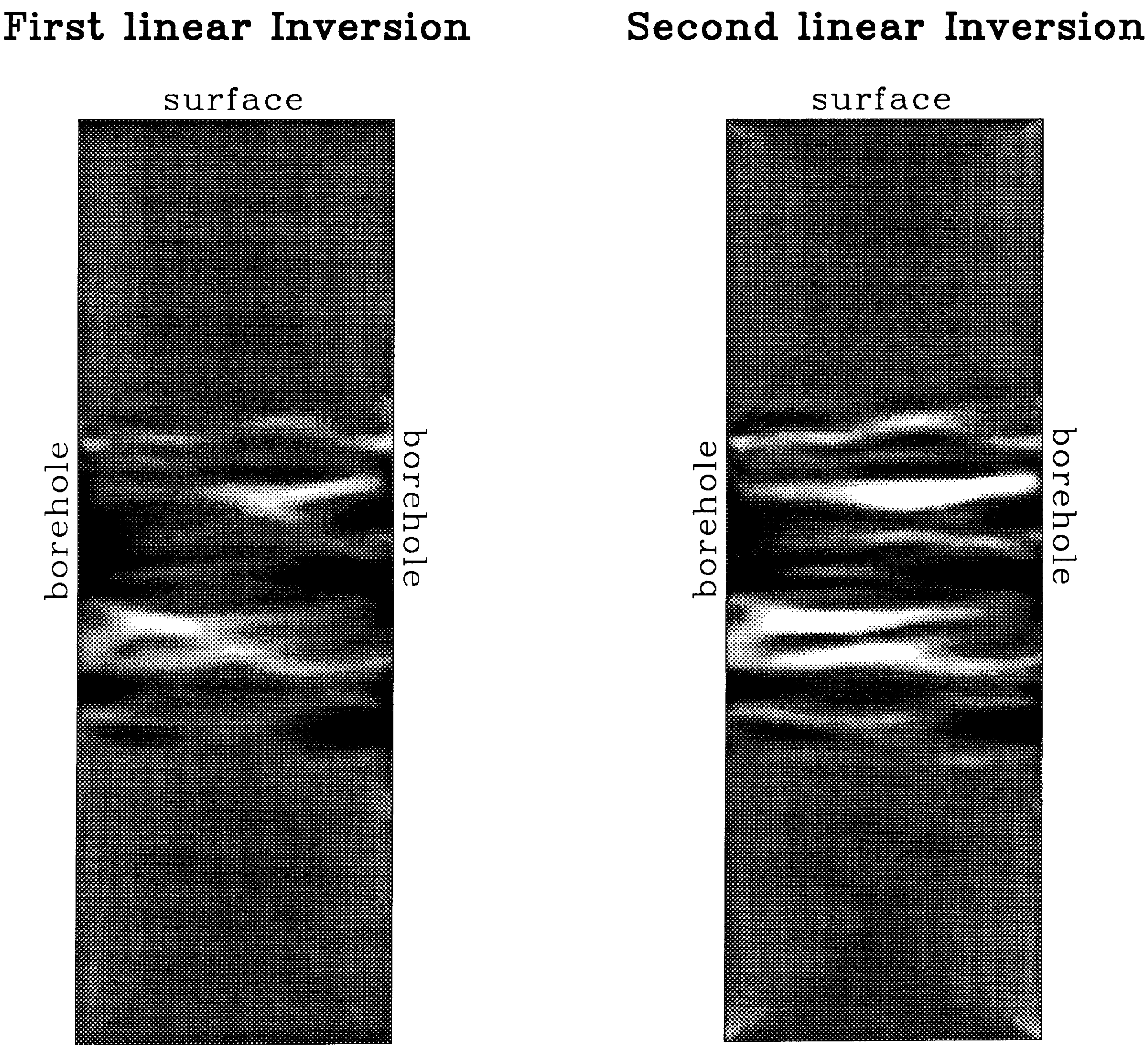
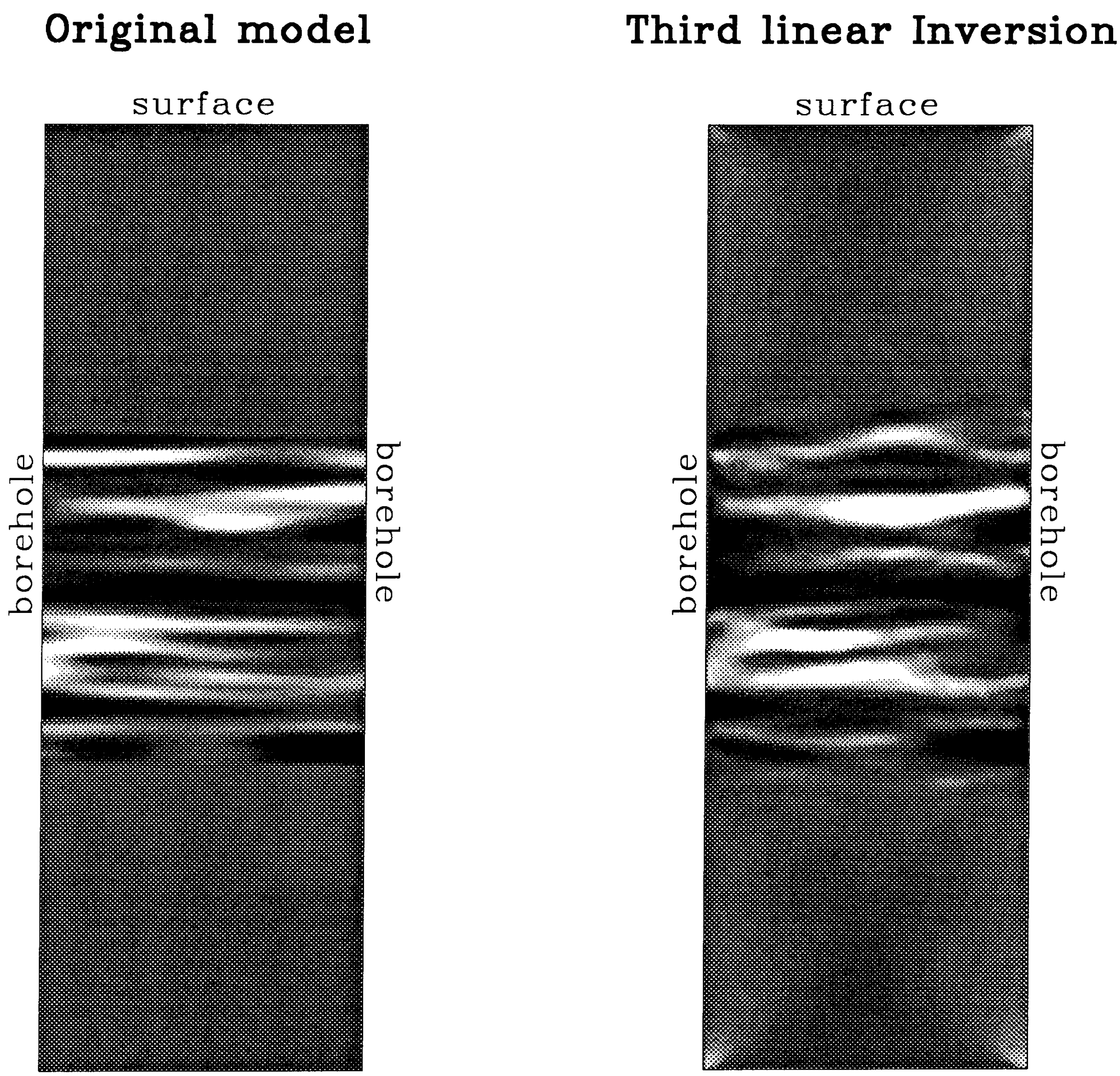


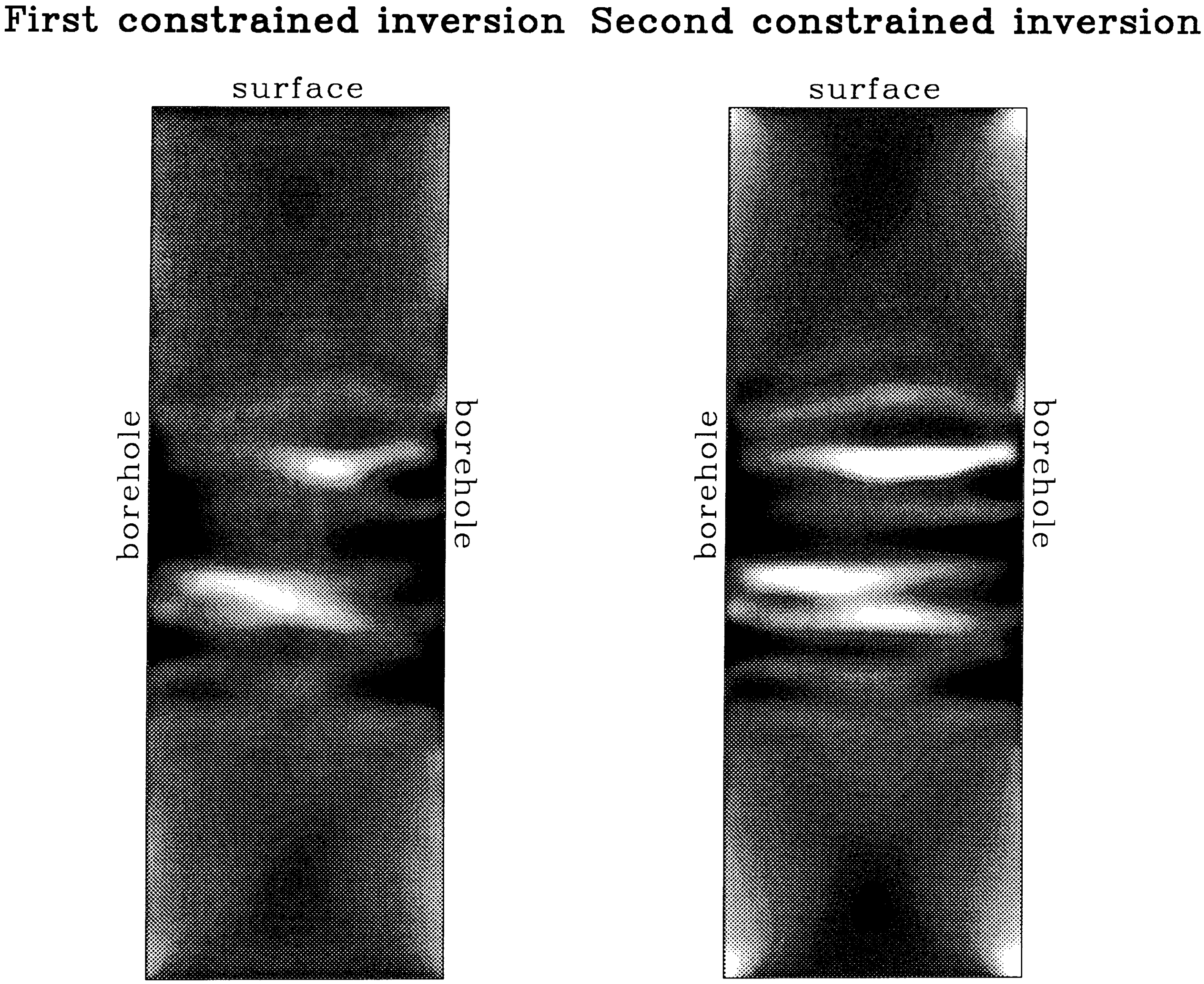
Figure 7: Ray coverage in extended model is more uniform.



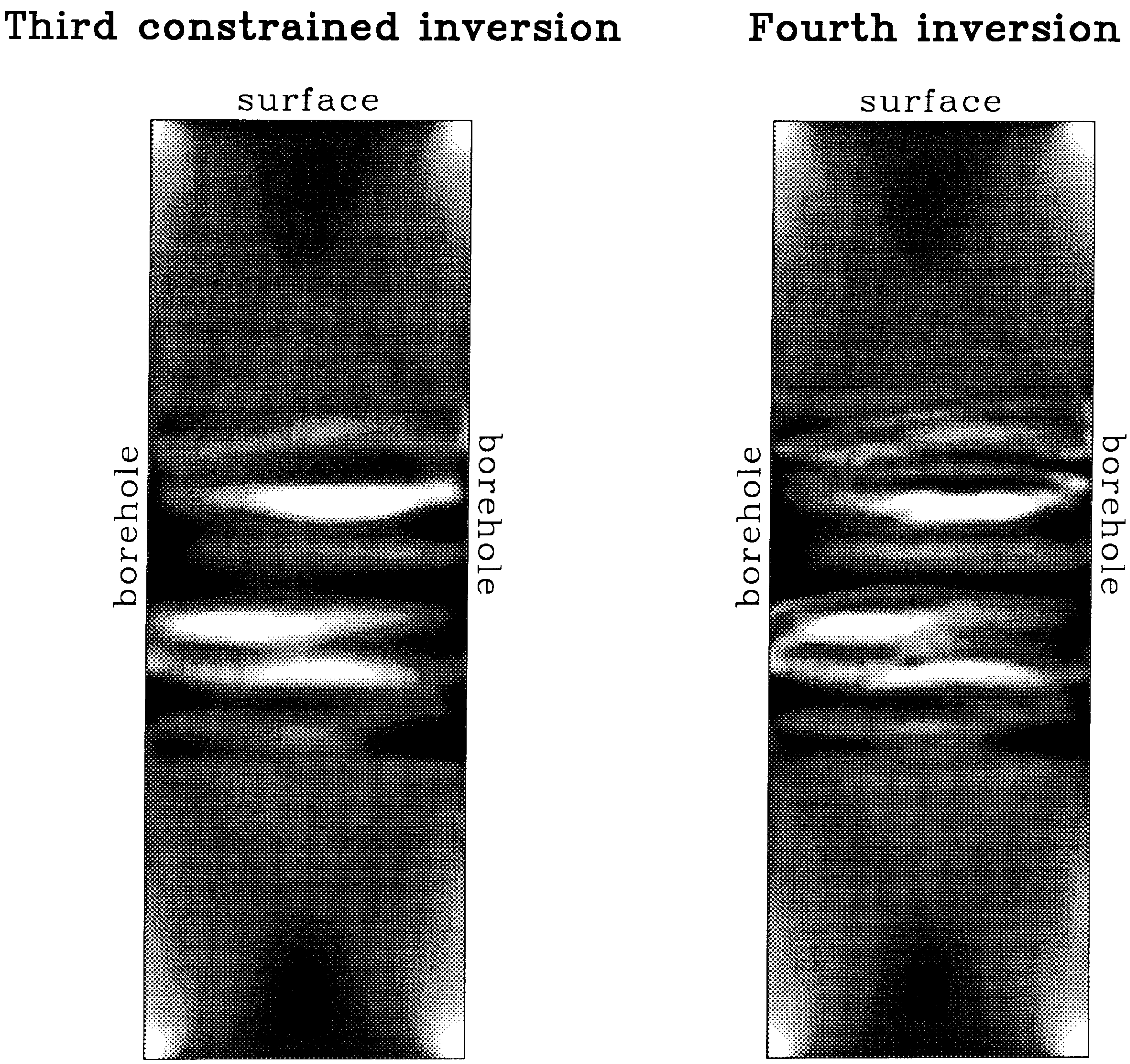
Figures 8a & b: Inversions of extended model with ray path errors. Result is much improved over analogous inversion in Figures 4a & b. The re-ray tracing of the second linear inversion has improved the result significantly.



Figures 8c & d: Inversion after yet another ray tracing. Inversion is not improved much over that of Figure 8b. Original model is plotted again for comparison.



Figures 9a &b: Inversions with ray path errors constrained to allow only broad velocity variations. Second inversion considerably improves result.



Figures 9c & d: Third constrained inversion after re-ray tracing through second constrained inversion shows little improvement. The fourth inversion is unconstrained.

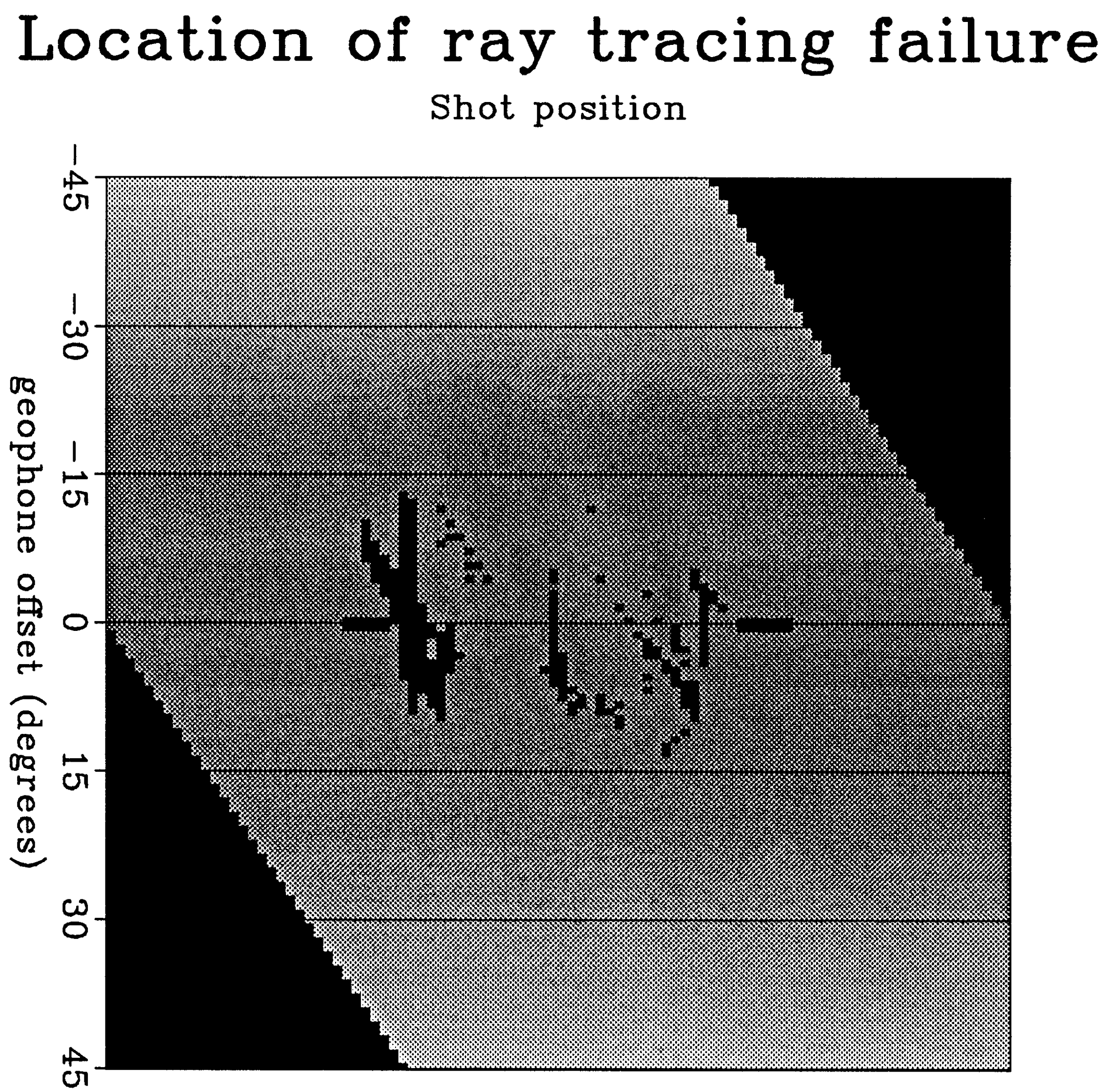
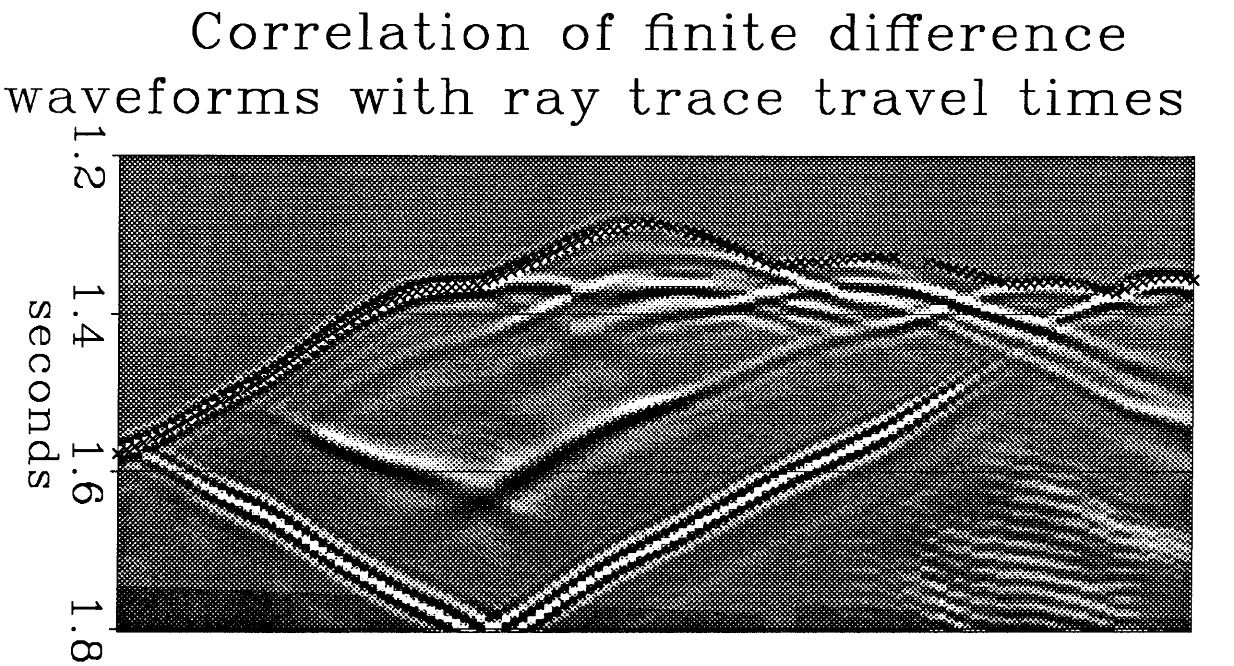
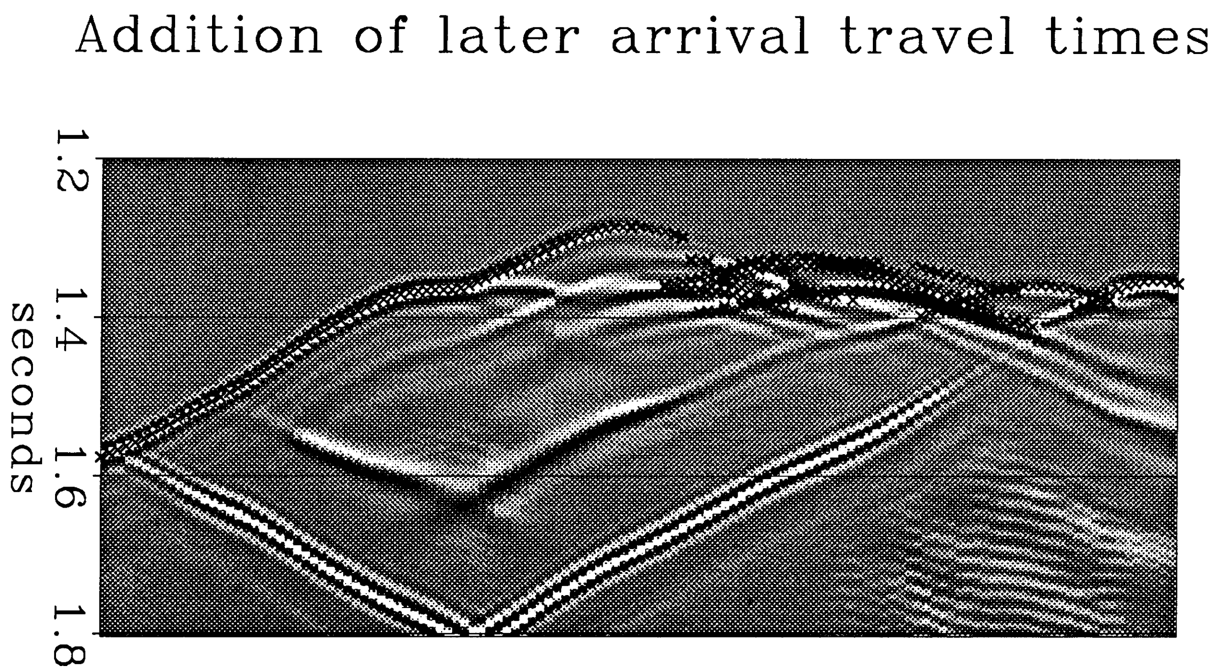
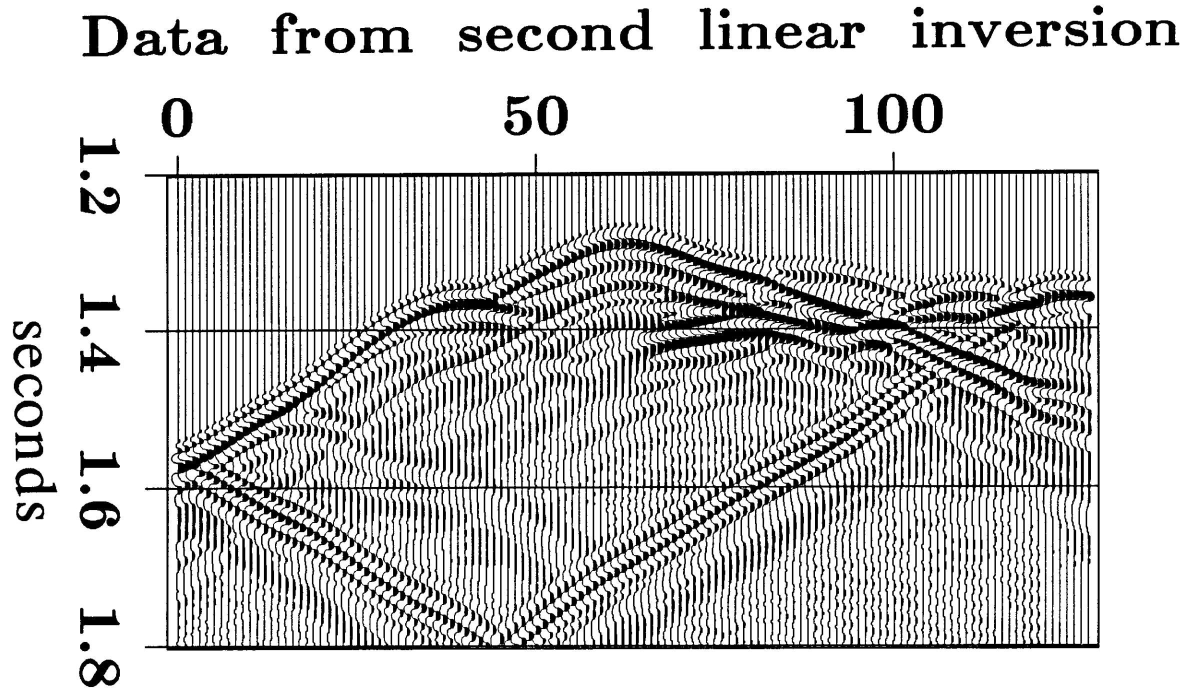


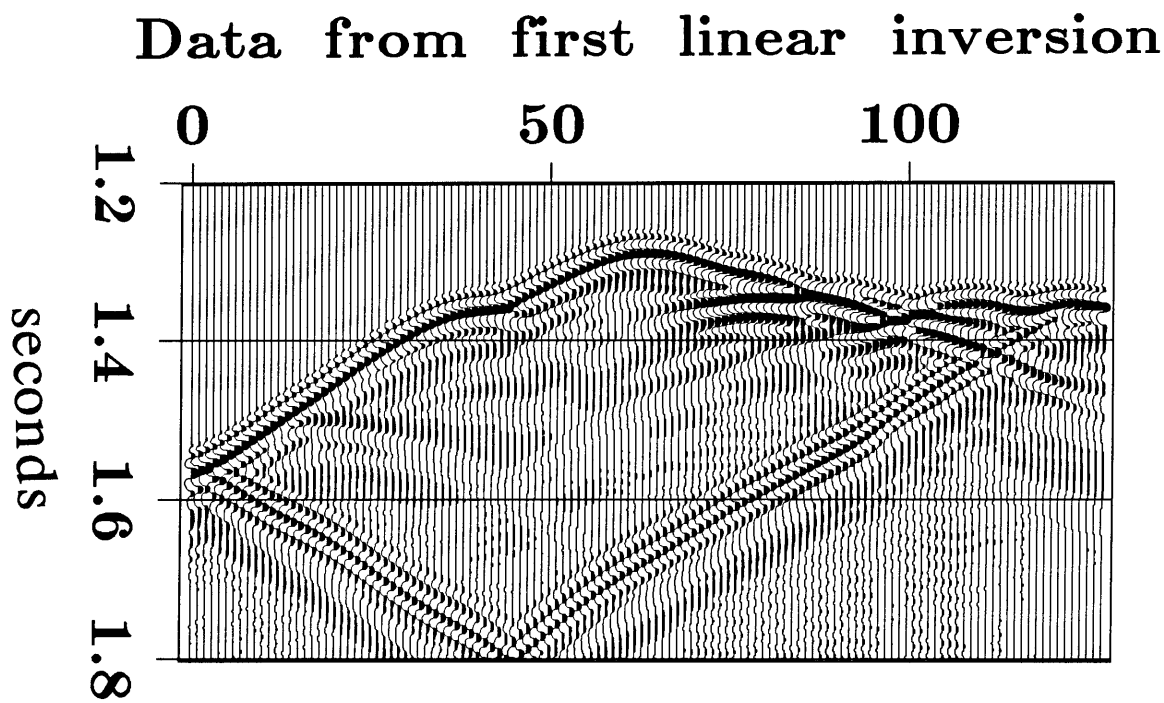
Figure 10: Black zones indicate location where no rays were traced. Upper-right and lower-left triagnle are the roll-in and roll-out where cable was outside of model. Other locations are where ray tracing program failed for numerical reasons.





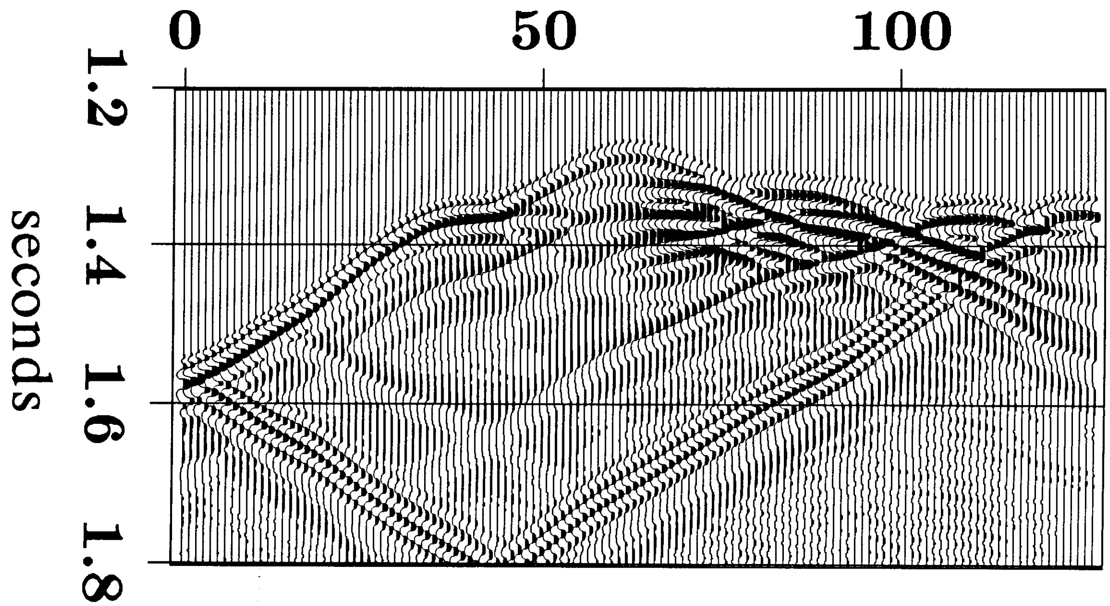
Figures 11a & b: Comparison of travel times from ray tracing code with the wave forms from finite difference. The two methods are quite consistent. Wave forms contain additional information.



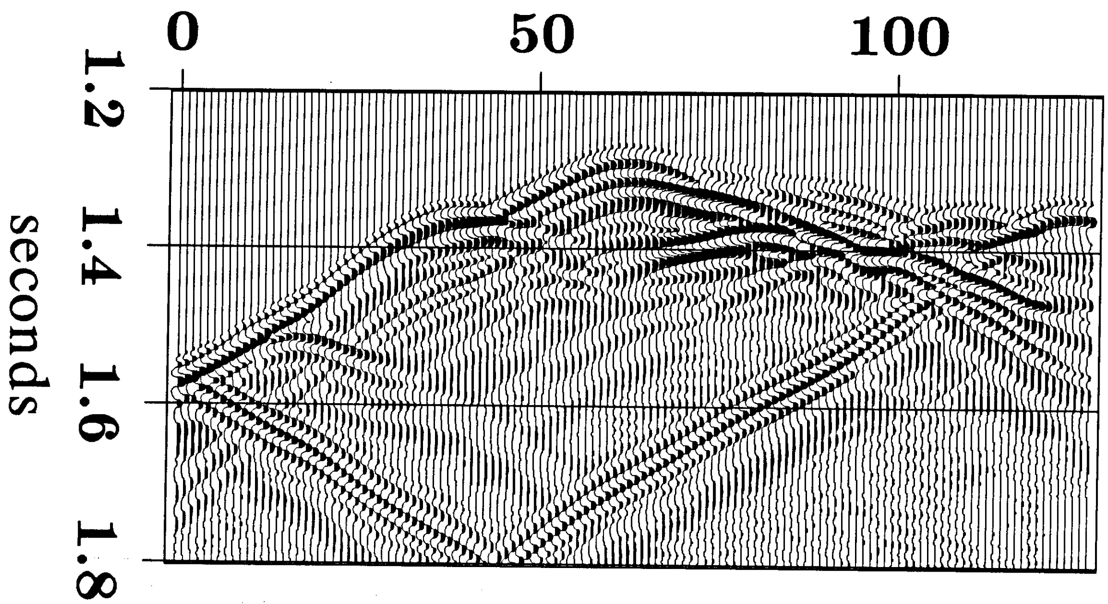


Figures 12a & b: Finite difference wave forms through the previous inversions. Wave forms through original model are in Figure 12c.

Cross-hole data from original model

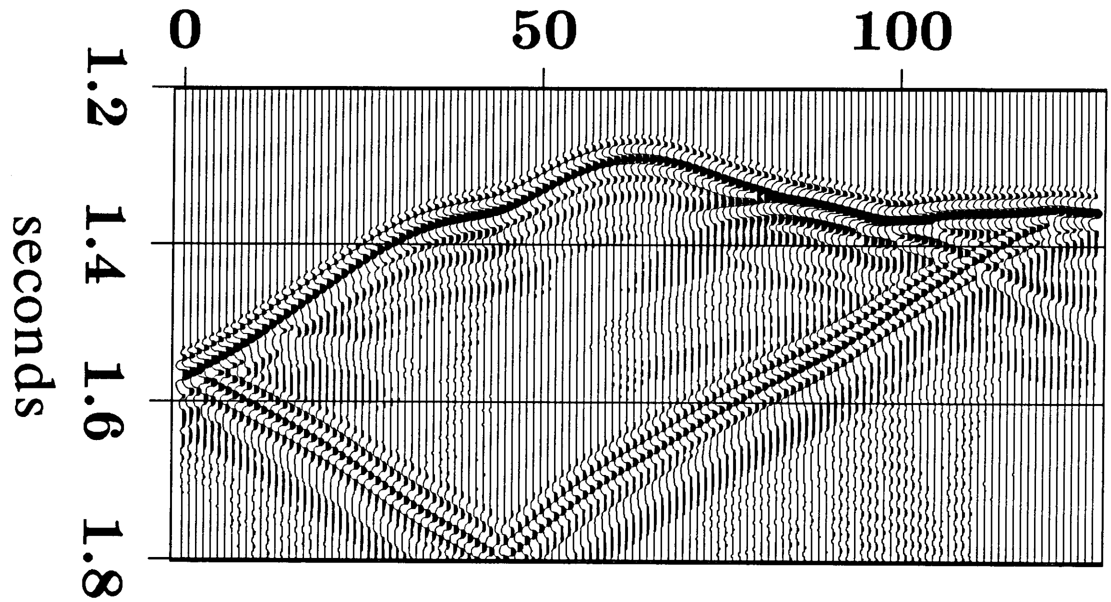


Data from third linear inversion

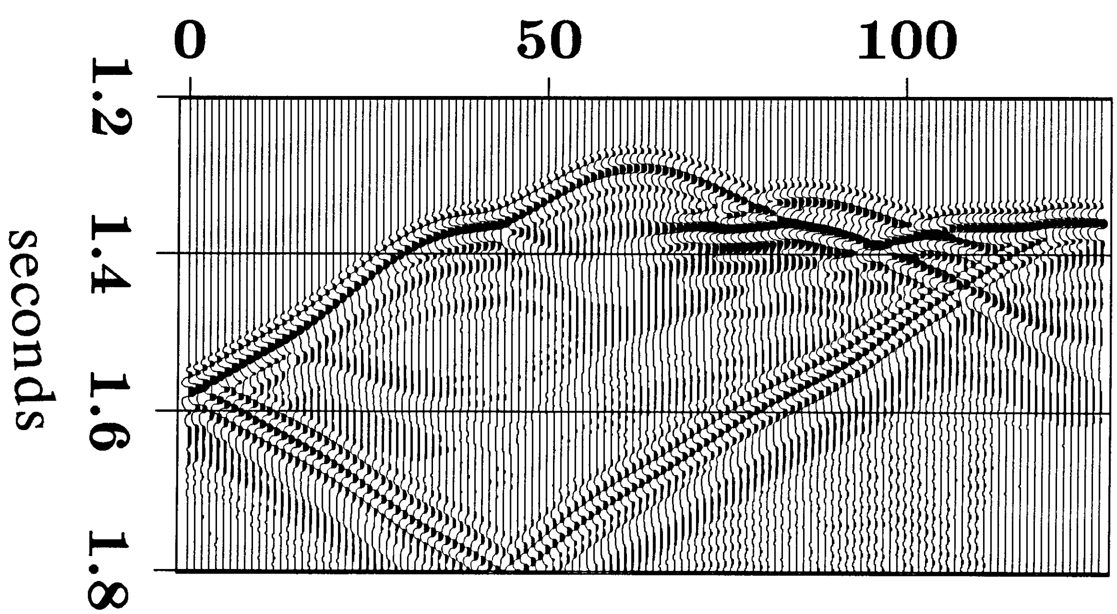


Figures 12c & d

Data from first constrained inversion

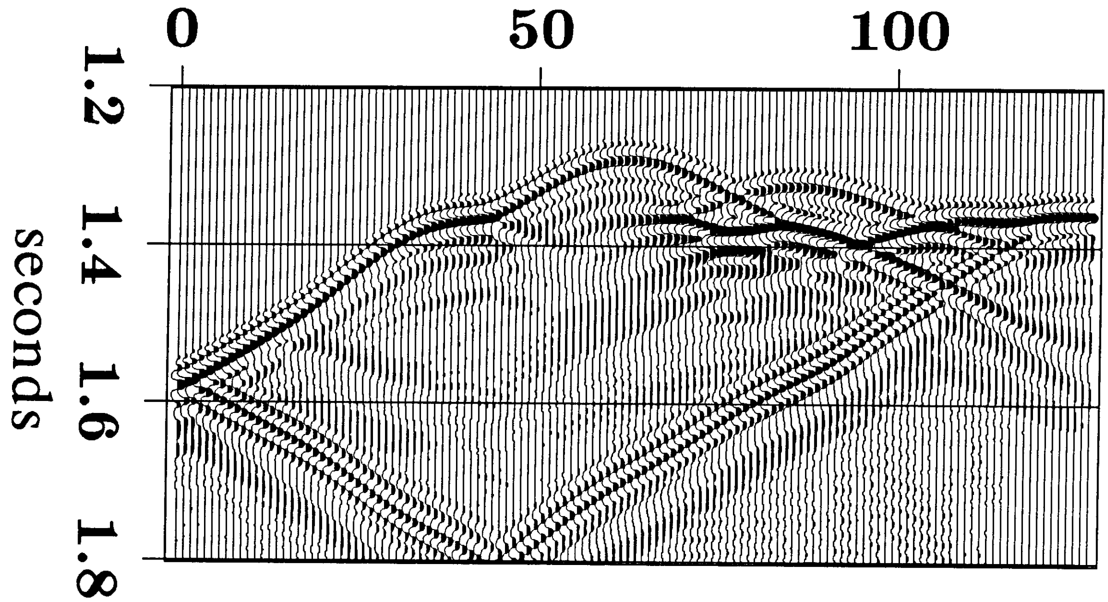


Data from second constrained inversion

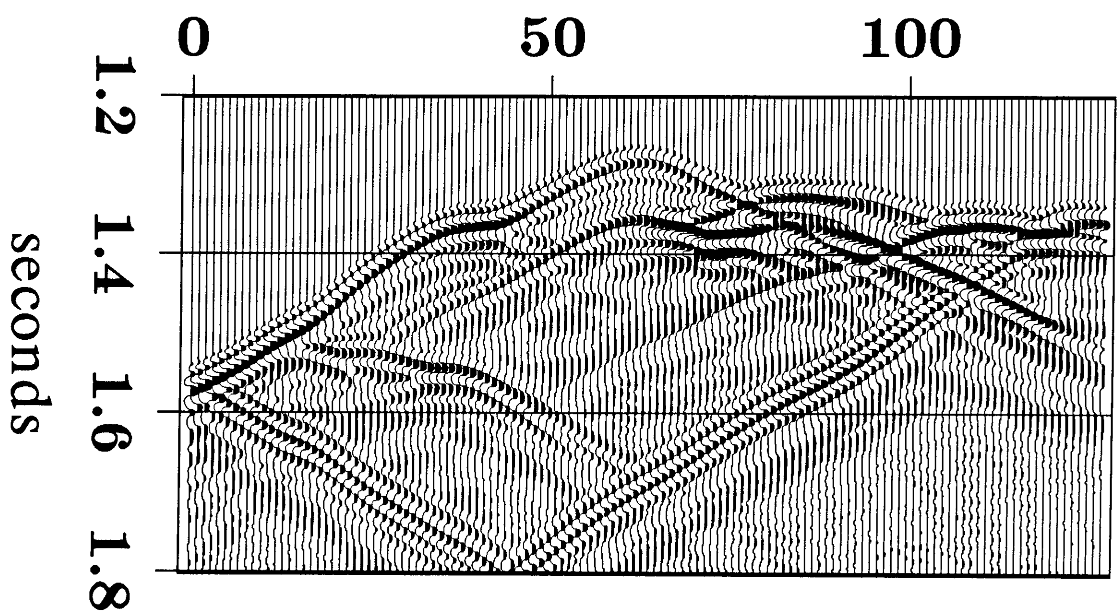


Figures 12e & f

Data from third constrained inversion

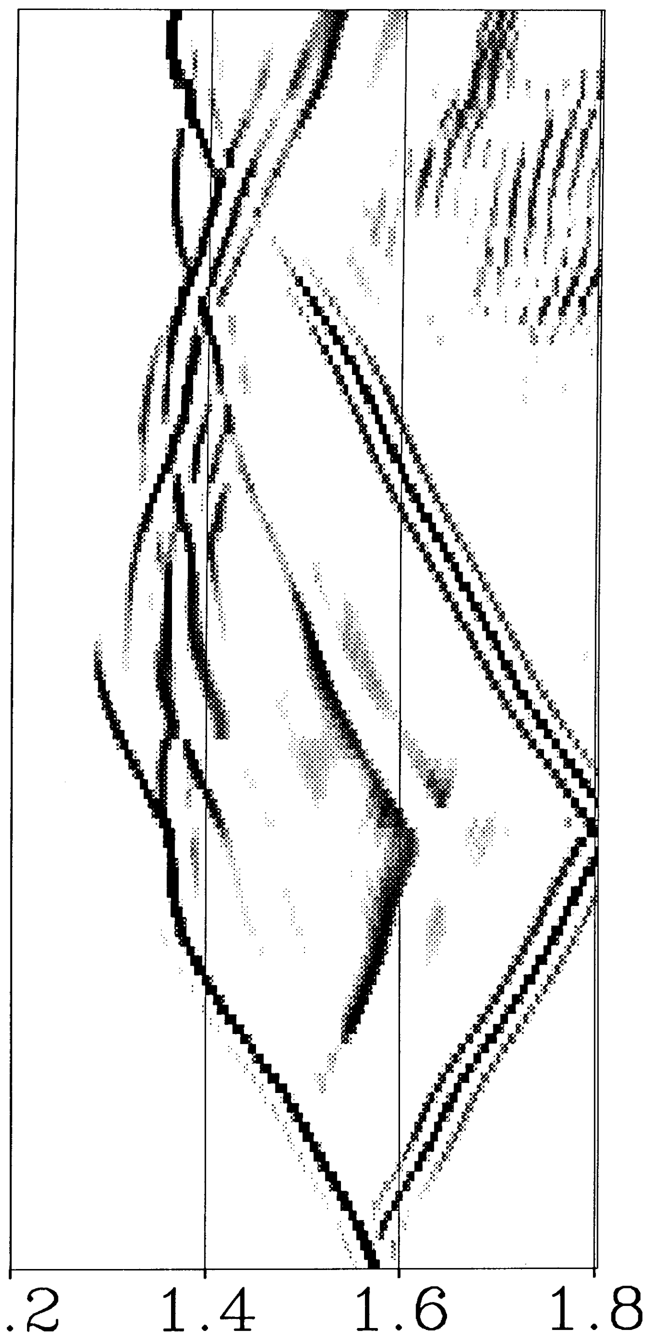


Data from fourth inversion

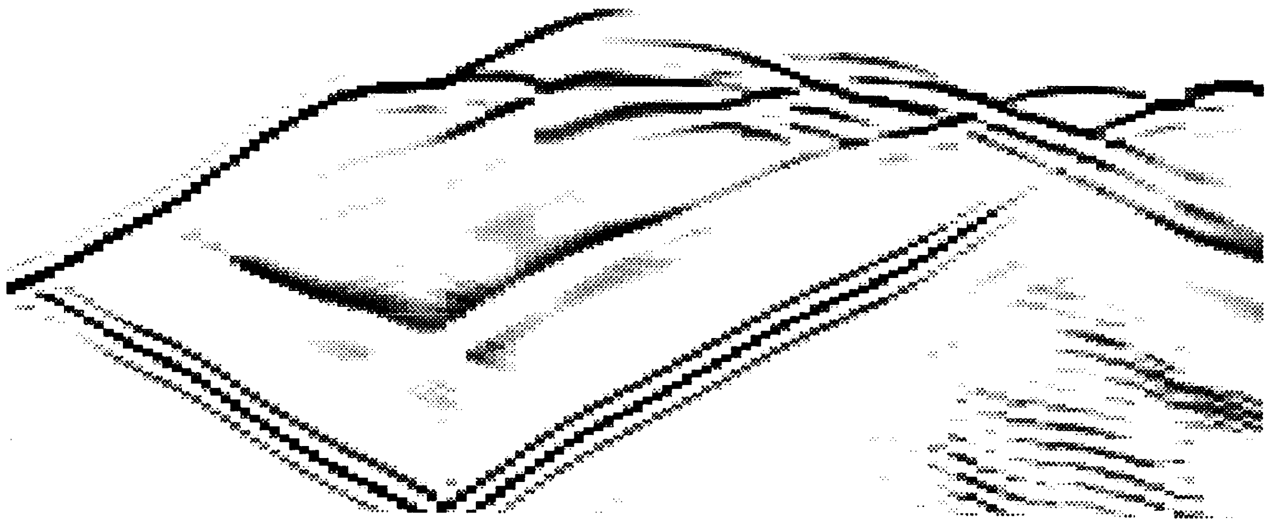


Figures 12g & h

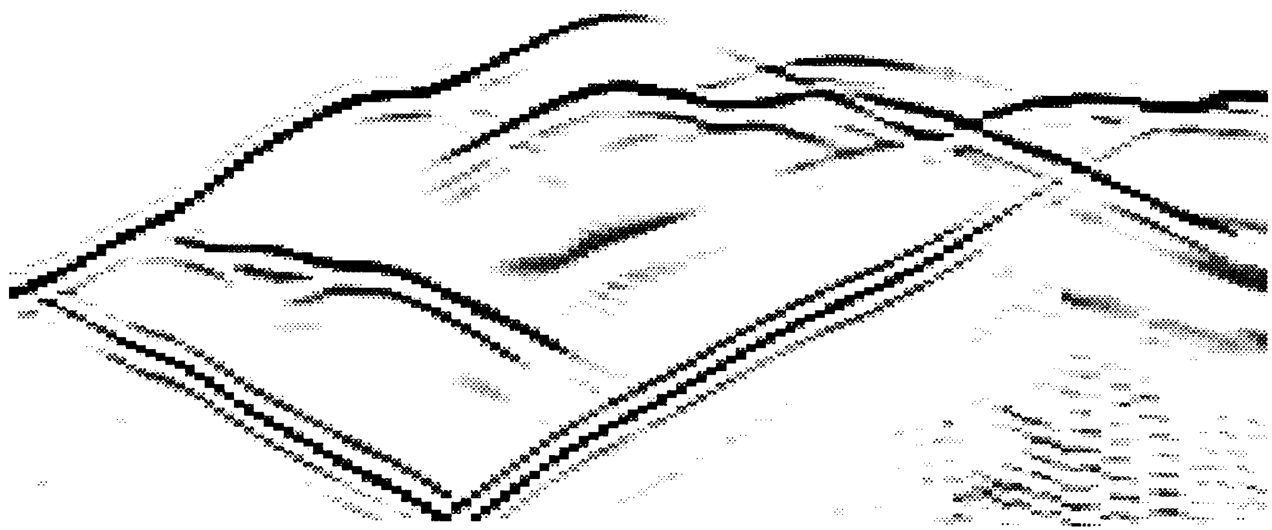
Data from original model overlayed with data from fourth inversion



seconds



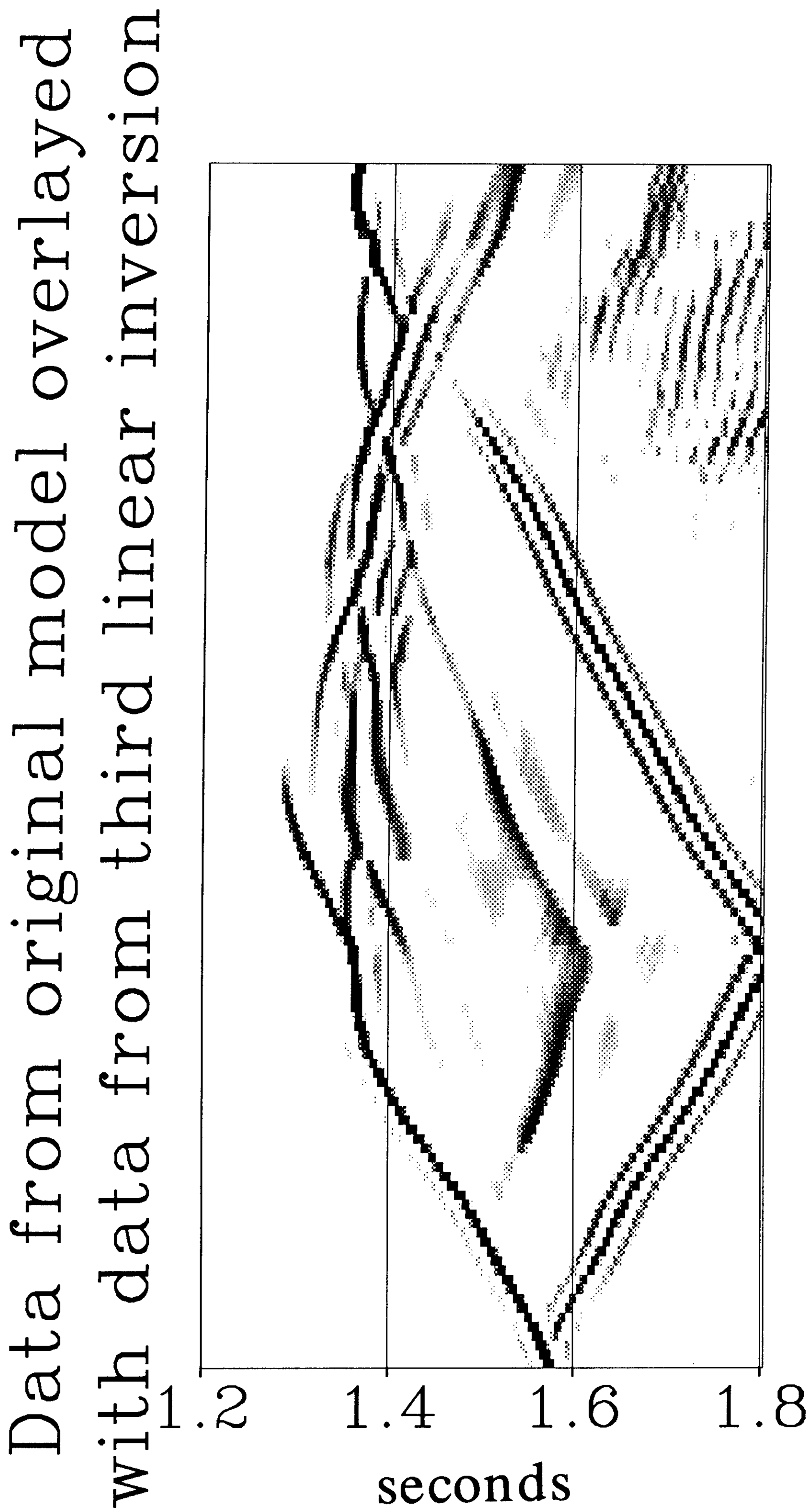
yellow



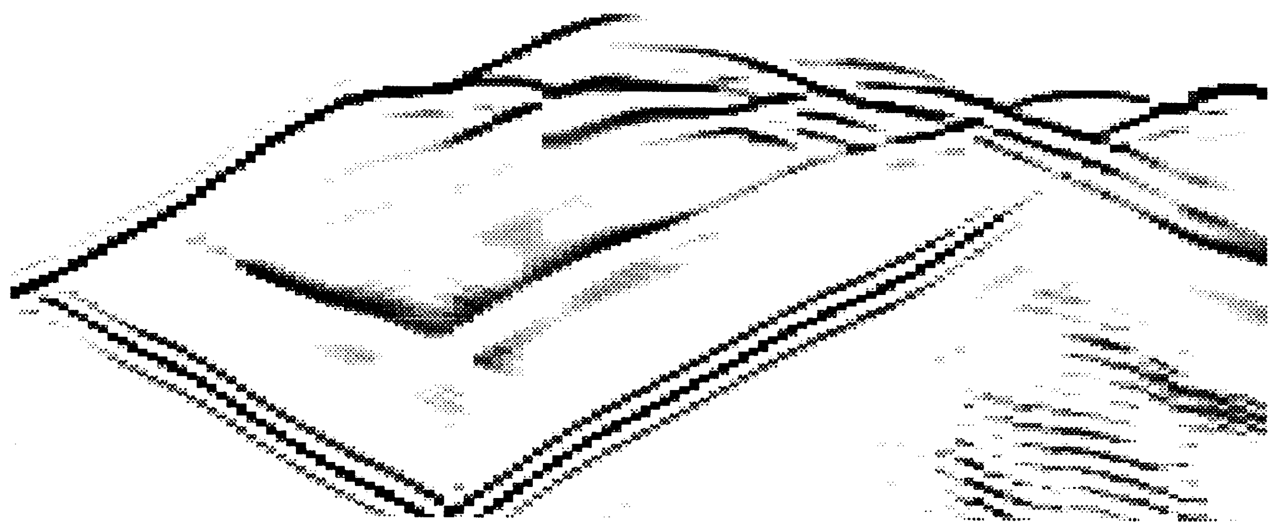
magenta

black

Figure 14:
Positive values of data in Figure 12c from original model are plotted green. Positive values of data in Figure 12h from fourth inversion are plotted purple. Where they overlap, they appear as brown. This method of overlaying data is an effective way of correlating arrivals between data and synthetics when the arrivals do not overlap.


Data from original model overlayed
with data from third linear inversion
1.2
1.4
1.6
1.8
seconds

cyan



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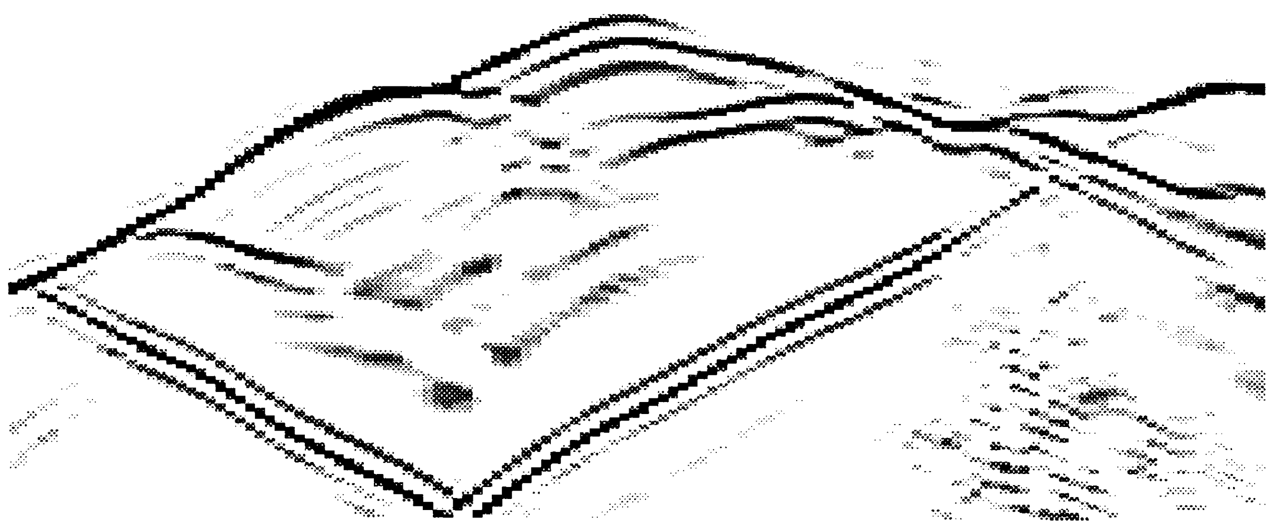

magenta

black

Figure 13:
Positive values of seismograms through original model are plotted green. Positive values of seismograms through third unconstrained linear inversion are plotted purple. Where the seismograms overlap, the two colors produce brown. Born only uses transmitted data when wave forms overlap, but some arrivals can be identified as corresponding to each other even when they do not overlap.