Sensitivity of acoustic data to the geometry of the earth model

Niki Serakiotou

ABSTRACT

We consider the case in which an interesting formation that is found in an area of generally known geology needs to be further resolved. Assuming knowledge of its acoustic properties (velocity,density), we focus on the shape of the formation. We describe the formation by a set of geometrical parameters and, using a finite difference scheme, we compute the 2-D acoustic seismic response of an area around the formation. We also compute the area's responses made when each of the parameters is perturbed the same amount of distance. We then formulate a quantitative method to compare the influence of the perturbation of different parameters on the synthetic data. Our purpose is to see which parameters are more influencial (and thus resolvable) for the specific case at hand, as well as get some indication about general properties of the 2-D acoustic data (for example vertical versus horizontal resolution).

INTRODUCTION

Imagine a case in which we want to resolve the details of an interesting structure that is found in an area of generally known geology. Interpreters with access to interactive facilities could start with an initial model of the structure and try to perturb this model until they get a good fit of its (synthetic) seismic response with the data they have available. While they "play" with the shape of the structure, they could use some information about the ways in which the seismic response is influenced by the different parameters that define the shape.

For example one expects that a parameter that significantly controls the volume of the structure will also have a major influence on its seismic response. So a perturbation in the height of a pyramid-like structure will influence the synthetic data more than will a perturbation of the position of its top in the horizontal direction.

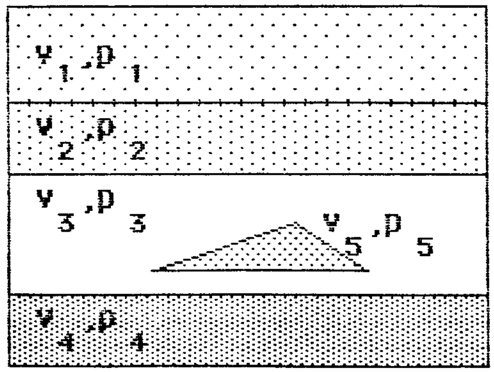

FIG. 1. The velocity-density model for our simple case study.

A study of the sensitivity of the seismic response to all such parameters should give us a quantitative measure of each specific parameter's influence on the seismic response; this study should also reveal general tendencies (for example, that horizontal perturbations of the shape are less influential than vertical ones).

Such information would be useful also in an inversion scheme that uses these geometrical parameters as model parameters to be identified.

We conducted such a sensitivity study for the unrealistically simple case of a formation that presents itself as a triangle in 2-D and that is embedded in the third of four horizontal layers, as is shown in Figure ??.

THE SENSITIVITY STUDY

To describe the shape of the triangular formation of Figure ?? we choose as parameters the (x, z) coefficient of its vertices (As an aid to memory, the numbering order of the vertices is clockwise: (x_1, z_1) is the top vertex, (x_2, z_2) the vertex to its right and (x_3, z_3) the third one). So our set of parameters is completely determined by the vector

$$\mathbf{p} = \begin{bmatrix} x_1 \\ z_1 \\ x_2 \\ z_2 \\ x_3 \\ z_3 \end{bmatrix} \tag{1}$$

We assume that the problem is linear, so

$$\mathbf{d}(\ \mathbf{p}) = \ \mathbf{d}(\mathbf{p}_0) + \frac{\partial \mathbf{d}}{\partial \mathbf{p}}(\mathbf{p}_0)\delta \mathbf{p} \tag{2}$$

where

$$\mathbf{d}(\mathbf{p}) = \mathbf{d}(\mathbf{p}; s, g, t)$$

is the "cube" of data produced when the formation is described by the parameter vector $\mathbf{p}$. In our case there are 10 shot gathers of acoustic synthetics computed by a finite-difference algorithm.

Also, for our study, $\mathbf{p}_0$ is our initial vector of parameters while $\delta\mathbf{p}$ is the vector of perturbations

$$\delta\ \mathbf{p} = \begin{bmatrix} \delta p_1 \\ \delta p_2 \\ \delta p_3 \\ \delta p_4 \\ \delta p_5 \\ \delta p_6 \end{bmatrix} = \begin{bmatrix} \delta x_1 \\ \delta z_1 \\ \delta x_2 \\ \delta z_2 \\ \delta x_3 \\ \delta z_3 \end{bmatrix} \tag{3}$$

To compute the Jacobian $\frac{\partial \mathbf{d}}{\partial \mathbf{p}}(\mathbf{p}_0)$ we use the very simple finite-difference scheme

$$\frac{\partial \mathbf{d}}{\partial p_i} = \frac{\mathbf{d}(p_0+[0,\ldots,\delta p_i,\ldots,0]^{\top}) - \mathbf{d}(p_0)}{\delta p_i} \tag{4}$$

Next we form the following measure for the effect of a perturbation on the seismic response

$$\begin{aligned} R(\delta p) &= \left\|\frac{\partial \mathbf{d}}{\partial \mathbf{p}}(\mathbf{p}_0)\delta\mathbf{p}\right\|^2_{L^2} &(5)\\ &= <\frac{\partial \mathbf{d}}{\partial \mathbf{p}}(\mathbf{p}_0)\delta\mathbf{p}, \frac{\partial \mathbf{d}}{\partial \mathbf{p}}(\mathbf{p}_0)\delta\mathbf{p}> &(6)\\ &= <\frac{\partial \mathbf{d}}{\partial \mathbf{p}}^{\top}\frac{\partial \mathbf{d}}{\partial \mathbf{p}}(\mathbf{p}_0)\delta\mathbf{p}, \delta\mathbf{p}> &(7)\\ &= <\mathbf{H}\delta\mathbf{p}, \delta\mathbf{p}> &(8) \end{aligned}$$

where

$$[\mathbf{H}]_{ij} = \sum_s\sum_g\sum_t \frac{\partial \mathbf{d}}{\partial p_i}(s,g,t)\frac{\partial \mathbf{d}}{\partial p_j}(s,g,t)$$

is a symmetric (6×6) matrix. If we write

$$\mathbf{H} = \begin{bmatrix} y_1 & \cdots & y_6 \end{bmatrix} \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_6 \end{bmatrix} \begin{bmatrix} y_1^{\top} \\ \vdots \\ y_6^{\top} \end{bmatrix} \tag{9}$$

where λ_i and y_i are the eigenvalues and (column) eigenvectors of $\mathbf{H}$, then

$$\begin{aligned} R(\delta\mathbf{p}) &= <\mathbf{H}\delta\mathbf{p}, \delta\mathbf{p}> \\ &= < \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_6 \end{bmatrix} \begin{bmatrix} y_1^{\top}\delta\mathbf{p} \\ \vdots \\ y_6^{\top}\delta\mathbf{p} \end{bmatrix}, \begin{bmatrix} y_1 & \cdots & y_6 \end{bmatrix}^{\top}\delta\mathbf{p} > &(10)\\ &= \lambda_1(y_1^{\top}\delta\mathbf{p})^2 + \lambda_2(y_2^{\top}\delta\mathbf{p})^2 + \cdots + \lambda_6(y_6^{\top}\delta\mathbf{p})^2 \end{aligned}$$

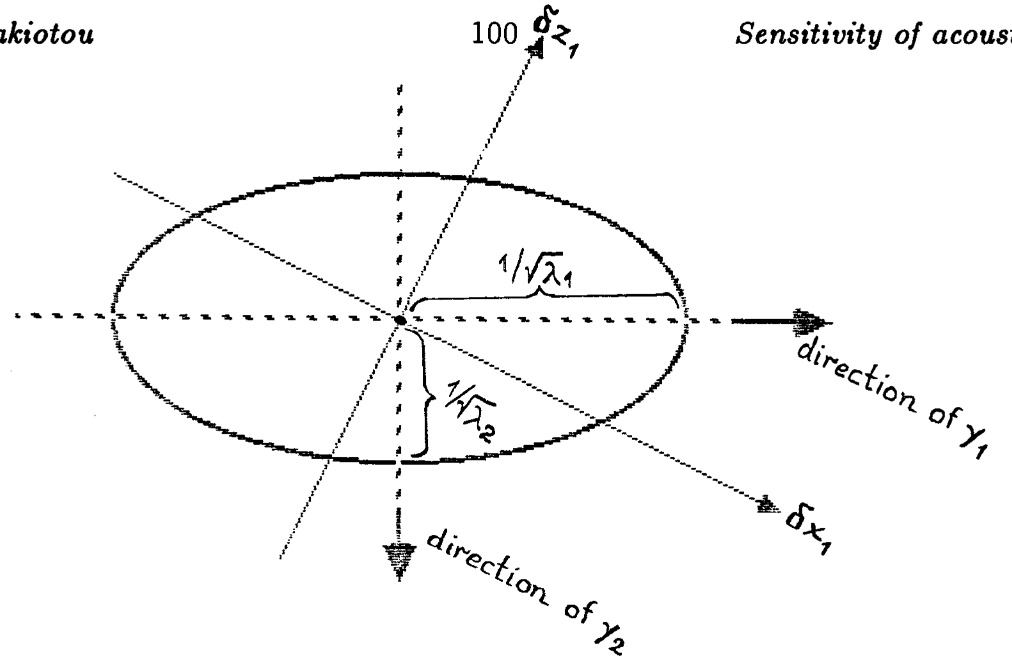

FIG. 2. The contours of $R(.)$ are ellipses in the space of δx_1 and δz_1.

Note that

$$\begin{aligned} R(\delta p_i) &= R(\delta \mathbf{p})\,[0,\ldots,\delta p_i,\ldots,0]^{\top} \\ &= (\lambda_1 y_{1i}^2 + \cdots + \lambda_6 y_{6i}^2)\delta p_i^2 \\ &= F(\delta p_i)\delta p_i^2 \end{aligned} \qquad (11)$$

and we can compute the factors $F(\delta p_i)$ for each of the $i = 1,\ldots,6$ parameters from the eigenvalues and eigenvectors of $\mathbf{H}$.

To realize the meaning of these factors, suppose that we had only a two-element vector of parameters, for example

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ z_1 \end{bmatrix}$$

Then

$$R(\delta \mathbf{p}) = \lambda_1 (y_1^{\top} \delta \mathbf{p})^2 + \lambda_2 (y_2^{\top} \delta \mathbf{p})^2 = < \mathbf{H}\delta \mathbf{p}, \delta \mathbf{p} > = (\delta \mathbf{p})^{\top} \mathbf{H} \delta \mathbf{p}$$

We know from linear algebra (Strang, 1976) that the contours of $R(\delta p)$ on the plane (x_1, z_1) are ellipses, with axes along the directions of the eigenvectors y_1, y_2 of $\mathbf{H}$ and half-lengths equal to $1/\sqrt{\lambda_1}$ and $1/\sqrt{\lambda_2}$, where λ_1 and λ_2 are the eigenvalues of $\mathbf{H}$. (See Figure 2).

We can see now that $R(\delta \mathbf{p})$ shows the rate by which the seismic response is affected when we perturb the parameters by $\delta \mathbf{p}$. If we only perturbed δx_1 (for example), while $\delta z_1 = 0$, that is if $\delta \mathbf{p} = [\delta x_1, 0]^{\top}$ then

$$R(\delta p_1) = (\lambda_1 y_{11}^2 + \lambda_2 y_{21}^2)\delta x_1^2 = F(\delta x_1)\delta x_1^2,$$

so the factor $F(\delta x_1)$ shows the rate by which the seismic response is affected when we only perturb δx_1. $F(\delta x_1) < F(\delta z_1)$ means that the seismic response is less affected by a perturbation of x_1 than by an equally sized perturbation of z_1. We notice that there is a correspondance between this effect and the tilt of the axes of the ellipse in Figure ?? with respect to the δx_1,δz_1 axes. If the two pairs of axes are 45^o apart then both parameters have exactly the same effect on the seismic response. If the angle between the pairs of axes is less than 45^o, then a perturbation of the parameter whose axis is closer to the smaller axis of the ellipse has a greater effect on the seismic response, than does perturbation of the other parameter.

Although it is impossible to imagine an n-dimensional ellipsoid with $n > 3$, we can see from the previous discussion how the factors $F(\delta p_i)$, $i = 1, \ldots, 6$ can enable us to compare the influences of the parameters on the seismic response.

RESULTS FOR THE SIMPLE EXAMPLE

We carried out the previous analysis for the case of Figure ??, to get the following factors:

$$F(\delta x_1) = 56$$
$$F(\delta x_2) = 38$$
$$F(\delta x_3) = 17$$
$$F(\delta z_1) = 1274$$
$$F(\delta z_2) = 5864$$
$$F(\delta z_3) = 5864$$

These indicate that

- Perturbations of parameters of the vertical direction are much more important than those of the horizontal direction.
- All the perturbations of the horizontal direction have about the same effect.
- There is a considerable difference between the effects of δz_1 on the one hand and of δz_2 and δz_3 on the other. This needs some explanation from our part. Because we were using a somewhat cumbersome modeling program, we kept the bottom side of the triangle always horizontal (in the velocity-density model). So z_2 and z_3 were always simultaneously perturbed and by the same amount of distance. This explains their identical F factors. Though this is actually a defect of the study it accidentally revealed something: note that these simultaneous perturbations change the volume (area) of the triangular formation significantly more than does a perturbation of z_1 only. This explains the difference between $F(\delta z_1)$ and $F(\delta z_2)$.

CONCLUSION

We have developed a mathematical framework to reveal the sensitivity of seismic responses to the geometrical shape of the perturbations in density and velocity.

This theory may seem mathematically too complex, especially for dealing with the situation presented as a numerical example. We believe however that the theory could be very useful when applied to more complicated situations.

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REFERENCES

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