Prestack depth migration velocity analysis: linear theory revised

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ABSTRACT

A residual moveout correction applied to prestack-depth-migrated constant-offset sections approximates remigrating the constant-offset sections with a perturbed interval-slowness model. The residual moveout correction is applied to each offset for all points in the migrated image, and a moveout corrected stacked section is made for a range of residual slownesses. The result constitutes the data space for an optimization problem that estimates perturbations to the interval-slowness model that will result in the most coherent migrated image. Changes in the interval-slowness model can be linearly related to changes in the residual moveout curves that best approximate the offset-stacking curves required to form the migrated image produced by the perturbed interval-slowness model. The linear relation between changes in the interval-slowness model and the best fitting residual moveout curves is used to compute the gradient of the total coherence of the image with respect to the interval-slowness model from the precomputed data space. An iterative optimization algorithm driven by this gradient will estimate the interval-slowness model that produces the most coherent prestack depth migrated image of the data.

INTRODUCTION

Toldi (1985) introduced an operator that relates laterally varying interval slownesses to observed laterally varying stacking slownesses for horizontal reflectors. Fowler (1985, 1988) took a similar approach and derived an operator that relates laterally varying interval slownesses to dip-moveout corrected stacking slownesses and to prestack time migration slownesses. Al-Yahya (1987) introduced a residual moveout correction applied to migrated shot profiles that approximates the result of migrating with a new slowness model. In my previous report (Etgen, 1988), I extended Al-Yahya's residual moveout correction to account for more general structure. I also derived a linear operator similar to the ones used by Fowler and Toldi that related changes in the interval-slowness model used for shot profile migration to changes in the curvature of the residual moveout operator.

In this paper, I apply residual moveout to prestack-migrated constant-offset sections rather than prestack-migrated shot profiles. The residual moveout correction converts migrated constant-offset sections from images formed with an initial interval-slowness model to images formed with a perturbed interval-slowness model. The residual moveout correction can be decomposed into two corrections. The first correction is a differential moveout over offset much like NMO; the second is a residual zero-offset migration. The residual zero-offset migration is the same at a given point for all constant-offset sections, thus it will not change the offset coherence of any event. Moreover, Fowler (1988) found that the zero-offset migration component can confuse the velocity analysis by moving events. I now apply the residual moveout correction to only change the curvature of events over offset and not perform the residual zero-offset migration. For horizontal reflectors, this is equivalent to performing NMO and stack at fixed zero-offset traveltimes rather than performing NMO and stack and converting the stacked image from time to depth.

To derive the linear operator, I find the changes in traveltimes to reflectors due to changes in the interval-slowness model using tomography. A change in the traveltime to a reflector implies that the stacking trajectory over offset must change to move the event. I approximate these changes in the stacking trajectories with changes in the derived residual moveout curve. Changes in the stacking trajectory can be fit with linearized version of the residual moveout equation using least squares. The least squares fit combined with the tomography calculation defines an operator that relates changes in interval-slowness to changes in the curvature (or residual slowness) of a moveout operator. This operator can be used to find the change in the migrated image due to a change in the interval-slowness model, and hence, the gradient of the coherence of the image with respect to the interval-slowness model. This gradient drives an iterative optimization procedure that should estimate the interval-slowness model that produces the most coherent image using prestack depth migration.

RESIDUAL MOVEOUT CORRECTIONS

The image of a dipping reflector (shown in Figure 1) on a constant-slowness migrated constant-offset section is the tangent line to an ellipse. The equation of the ellipse is equation (1).

$$t = w\sqrt{z^2 + (s-x)^2} + w\sqrt{z^2 + (g-x)^2} \tag{1}$$

If the slowness used for migration is changed the location of the migrated reflector will change as shown in Figure 2. The new position of the migrated reflector is the

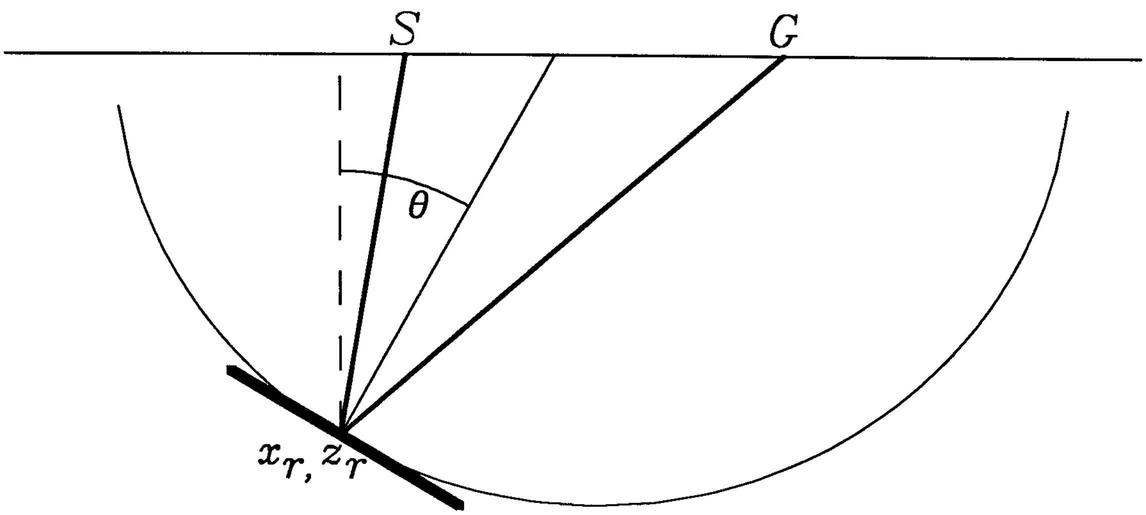


FIG. 1. The image of a constant-slowness-migrated dipping reflector is the tangent line to an ellipse with foci at source and receiver.

tangent line to another ellipse given by equation (2).

$$t = w_n\sqrt{z_n^2 + (s - x_n)^2} + w_n\sqrt{z_n^2 + (g - x_n)^2} \qquad (2)$$

To first order, a point on the original reflector will move in the direction normal to the reflector (Fowler 1987). In light of this, recast the equations for the migrated positions of the reflector to use the dip of the event θ and the arclength of the normal to the reflector σ at the given point as independent variables. For constant slowness the normal to the reflector is the zero-offset ray.

$$t = w\sqrt{\sigma^2\cos^2\theta + (s - \sigma\sin\theta)^2} + w\sqrt{\sigma^2\cos^2\theta + (g - \sigma\sin\theta)^2} \qquad (3)$$

$$t = w_n\sqrt{\sigma_n^2\cos^2\theta + (s - \sigma_n\sin\theta)^2} + w_n\sqrt{\sigma_n^2\cos^2\theta + (g - \sigma_n\sin\theta)^2} \qquad (4)$$

For a given event of interest the traveltime t is fixed. Combining equations (3) and (4) and solving for $\sigma_n(h)$ in terms of σ would give the position of the reflector originally at σ for different offsets after the slowness is changed. $h = (g - s)/2$ is called the half-offset. However, solving equations (3) and (4) for the arclength σ_n involves much algebra, and I was unable to obtain a useful expression for σ_n. I simplified equations (3) and (4), replacing the sum of the true shot to reflector and geophone to reflector distances with twice the root-mean-square of

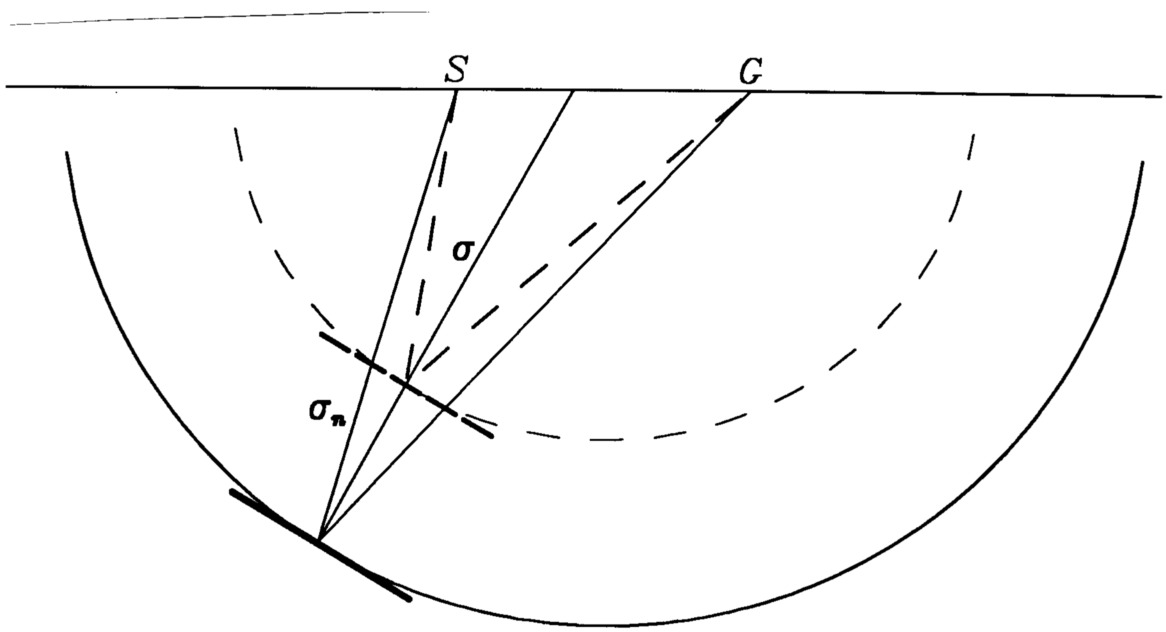


FIG. 2. The change in the position of a migrated reflector due to a change in migration slowness is in the direction normal to the reflector.

those distances. This turns the double square-root equations into single square-root equations. Equations (5) and (6) give the approximations to equations (3) and (4).

$$t = w\sqrt{\sigma^2 + \frac{s^2+g^2}{2} - (s+g)\sigma\sin\theta} \tag{5}$$

$$t = w_n\sqrt{\sigma_n^2 + \frac{s^2+g^2}{2} - (s+g)\sigma_n\sin\theta} \tag{6}$$

Combining equations (5) and (6) and solving for $\sigma(h)$ in terms of σ_n, w_n, w, s, and g tells which points in the migrated image to move to the output location σ_n when the slowness model changes from w to w_n. $\sigma(h)$ can be called a residual moveout curve or an offset-stacking trajectory.

$$\sigma(h) = \frac{-(s+g)}{2}\sin\theta + \sqrt{\gamma^2\sigma_n^2 + (\gamma^2-1)\frac{g^2+s^2}{2} + \gamma^2\sigma_n\sin\theta(g+s) + \frac{(g+s)^2\sin^2\theta}{4}} \tag{7}$$

The residual moveout equation does not depend on w_n or w directly, but only on their ratio $\gamma = w_n/w$. Equation (7) performs residual moveout in depth. An event on a zero-offset section moves from $\sigma_o = \sigma(0)$ to $\sigma_n(0) = w/w_n\ \sigma_o$. This movement is a residual zero-offset migration. The velocity information is contained in the differential moveout over offset; the residual zero-offset migration merely changes the location of the event not its coherence over offset. Figure 3 shows residual

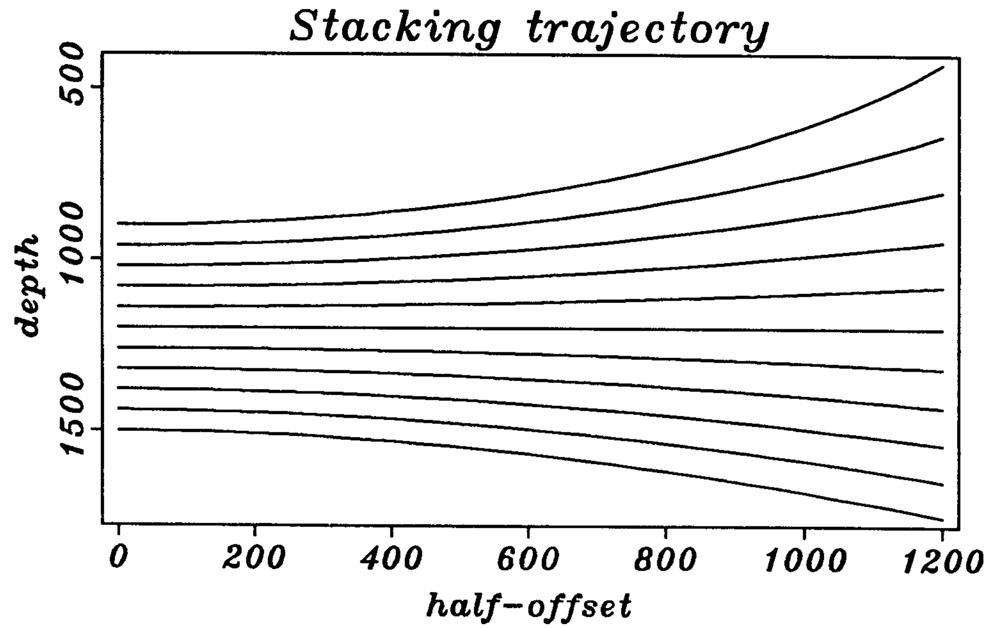


FIG. 3. Residual moveout curves for a horizontal reflector given by equation (7). Each curve is the offset stacking trajectory for a different γ. γ ranges between .8 and 1.2. The result of stacking along each curve is placed at depth 1 200. The zero-offset of one event appears at different depths for different γ.

moveout curves for a horizontal reflector. Figure 4 shows residual moveout curves for a dipping reflector. When the reflector is dipping, the residual moveout curves lie in a plane that is constant with offset containing the normal to the reflector.

Rather than stack the data and place the result at σ_n, I will stack the data and place the result at $\sigma_o = w_n/w\ \sigma_n$, the zero-offset of the stacking curve. Equation (8) gives offset-stacking curves that pass through the same location on the original migrated zero-offset section for all values of γ. This is analogous to stacking at fixed zero-offset traveltimes rather than performing NMO and stack for fixed locations in depth.

$$\sigma(h) = \frac{-(s+g)}{2}\sin\theta + \sqrt{\sigma_o^2 + (\gamma^2 - 1)\frac{g^2+s^2}{2} + \gamma\ \sigma_o \sin\theta(g+s) + \frac{(g+s)^2\sin^2\theta}{4}} \tag{8}$$

Again $\gamma = w_n/w$, which controls the curvature of the residual moveout operator. Figure 5 shows residual moveout curves (or offset-stacking trajectories) calculated with equation (8) for a horizontal reflector. For horizontal reflectors, these curves are simply residual NMO (normal moveout) curves. Figure 6 shows residual moveout curves for a dipping reflector. As in Figure 4, the residual stacking trajectories are contained in the plane that contains the normal to the reflector. As opposed to

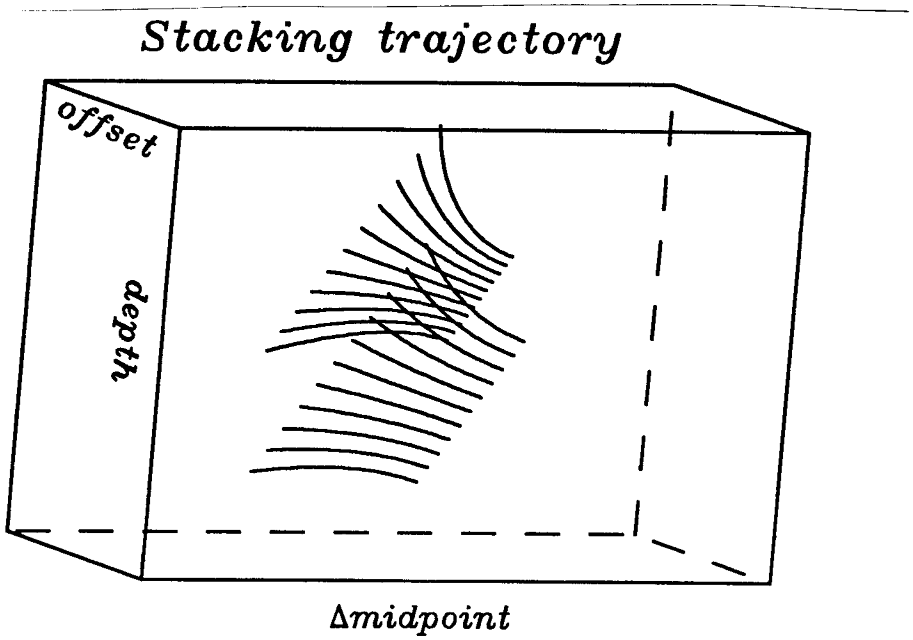


FIG. 4. Residual moveout curves for points on dipping reflectors given by equation (7). The dip of the reflector, $\theta = 26$ degrees. The stacking trajectories are now curves in 3-space. All curves are contained in the plane of the normal to the reflector. A given event will appear in different zero-offset locations as γ changes.

Figures 3 and 4, the offset-stacking trajectories for a given output location all pass through the same zero-offset location in the input.

LINEAR THEORY

This section describes a linear relation between changes in the interval-slowness model used for prestack depth migration and changes in the residual moveout curves governed by equation (8). Consider a trial reflector shown in Figure 7. A perturbation to the interval-slowness model will produce a change in travel time of any ray passing through the perturbation. Changes in the traveltime along any ray from a source to the trial reflector point to a receiver result in changes in the location of an event in the appropriate migrated constant-offset section. The change in the position of an event originally at the trial reflector point implies we must change the offset-stacking trajectory to find the new event that will move to the trial reflector point when the interval-slowness model is perturbed. This change in stacking trajectory can be linearly related to a change in the residual moveout curve described by equation (8). We can apply residual moveout corrections to the migrated constant-offset sections and form stacked images for a range of γ. The linear relation between changes in the interval-slowness model and changes in the best

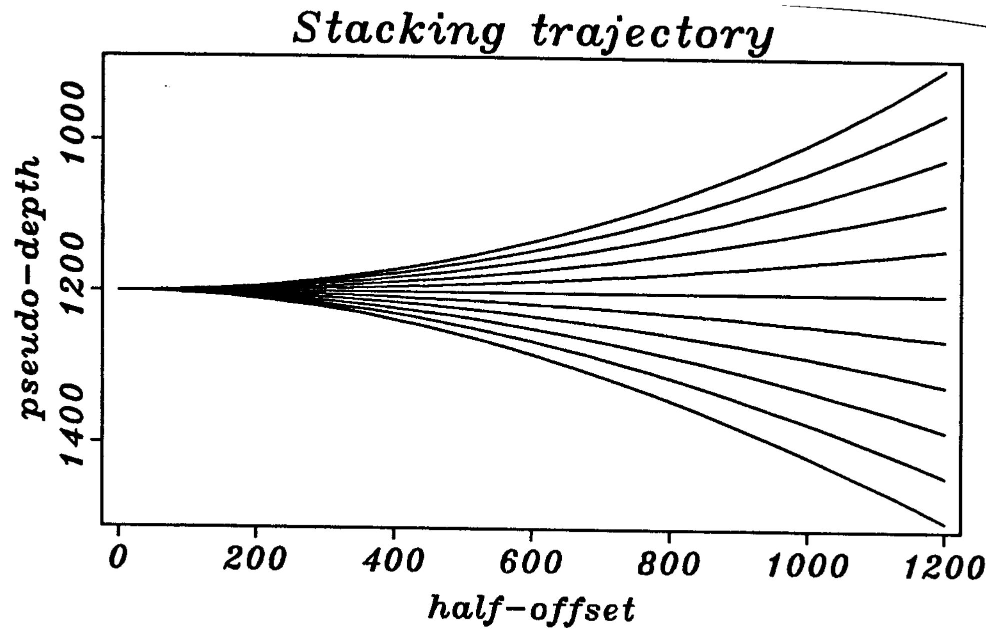


FIG. 5. Residual moveout curves for a horizontal reflector given by equation (8). Each curve is the offset stacking trajectory for a different γ. γ ranges between .8 and 1.2. The result of stacking along each curve is placed at the common zero-offset of all stacking curves. This residual moveout operator only changes the curvature of an event not its zero-offset location.

fitting residual moveout curves can be used to compute the gradient of objective function (total stack semblance) with respect to the interval-slowness model.

Δt due to $\Delta w(x, z)$

If the interval-slowness model $w(x, z)$ is perturbed by $\Delta w(x, z)$, the traveltime of any ray passing through the perturbation will change. Write the traveltime along a ray from any subsurface point to a surface point (a source or receiver) as

$$t_{ray} = \int_{ray} w(x, z) dS \quad . \tag{9}$$

For small perturbations to the interval-slowness model, $\Delta w(x, z)$, we can apply Fermat's principle and calculate the change in traveltime due to a perturbation to the interval-slowness model as:

$$\Delta t = \int_{ray} \Delta w(x, z) dS \ . \tag{10}$$

The integral of the slowness perturbation is evaluated along the unperturbed ray.

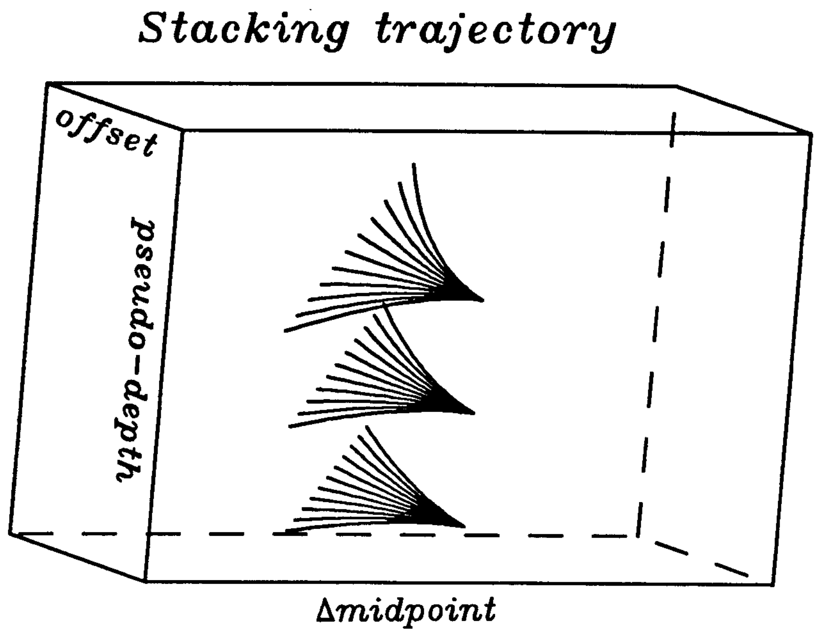


FIG. 6. Residual moveout curves for points on dipping reflectors using equation (8). As in Figure 4, the stacking trajectories are curves in 3-space contained in the plane of the normal to the reflector. The result of stacking over any curve is placed at the zero-offset location common to all the stacking curves. Events stay at fixed zero-offset locations.

$\Delta\sigma_s$ due to Δt

For a given reflector point and a given dip θ, the family of rays that obey Snell's law (angle of incidence = angle of reflection) at the reflector point are the rays along which most reflected energy travels. These rays are called specular rays. A change in traveltime along a specular ray caused by a change in interval slowness will lead to a change in the location of the image of the reflector. Conversely, to form the stacked image at a given point, we must change how the migrated constant-offset sections are stacked.

For a constant slowness medium, the derivative of the traveltime with respect to a change in the location of a reflector in the direction of its normal can be calculated using the chain rule. The arclength normal to the reflector is σ_r; $\Delta x_r = \sin\theta\ \Delta\sigma_r$; $\Delta z_r = \cos\theta\ \Delta\sigma_r$. Write the traveltime as:

$$t = w\sqrt{z_r^2 + (s - x_r)^2} + w\sqrt{z_r^2 + (g - x_r)^2}\ . \tag{11}$$

x_r, z_r are the coordinates of the reflector point. The derivative of traveltime with respect to movement normal to the reflector can be written as:

$$\frac{\partial t}{\partial \sigma_r} = \frac{\partial t}{\partial x_r}\frac{\partial x_r}{\partial \sigma_r} + \frac{\partial t}{\partial z_r}\frac{\partial z_r}{\partial \sigma_r}\ ;$$

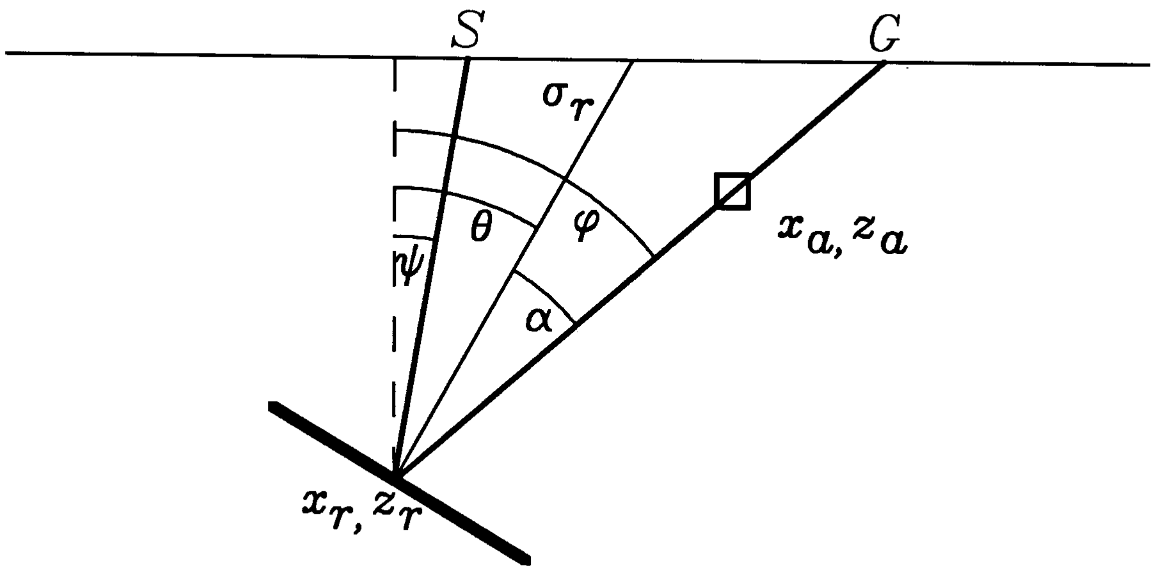


FIG. 7. Specular rays for a dipping reflector. If the specular ray for a given offset goes through the slowness anomaly, the position of the reflector will be perturbed.

where

$$\frac{\partial t}{\partial x_r}\frac{\partial x_r}{\partial \sigma_r} = w\Big(\frac{s - x_r}{\sqrt{z_r^2 + (s - x_r)^2}} + \frac{g - x_r}{\sqrt{z_r^2 + (g - x_r)}}\Big)\sin\theta \; ; \tag{12}$$

$$\frac{\partial t}{\partial z_r}\frac{\partial z_r}{\partial \sigma_r} = w\Big(\frac{z_r}{\sqrt{z_r^2 + (s - x_r)^2}} + \frac{z_r}{\sqrt{z_r^2 + (g - x_r)}}\Big)\cos\theta \; .$$

The derivative of t with respect to σ_r does not depend on t, x, or z explicitly, only on the dip and opening angles of the rays at the reflector (see Figure 7) and can be rewritten as:

$$\frac{\partial t}{\partial \sigma_r} = w(\cos\psi\cos\theta + \sin\psi\sin\theta + \cos\varphi\cos\theta + \sin\varphi\sin\theta) = 2\; w\cos\alpha \quad . \tag{13}$$

Since the total traveltime t_r of an event in the unmigrated data is fixed, any change in the traveltime Δt_r along the specular rays must be compensated by a change in traveltime due to a change in the position of the reflector.

$$\Delta t_r + \frac{\partial t}{\partial \sigma_r}\Delta\sigma_r = 0 \quad . \tag{14}$$

Solving for the change in the position of the reflector,

$$\Delta\sigma_r = -\frac{\partial \sigma_r}{\partial t}\Delta t_r \quad . \tag{15}$$

If the reflector originally at (x_r, z_r) moves when the interval-slowness model is perturbed, the stacking trajectory for the given location must change. To first order, the event can be moved by $\Delta\sigma_r$ if the stacking trajectory is perturbed by $-\Delta\sigma_r$. Write the change in the stacking trajectory as:

$$\Delta\sigma_s = \frac{\partial\sigma_s}{\partial t}\,\Delta t = \frac{1}{2\,w}\,\frac{1}{\cos\alpha}\,\Delta t \quad . \tag{16}$$

Although the derivation assumed slowness was constant, to an approximation, the derivative can be used for a non-constant slowness model if w is taken to be the slowness at the reflector, $w(x_r, z_r)$.

$$\Delta\sigma_s = \frac{\partial\sigma_s}{\partial t}\Delta t = \frac{1}{2\,w(x_r, z_r)}\frac{1}{\cos\alpha}\Delta t \qquad . \tag{17}$$

$\Delta\gamma$ and $\Delta\sigma_o$ due to $\Delta\sigma_s$

The change in the offset-stacking trajectory $\Delta\sigma_s$ computed in the last two sections can be related to a change in the best fitting residual moveout curve generated by equation (8). The best fitting residual moveout curve may require not only changes in γ, but also changes in σ_o the reference arclength normal to the reflector. Finding the derivatives of σ_s with respect to γ and σ_o linearizes equation (8) about a given value of γ and σ_o. A change in σ_s due to a change in γ and σ_o can be written as:

$$\Delta\sigma_s = \frac{\partial\sigma_s}{\partial\gamma}\Delta\gamma + \frac{\partial\sigma_s}{\partial\sigma_o}\Delta\sigma_o \quad . \tag{18}$$

The expressions for the derivatives $\partial\sigma_s/\partial\gamma$ and $\partial\sigma_s/\partial\sigma_o$ can be found in appendix A. Immediately following prestack depth migration before the slowness model is changed, linearize about the γ and σ_o that produce no residual moveout $\gamma = 1$ and $\sigma_o = z/\cos\theta$. As the slowness model is perturbed the reference values used for linearization change.

Rather than finding $\Delta\sigma_s$ for each offset from $\Delta\gamma$ and $\Delta\sigma_o$, we need to do the opposite, find the $\Delta\gamma$ and $\Delta\sigma_o$ that best fit the $\Delta\sigma_s(h)$ of the migrated constant-offset sections at a given point in space by minimizing:

$$E = \sum_{h_{min}}^{h_{max}} (\Delta\sigma_s(h) - \frac{\partial\sigma_s(h)}{\partial\gamma}\Delta\gamma - \frac{\partial\sigma_s(h)}{\partial\sigma_o}\Delta\sigma_o)^2 \tag{19}$$

using least squares. The solution to this least-squares problem for $\Delta\gamma$ and $\Delta\sigma_o$, is the solution to the normal equations:

$$\begin{pmatrix} \sum_{h_{min}}^{h_{max}} A_h^2 & \sum_{h_{min}}^{h_{max}} A_h B_h \\ \sum_{h_{min}}^{h_{max}} A_h B_h & \sum_{h_{min}}^{h_{max}} B_h^2 \end{pmatrix} \begin{pmatrix} \Delta\gamma \\ \Delta\sigma_o \end{pmatrix} = \begin{pmatrix} \sum_{h_{min}}^{h_{max}} A_h \Delta\sigma_s(h) \\ \sum_{h_{min}}^{h_{max}} B_h \Delta\sigma_s(h) \end{pmatrix} \quad . \tag{20}$$

A_h and B_h are $\partial\sigma_s(h)/\partial\gamma$ and $\partial\sigma_s(h)/\partial\sigma_o$ respectively, at a fixed spatial location. Equation (21), the solution to equation (20), gives changes in the residual moveout curve that best fits the changes in stacking trajectories.

$$\begin{pmatrix} \Delta\gamma \\ \Delta\sigma_o \end{pmatrix} = \begin{pmatrix} \Gamma_\gamma \\ \Gamma_{\sigma_o} \end{pmatrix} \Delta\sigma_s \tag{21}$$

Expressions for Γ_γ and Γ_{σ_o} can be found in appendix B.

Figures 8 and 9 show the operators Γ_γ and Γ_{σ_o} for one point on a horizontal reflector and a dipping reflector respectively. The value of the operator as a function of offset is the change in γ or σ_o due to a unit change $\Delta\sigma_s$ in the stacking trajectory at that offset. Figure 10 shows an example of best fitting residual moveout curves for perturbations to the offset-stacking trajectory at small and large offsets. Figures 8, 9, and 10 show that the effect of a perturbation to the stacking trajectory at small offsets is opposite to the effect of a perturbation at large offsets. A positive perturbation to the stacking trajectory at inner offsets leads to a negative change in the apparent residual slowness, $\gamma = w_n/w$. The perturbation at inner offsets leads to a pull down of the zero-offset of the best fitting residual moveout curve. A positive perturbation to the stacking trajectory at outer offsets leads to a positive change in the apparent residual slowness, $\gamma = w_n/w$. The perturbation at outer offsets leads to a pull up of the zero-offset of the best fitting residual moveout curve.

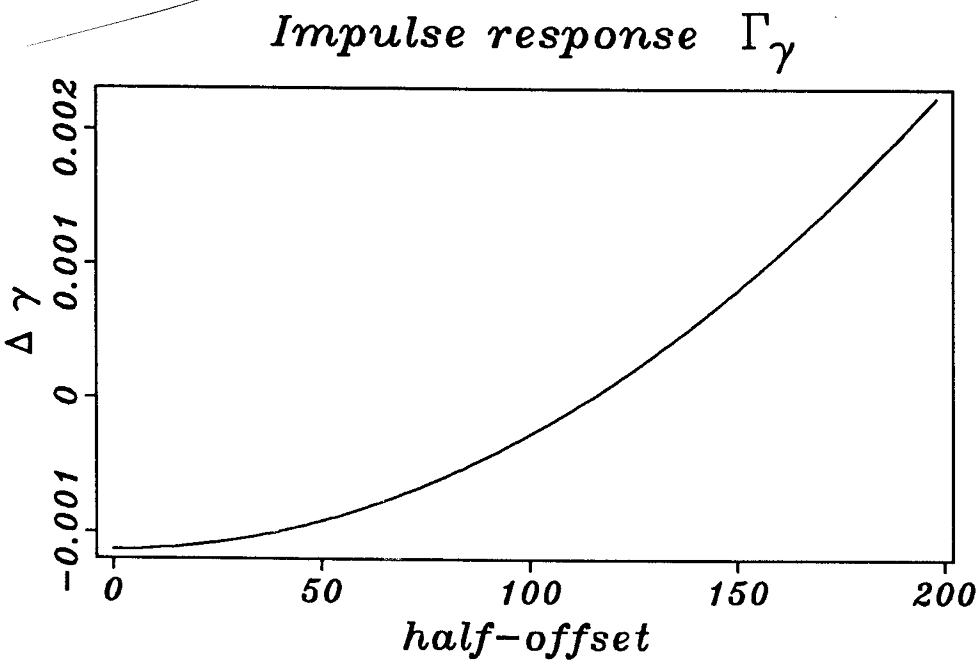
Equation 21 can be combined with the operators described in the previous two sections to write one operator that relates changes in interval slowness to changes in the parameters that describe the best fitting residual moveout curve at a given point in space. $\partial t/\partial w$ converts changes in interval slowness to changes in traveltime. It is a tomography operator (Fowler, 1987); call it T. $\partial\sigma_s/\partial t$ converts changes in traveltime to changes in the offset-stacking trajectory; call it Θ. Writing the operator in compact notation,

$$\begin{pmatrix} \Delta\gamma \\ \Delta\sigma_o \end{pmatrix} = \begin{pmatrix} \Gamma_\gamma \\ \Gamma_{\sigma_o} \end{pmatrix} \Theta\, T\, \Delta w \;=\; \begin{pmatrix} G_\gamma \\ G_{\sigma_o} \end{pmatrix} \Delta w \;. \tag{22}$$

The linear operator in equation (22) relates changes in the interval-slowness model to changes in the best fitting residual moveout curve at a fixed spatial location. However, events in the residual-moveout corrected images obtained from equation (8) are not at fixed spatial locations. The spatial location of a fixed event on the residual-moveout corrected images depends on γ and σ_o. The next section describes how to evaluate the the operator at fixed points in the data space rather than at fixed spatial locations.

Data space and model space grids

Residual moveout governed by equation (8) corrects migrated constant-offset sections for differential moveout over offset due to a change in the slowness model



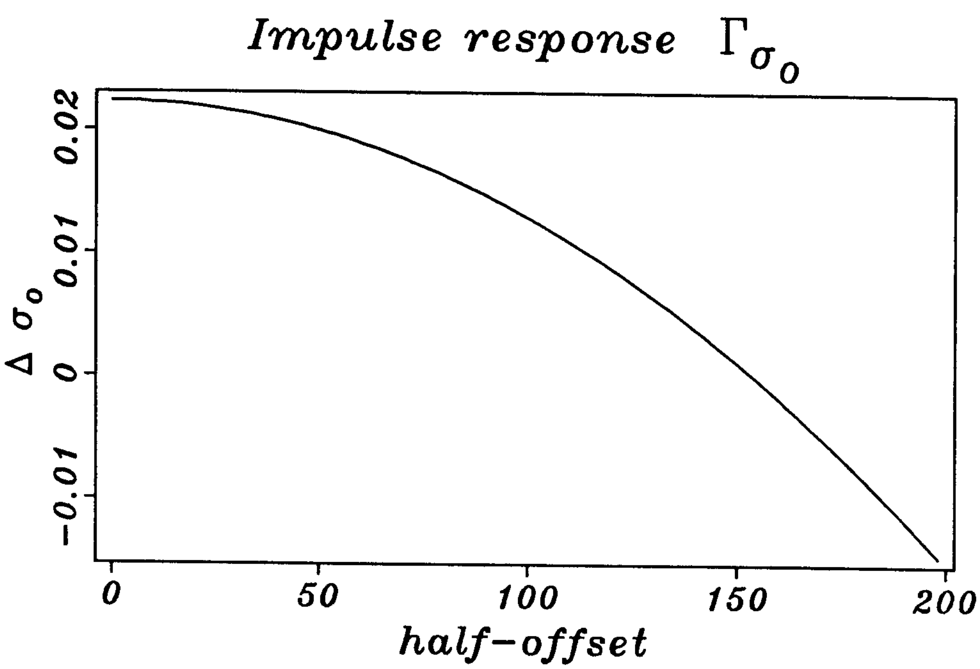
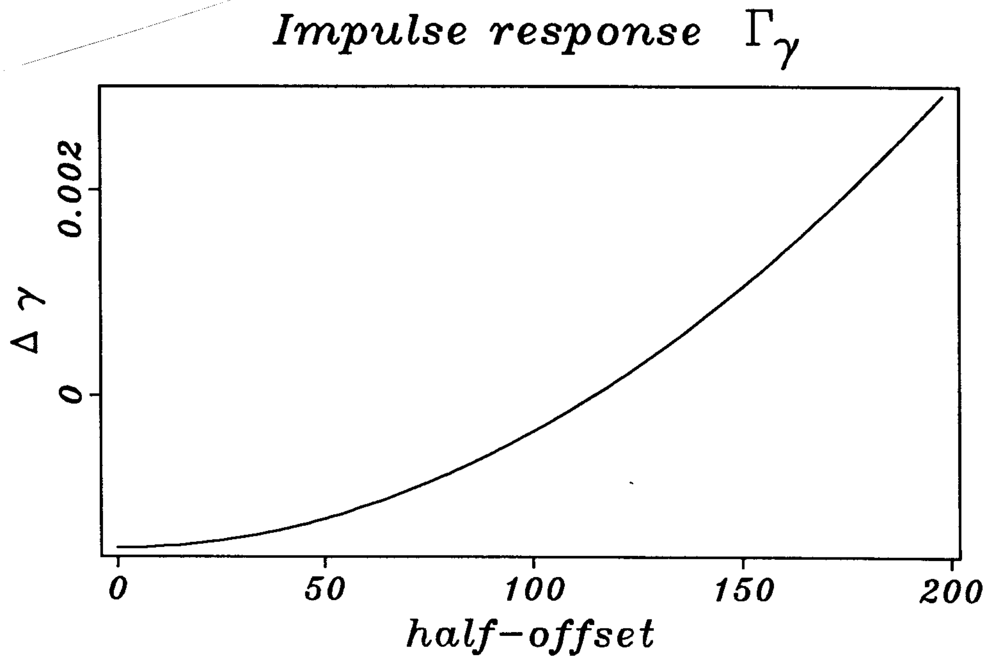


FIG. 8. The top graph shows the change in γ for each offset due to a unit change in the stacking trajectory at that offset $(\Delta\sigma_s = \delta(h - h_o))$ for a horizontal reflector. Perturbations at inner offsets give a negative response, while perturbations at outer offsets give a positive response. The bottom graph shows the change in σ_o for each offset due to a unit change in the stacking trajectory at that offset. Inner offsets show a positive response and outer offsets a negative response.



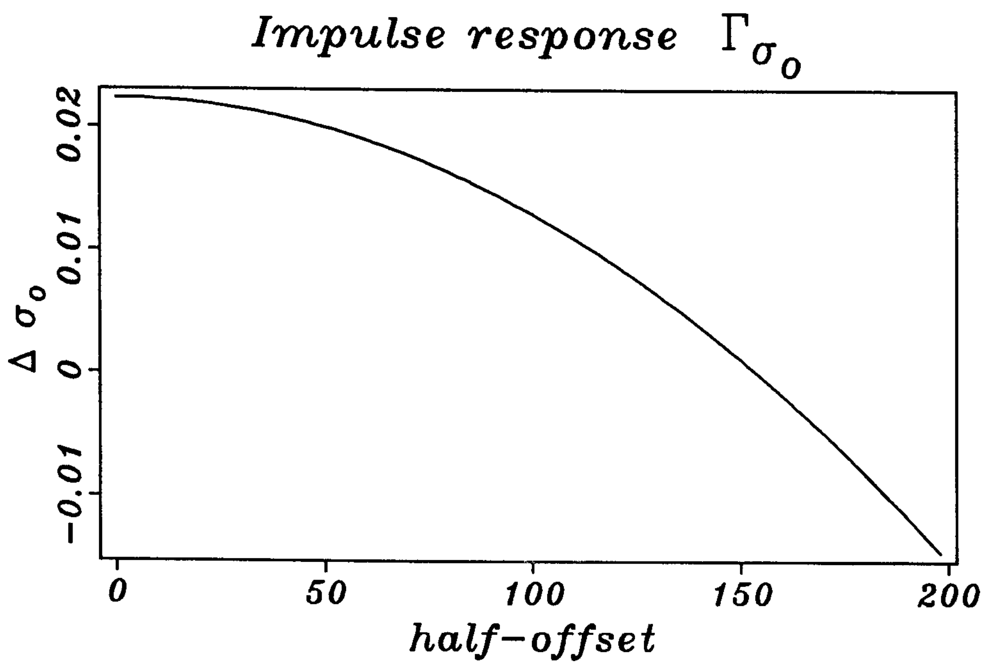
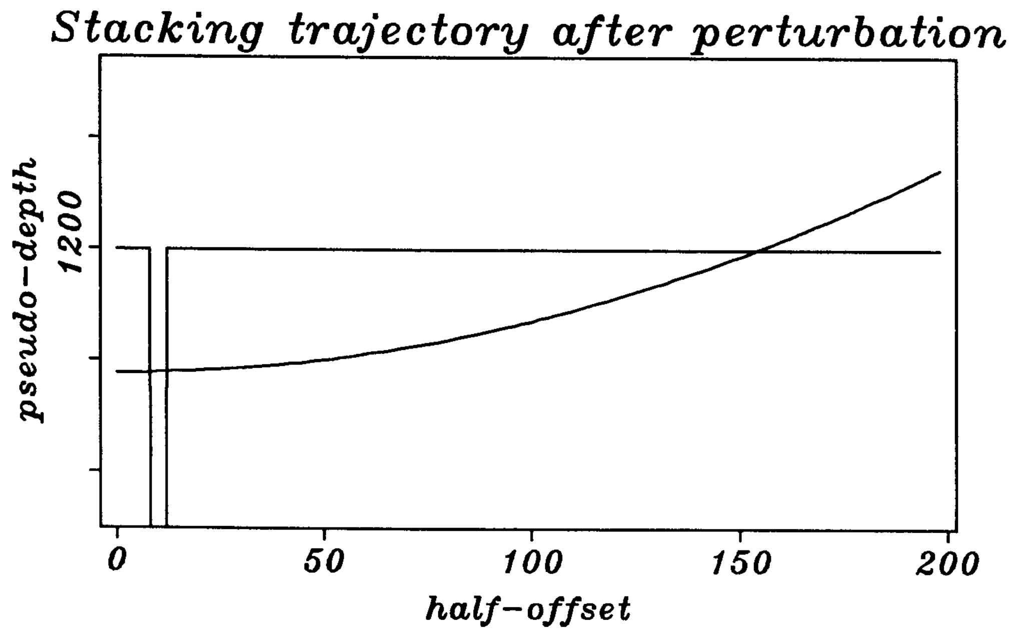


FIG. 9. The top graph shows the change in γ for each offset due to a unit change in the stacking trajectory at that offset ($\Delta\sigma_s = \delta(h - h_o)$) for a dipping reflector, $\theta = 65$ degrees. The bottom graph shows the change in σ_o for each offset due to a unit change in the stacking trajectory at that offset. The operators are similar to the Γ_γ and Γ_{σ_o} operators in Figure 8. The magnitudes of the operators change slightly.



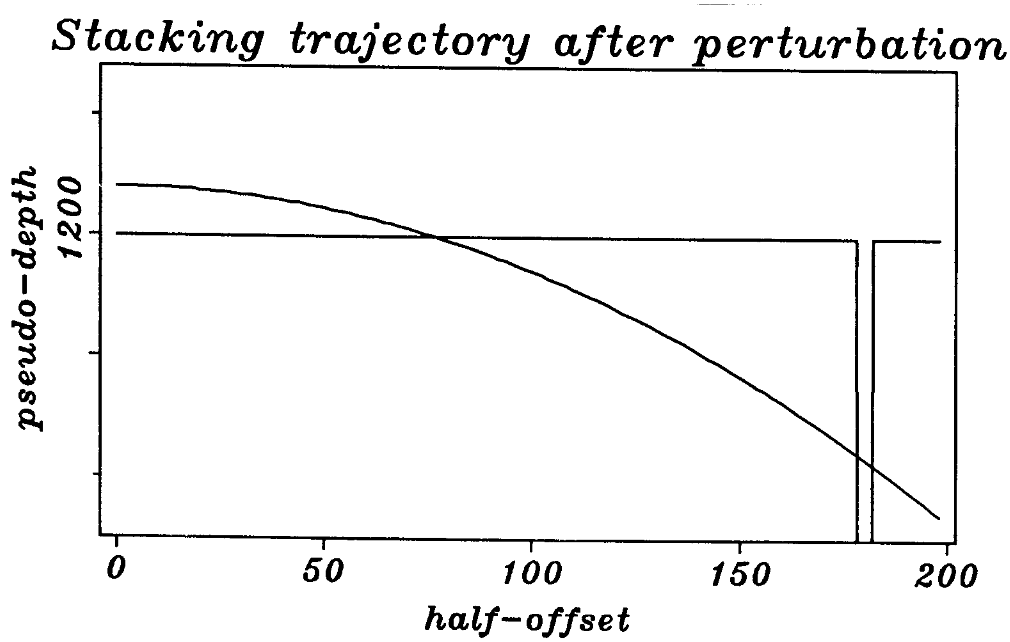


FIG. 10. Best fitting residual moveout curves for perturbations to an inner offset and to an outer offset. Both cause changes to the curvature and the zero-offset location of the best fitting stacking trajectory. The plot scale is expanded to show the effect more clearly. The perturbation is a unit change in stacking trajectory for one offset.

and does not perform residual zero-offset migration. The zero-offset location of an event initially at σ_o stays at σ_o after residual moveout for all γ. When $\gamma = 1$, the true spatial location of the event is σ_o. When $\gamma \neq 1$, the true spatial location changes from σ_o to $\sigma_n = 1/\gamma\ \sigma_o$. I will call the output space of residual-moveout corrected stacks as a function of γ the pseudo-depth space. This "space" is the data space for estimating changes to the interval-slowness model. Spatial coordinates in the pseudo-depth space are denoted by coordinate pairs (ξ, η); derivatives evaluated at fixed locations in pseudo-depth will be denoted by the notation $\delta\gamma/\delta\xi$ for example. When $\gamma \neq 1$, events are not at their true spatial locations but at some scale factor (γ) times their true spatial locations.

The interval-slowness model and the linear operator just described are evaluated at true spatial locations. I will denote true spatial locations with coordinate pairs (x, z). Derivatives evaluated at fixed true spatial locations will be denoted by the usual partial derivative notation $\partial\gamma/\partial w$ for example. To apply the linear operator to relate changes in interval slowness to changes in the residual moveout curve for some event (in the fixed pseudo-depth space), we maintain a map between the pseudo-depth location of an event (ξ, η) and its true depth (x, z). As the interval-slowness model changes, the true spatial location (x, z) of a fixed event in (ξ, η) space will change.

The linear operator that relates changes in the interval-slowness model to changes in γ is defined at fixed (x, z). Perturbing the interval-slowness model at (x_a, z_a) will change the best fitting γ value of an event at (ξ, η) in the data space corresponding to the (x, z) location under consideration, and also cause (x, z) to map to a new value in the data space (ξ', η'). Let γ, ξ, and η denote values before the interval slowness is perturbed and γ' ,ξ', and η' denote values after the interval slowness is perturbed. Applying the linear operator to the perturbation of the interval-slowness model write:

$$\gamma'(\xi', \eta') = \gamma(\xi, \eta) + \sum_{x_a, z_a} \frac{\partial\gamma}{\partial w} \Delta w_{x_a, z_a} \quad . \tag{23}$$

We are interested however, in knowing how γ changes for a fixed location (ξ, η) in the data space. To first order,

$$\gamma'(\xi, \eta) = \gamma'(\xi' - \Delta\xi, \eta' - \Delta\eta) = \gamma'(\xi', \eta') - \frac{\delta\gamma}{\delta\xi}\Delta\xi - \frac{\delta\gamma}{\delta\eta}\Delta\eta \quad . \tag{24}$$

The $\Delta\xi$ and $\Delta\eta$ in equation (24) represent the change in where a point (x, z) in the true depth space maps in (ξ, η) space due to the change in the interval-slowness model. Combining the above equations, we can solve for the change in γ at a fixed point (ξ, η).

$$\gamma'(\xi, \eta) = \gamma(\xi, \eta) + \sum_{x_a, z_a} \frac{\partial\gamma}{\partial w} \Delta w_{x_a, z_a} - \frac{\delta\gamma}{\delta\xi}\Delta\xi - \frac{\delta\gamma}{\delta\eta}\Delta\eta \quad ; \tag{25}$$

where

$$\Delta\xi = \sin\theta\ \Delta\sigma_o \quad ; \quad \Delta\eta = \cos\theta\ \Delta\sigma_o,$$

and

$$\Delta\sigma_o = \sum_{x_a, z_a} \frac{\partial \sigma_o}{\partial w} \Delta w_{x_a, z_a} \quad .$$

We can now write the linear operator that relates perturbations in the interval-slowness model, on a fixed (x, z) grid, to changes in γ the parameter of the best fitting residual moveout curve on a fixed (ξ, η) grid.

$$\Delta\gamma(\xi, \eta) = \frac{\partial \gamma}{\partial w} \Delta w - \left[\frac{\delta \gamma}{\delta \xi} \sin\theta + \frac{\delta \gamma}{\delta \eta} \cos\theta \right] \frac{\partial \sigma_o}{\partial w} \Delta w \quad . \tag{26}$$

Expressing the derivatives above as the operators of equation (20) write

$$\Delta\gamma(\xi, \eta) = \frac{\delta \gamma}{\delta w} \, \Delta w = \left[G_\gamma - \left[\frac{\delta \gamma}{\delta \xi} \sin\theta + \frac{\delta \gamma}{\delta \eta} \cos\theta \right] G_{\sigma_o} \right] \Delta w \quad . \tag{27}$$

Gradient of the objective function

Residual moveout correction and stacking is applied to the migrated constant-offset sections for a range of γ building the data space. The objective function I seek to maximize is the sum of the stack semblance over all pseudo-depth locations This objective function Q is evaluated for fixed events rather than for fixed spatial locations.

$$Q(\gamma(\xi, \eta)) = \sum_{\xi, \eta} S(\gamma(\xi, \eta), \xi, \eta) \tag{28}$$

S is the stack semblance at a given location (ξ, η) in the data space for a given γ. Applying the chain rule, write the *ith* component of the gradient of the objective function with respect the interval-slowness model.

$$\nabla_{w_i} Q \;=\; \sum_{\xi, \eta} \frac{\delta S(\gamma, \xi, \eta)}{\delta w_i} \;=\; \sum_{\xi, \eta} \frac{\delta S(\xi, \eta)}{\delta \gamma} \frac{\delta \gamma(\xi, \eta)}{\delta w_i} \quad . \tag{29}$$

Since we have precomputed values of S for each ξ, η, and γ, the derivative of the objective function with respect to γ can be calculated by finite differences.

$$\frac{\delta S(\xi, \eta)}{\delta \gamma} = \frac{1}{\Delta \gamma} \left[S(\gamma(\xi, \eta) + \Delta\gamma, \xi, \eta) - S(\gamma(\xi, \eta), \xi, \eta) \right] \tag{30}$$

Multiplying $\delta S / \delta \gamma$ by $\delta \gamma / \delta w$ is equivalent to multiplying by the transpose of the linear operator of the previous section.

$$\nabla_w Q \;=\; \left[G_\gamma^T - G_{\sigma_o}^T \left[\frac{\delta \gamma}{\delta \xi} \sin\theta + \frac{\delta \gamma}{\delta \eta} \cos\theta \right] \right] \nabla_\gamma Q \quad . \tag{31}$$

Since G_γ and G_{σ_o} contain a tomography operator, the gradient of equation (31) is a filtered tomographic back-projection (Fowler, 1988). The filter converts changes in stack semblance to traveltimes that can be used to update the interval-slowness model.

OPTIMIZATION

The gradient just described can be used to drive an iterative optimization method to find the interval-slowness model that maximizes our objective function, the total stack semblance. For simplicity sake I will outline a steepest ascent algorithm that uses the linear theory of this paper to find the best interval-slowness model. In practice, more sophisticated optimization techniques such as the conjugate-gradient method should be used.

Set initial interval-slowness model $w(x,z)$

Migrate each constant-offset section using $w(x,z)$

Estimate dips on stacked image $\theta(\xi,\eta)$

Apply residual moveout and stack for range of γ making $S(\gamma,\xi,\eta)$

Iterate until $\nabla_w Q$ is small

{

1. Compute $\nabla_\gamma Q$, $\delta\gamma/\delta\xi$, and $\delta\gamma/\delta\eta$ by finite differences
2. Compute G_γ and G_{σ_o}
3. Compute $\nabla_w Q = \left[G_\gamma^T - G_{\sigma_o}^T\left[\frac{\delta\gamma}{\partial\xi}\sin\theta + \frac{\delta\gamma}{\partial\eta}\cos\theta\right]\right]\nabla_\gamma Q$
4. Line search for β that maximizes $Q(w+\beta\nabla_w Q)$
5. Update interval-slowness model, γ, and σ_o

   $\Delta w = \beta\nabla_w Q$

   $w(x,z) = w(x,z) + \Delta w$

   $\gamma(\xi,\eta) = \gamma(\xi,\eta) + \left[G_\gamma \; - \left[\frac{\delta\gamma}{\delta\xi}\sin\theta + \frac{\delta\gamma}{\delta\eta}\cos\theta\right]G_{\sigma_o}\right]\;\Delta w$
6. Compute $\xi(x,z)$ and $\eta(x,z)$

   $\xi(x,z) = \xi(x,z) + \sin\theta\; G_{\sigma_o}\Delta w$

   $\eta(x,z) = \eta(x,z) + \cos\theta\; G_{\sigma_o}\Delta w$
7. Compute $x(\xi,\eta)$ and $z(\xi,\eta)$ by inverse interpolation

}

If the starting interval-slowness model is far from the best interval-slowness model, the outlined optimization method may not be able to reach the best interval-slowness model. Residual moveout and stack may not be a good approximation to remigrating the data when large perturbations to slowness model are necessary. The optimization can be reinitialized by remigrating the data and building a new data space when the perturbations to the interval-slowness model become too large.

SUMMARY

Migration with interval-slowness model $w(x, z)$ followed by residual moveout and stack approximates migration and stacking with interval-slowness model $w(x, z) + \Delta w(x, z)$. The residual moveout operator has two parts. The first changes the curvature of events at a fixed zero-offset location. The second is a residual zero-offset migration. The residual zero-offset migration does not change the coherence of events over offset so I neglect it when computing the stack semblance. Changes in the interval-slowness model can be linearly related to changes in the best fitting residual moveout curve. This linear relation is used to compute the gradient of the stack semblance of the migrated image with respect to the interval-slowness model. The gradient drives an iterative optimization to find the interval-slowness model that gives the most coherent migrated image.

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REFERENCES

Al Yahya, K., 1987, Velocity analysis by profile migration: Ph.D. Thesis, Stanford University, also SEP–**53**, 11-30.

Etgen, J., 1988, Velocity analysis by prestack depth migration: linear theory: SEP–**57**, 77–98.

Fowler, P., 1988, Seismic velocity estimation using prestack migration: Ph.D. Thesis, Stanford University, also SEP–**58**

Fowler, P., 1987, Analysis of a dip-dependent operator relating migration velocities and interval velocities: SEP–**51**, 63–84.

Fowler, P., 1985, Migration velocity analysis: linear theory: SEP–**44**, 1–20.

Toldi, J., 1985, Velocity analysis without picking: Ph.D. Thesis, Stanford University, also SEP–**43**, 1-103.

APPENDIX A

The derivatives of $\sigma_s(\gamma, \sigma_o)$ at fixed (x_r, z_r) with respect to γ and σ_o for each migrated constant-offset section are used to linearize equation (8) of the text. Restating equation (8), write

$$\sigma_s(h) = \frac{-(s+g)}{2}\sin\theta + \sqrt{\sigma_o^2 + (\gamma^2-1)\frac{g^2+s^2}{2} + \gamma\,\sigma_o \sin\theta\,(g+s) + \frac{(g+s)^2\sin^2\theta}{4}}\,. \qquad (A.1)$$

Differentiating with respect to γ

$$\frac{\partial\sigma_s(h)}{\partial\gamma} = \frac{1}{2}R^{-\frac{1}{2}}\left[2\gamma\frac{(g^2+s^2)}{2} + \sigma_o\ \sin\theta\ (g+s)\right]\ . \qquad (A.2)$$

Differentiating with respect to σ_o

$$\frac{\partial\sigma_s(h)}{\partial\sigma_o} = \frac{1}{2}R^{-\frac{1}{2}}\left[2\sigma_o + \gamma\ \sin\theta\ (g+s)\right]\ ; \qquad (A.3)$$

In equations (A.1) and (A.2)

$$R = \sqrt{\sigma_o^2 + (\gamma^2-1)\frac{g^2+s^2}{2} + \gamma\,\sigma_o\sin\theta\,(g+s) + \frac{(g+s)^2\sin^2\theta}{4}}\ .$$

APPENDIX B

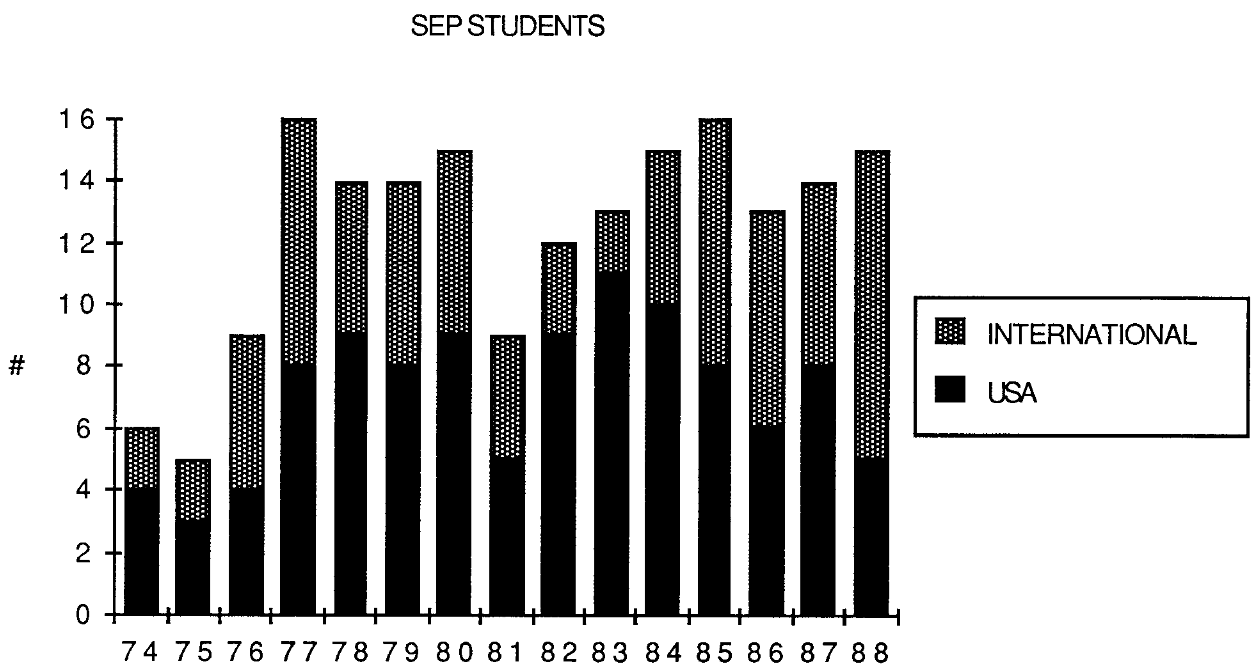
Write the normal equations to solve for $\Delta\gamma$ and $\Delta\sigma_o$, the change in the parameters of the best fitting residual moveout curve as

$$\begin{pmatrix} \sum_{h_{min}}^{h_{max}} A_h^2 & \sum_{h_{min}}^{h_{max}} A_h B_h \\ \sum_{h_{min}}^{h_{max}} A_h B_h & \sum_{h_{min}}^{h_{max}} B_h^2 \end{pmatrix} \begin{pmatrix} \Delta\gamma \\ \Delta\sigma_o \end{pmatrix} = \begin{pmatrix} \sum_{h_{min}}^{h_{max}} A_h\Delta\sigma_s(h) \\ \sum_{i=1}^{h_{max}} B_i\Delta\sigma_s(h) \end{pmatrix}\ , \qquad (B.1)$$

where $A_h = \partial\sigma_s(h)/\partial\gamma$ and $B_h = \partial\sigma_s(h)/\partial\sigma_o$. The solution to the normal equations is:

$$\begin{pmatrix} \Delta\gamma \\ \Delta\sigma_o \end{pmatrix} = \begin{pmatrix} \Gamma_\gamma \\ \Gamma_{\sigma_o} \end{pmatrix} \Delta\sigma_s(h) =$$

$$\frac{1}{\sum_{h_{min}}^{h_{max}} A_h^2 \sum_{h_{min}}^{h_{max}} B_h^2 - (\sum_{h_{min}}^{h_{max}} A_h B_h)^2} \begin{pmatrix} \sum_{h_{min}}^{h_{max}} B_h^2 & -\sum_{h_{min}}^{h_{max}} A_h B_h \\ -\sum_{h_{min}}^{h_{max}} A_h B_h & \sum_{h_{min}}^{h_{max}} A_h^2 \end{pmatrix} \begin{pmatrix} \sum_{h_{min}}^{h_{max}} A_h\Delta\sigma_s(h) \\ \sum_{h_{min}}^{h_{max}} B_h\Delta\sigma_s(h) \end{pmatrix}\ . \qquad (B.2)$$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