Velocity analysis by prestack depth migration: linear theory

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ABSTRACT

Imaging of seismic reflection data is accomplished by prestack depth migration of individual shot records followed by stacking the results at common surface locations. If the velocity model used for shot record migration does not correctly model the traveltimes of reflection events, the images of migrated shot profiles at a constant surface location (CSL) will be distorted from one another. This distortion can be used to estimate the error in the velocity model. The distortion among the images of the shot profiles can be removed with a residual moveout correction. In constant velocity medium the change in depth of images in a CSL gather due to a change in the velocity model can be found by applying a residual moveout correction in depth. The moveout is a function of the ratio of the imaging velocities, the original depth of the image, and the source and receiver locations. For perturbations to a general interval velocity model, the change in migrated depth of the migrated events in a CSL gather can be calculated through ray tracing. Applying Fermat's principle linearizes the relation between changes in the velocity model and changes in traveltime. Linearized geometrical relations describe how to map the change in traveltime to a change in migrated depth. Linear least squares theory can then be applied to find a change to the residual moveout curve that best fits the calculated changes in migrated depth observed in a CSL gather. The result is a linear operator that relates changes in interval velocity to changes in the parameters describing residual moveout using the intermediaries of traveltimes and migrated depth. The change of the stacking semblance of the images of migrated shot profiles due to a change of interval velocity can be computed without need to remigrate the data.

INTRODUCTION

Normally, seismic data are recorded redundantly; a given point in the subsurface is illuminated by many different shot profiles. Velocity analysis uses some characteristic of the redundant seismic data to estimate the velocities at which acoustic or elastic waves propagate through the subsurface. The characteristic usually measured is the variation in traveltime of waves from the sources to a reflecting point and back to receivers. If the subsurface reflectors are horizontal and velocity does not vary laterally, normal moveout (NMO) provides a measure of the root-mean-square average of the velocity of the rocks above a given reflector. If the velocity varies laterally, and the geologic structure is complicated (the two often go hand in hand), then NMO does not adequately account for the traveltimes from source to reflecting point to receiver. More sophisticated imaging and velocity analysis methods are needed. If reflectors are horizontal, but velocity varies laterally, the variations in NMO velocity can be used to estimate the laterally varying interval velocities (Toldi, 1985). If structure is complex, but lateral velocity variation not severe, variations in prestack time migration velocities can be used to estimate laterally varying interval velocities (Fowler, 1985). When lateral velocity variation is severe, images obtained by prestack time migration or NMO are mispositioned. Prestack depth migration must be used to obtain a focused image of the subsurface structure. Prestack depth migration uses interval velocities directly, there are no intermediaries such as NMO velocity or time migration velocity. This is unfortunate, because NMO or prestack time migration velocities can often be determined reliably even in noisy data.

Al Yahya, (1987, 1985) introduced a simple measure of the accuracy of an interval velocity model used for prestack depth migration. If the interval velocity model used for prestack depth migration predicts the traveltimes of reflection events accurately, the migrated images of different shot profiles at a constant surface location (CSL) will be nearly the same. Specifically, the migrated depth of a reflector will be the same on all migrated profiles. A gather of the images of different migrated shot profiles at a constant surface location is a constant surface location gather. If the estimate of the interval velocity model is in error, the images of different migrated shot profiles in a CSL gather will be distorted from each other. Commonly, this distortion is a residual curvature in depth of the migrated images of the images of reflectors in a CSL gather. Curvature of migrated events at a constant surface location can be related to errors in the interval velocity model, and corrected by a residual moveout applied to the images of the different shots. Al Yahya's (1987) residual moveout correction is only exact for horizontal reflectors and constant velocity, but can be extended to dipping beds, general velocity, and built into an optimization framework much like Toldi (1985) did with stacking velocities and Fowler (1985) did with prestack time migration velocities.

Stack semblance can be used to measure the distortion or lack of distortion present among the images of several migrated shot profiles in a CSL gather. When stack semblance or power is maximized, the images are the least distorted from each other.

In SEP–51 (Etgen, 1987), I proposed a velocity analysis method that uses prestack depth migration to estimate interval velocities by optimization. The method works as follows: First, image the seismic data using prestack depth migration with an estimate of the interval velocity model. Second, find a change to the current model that improves the objective function (a gradient direction). Third, find a new model that is a combination of the current model and the search direction that extremizes the objective function. This process is iterated until the objective function is globally optimized. In SEP–51, I proposed that the interval velocity model be constructed from a sparse set of basis functions and that the gradient of the second step be computed directly by finite differences. As long as the number of basis functions is small, this can be done. This may be practical if the velocity varies only smoothly laterally. However, if the lateral velocity gradient is small, then it is probably not worthwhile to use prestack depth migration, as it is too costly and other less expensive methods will provide satisfactory imaging and velocity analysis. My present feeling is that the number of basis functions needed to parametrize a velocity model with strong lateral velocity variation will drive the cost of the finite-difference gradient approach too high to be useful.

The purpose of this paper is to derive a linear theory that relates changes in the interval velocity model used for prestack depth migration to changes in the prestack depth migrated image; and thus, to changes in the objective function that measures the quality of the migrated image. I will describe how to incorporate this linear theory into a velocity analysis method based upon prestack depth migration. The linear theory will compute the gradient of the objective function more rapidly than computing the gradient by finite differences when the number of velocity parameters is large. Moreover, the linear operator allows a more detailed analysis of the changes in migrated images due to changes in interval velocity than can be obtained by applying finite-difference formulae.

ERRORS IN MIGRATED IMAGES DUE TO Δv

Prestack depth migration requires specifying an interval velocity model. Depending on the characteristics of this velocity model and the specifics of the algorithm used, changes in the prestack migrated image will be related to changes of the velocity model in a complicated way. Rather than compute changes in images due to changes in the interval velocity model by remigrating the data, it is possible to make simplifying restrictions to the problem.

From waves to traveltimes

Normally the highest wavenumber components of an interval velocity model are excluded when migrating because their inclusion will generate spurious events in the migrated image as the downward continued (or reverse time extrapolated) waves scatter from the sharp contrasts in the model (Stolt and Benson, 1986), (Etgen, 1987). Omitting the high wavenumber content of the interval velocity model often leads to little or no degradation of the imaging capability of prestack depth migration. Indeed, it may reduce artifacts. Furthermore, as long as the wavelength of the waves propagated by prestack depth migration is small compared to the scale of variations of the velocity model, wave propagation can be approximated by delaying the source waveform by ray-theoretic traveltime and scaling with ray-theoretic amplitude. Relating changes in interval velocity to changes in prestack migrated images becomes much simpler. Changing the interval velocity model repositions and scales migrated events. Wave effects, such as velocity dispersion due to heterogeneity and scattering are excluded. When the scale of the velocity variations is the same size as the seismic wavelength, the ray-theory approximation breaks down and wave equation based techniques should be used to estimate changes in the wave field due to changes in velocity (Woodward, 1987, 1988). I will assume the ray-theory approximation is valid, so the task of relating changes in velocity to changes in migrated images becomes a simpler, tomography-like problem.

Residual moveout (RMO) in migrated depth

Before introducing the theory that relates general changes in interval velocity to changes in migrated images, it is worthwhile to examine changes in migrated images in a simple situation. In a constant velocity medium, the image of a dipping reflector segment in a migrated shot profile is the tangent line to an ellipse whose foci are the source and receiver as shown in Figure 1. The straight lines between source, reflecting point and receiver are the rays along which the energy released at the source, reflected from the image point, and recorded at the receiver propagates. The equation of the ellipse is given by equation 1, where v is the imaging velocity, t is the traveltime of the reflected event from source to reflector to receiver, z_r and x_r are the coordinates of the reflecting point, and h one-half the distance between source and receiver. The origin of coordinates is taken to be at the surface in z and half-way between the source and receiver in x for notational convenience.

$$vt = \sqrt{z_r^2 + (x_r - h)^2} + \sqrt{z_r^2 + (x_r + h)^2} \tag{1}$$

Changing the imaging velocity from v to v_n, will move the image point in x and z. The traveltime t of the reflected event is fixed since we are considering the movement of the image of a specific part of the data. Figure 2 depicts the movement of the dipping reflector segment after a change in velocity. We can follow one fixed reflecting point as it moves in both x and z, or fix our observation horizontally at x_r and note the change in depth of the image. If we follow the reflecting point, a residual correction has to move the event in both x and z. If the horizontal location at which the image is examined is held constant, the movement is only a change in the depth as shown in Figure 3. The change in depth is a function of the source to

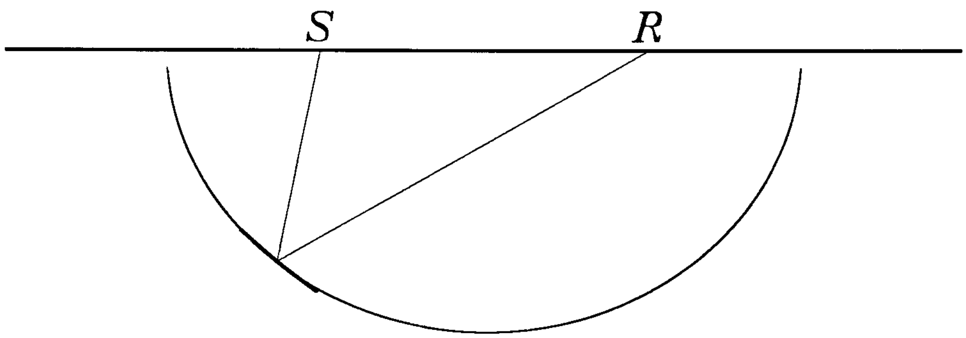

FIG. 1. Migrated image of a dipping reflector in a constant velocity medium. The image is tangent to the ellipse shown. The ellipse has foci at the source and receiver.

receiver offset, the original depth of the reflector segment and the change in velocity. The new image observed at the fixed horizontal distance x_r imaged with velocity v_n, will be tangent at x_r to the ellipse with equation

$$v_n t = \sqrt{z_n^2 + (x_r - h)^2} + \sqrt{z_n^2 + (x_r + h)^2} \ . \tag{2}$$

Equations 1 and 2 above can be combined to eliminate dependence on t and solved for z_n in terms of z_r, v, v_n, x_r, and h.

$$z_n = \sqrt{\frac{v_n^2}{v^2}\frac{X}{4} + (x_r^2 + h^2) + \frac{v^2}{v_n^2} 4 \frac{h^2 x_r^2}{X}} \ , \tag{3}$$

where

$$X = 2z_r^2 + 2(x_r^2 + h^2) + 2\sqrt{z_r^4 + 2z_r^2(h^2 + x_r^2) + (x_r^2 - h^2)^2} \ .$$

The movement of the dipping reflector segment from z_r to z_n at fixed x_r as a function of source receiver offset (which is a function of dip) and depth is a residual moveout (RMO) correction. Al Yahya (1987), derived the same expression (in slightly different notation), and introduced the parameter γ, as the ratio v_n/v (see Al Yahya, 1987 eqns 2.10, 2.11). Rewritting equation 3, it can be seen that the residual moveout does not explicitly depend on the velocities v or v_n, only on their ratio γ.

$$z_n = \sqrt{\gamma^2 \frac{X}{4} + 4(x_r^2 + h^2) + \frac{1}{\gamma^2}\frac{4h^2 x_r^2}{X}} \ . \tag{4}$$

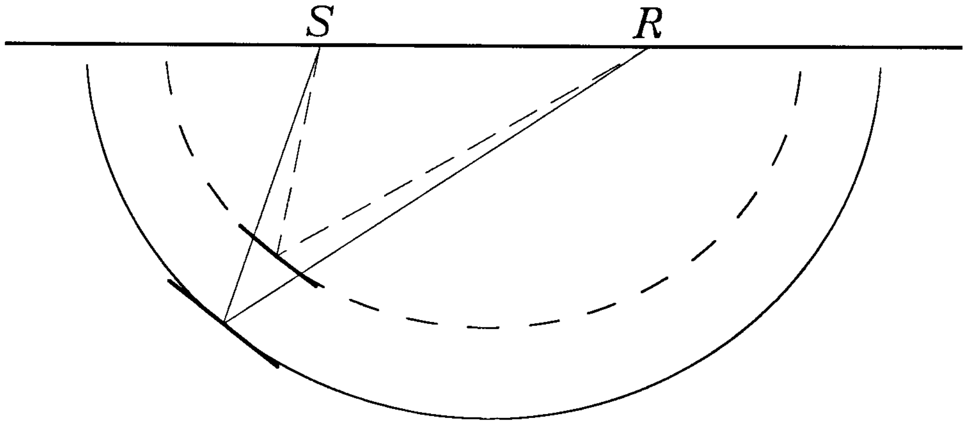

FIG. 2. Movement of the migrated image of a dipping reflector when the velocity is changed (increased). The new image is also tangent to an ellipse with foci at source and receiver. In this case we followed the movement of a given reflecting point.

When velocity varies vertically or laterally, the movement of the image of a dipping reflector segment is more complicated; thus, the reflector segment will not be tangent to an ellipse that has foci at the source and receiver. The equation of an ellipse is not useful in finding the traveltime from source to reflector to receiver because the raypaths are not straight. Figure 4 depicts the image of a dipping reflector segment imaged with a general velocity model. The source location is S, the receiver position is R, and the image location is x_r, z_r. If the interval velocity model $v(x,z)$ used for imaging is changed to $cv(x,z)$ where c is a constant, the image of dipping reflector segment will move in x and z as shown in Figure 5. When analyzing the change of the image with a change in velocity, I fix the observation location at x_r and observe the change in apparent depth of the event from z_r to z_n. The change in apparent depth from z_r to z_n is a function of the arc length of the rays, the opening angle of the rays from source to image point to receiver, and the magnitude of the change in velocity. The greater the arc length, the more error there will be in the traveltime to the original depth, z_r; thus, the further z_n will move from z_r. Also, the greater the opening angle of the rays, the further z_n will move from z_r. Finally, the greater the change in velocity the further the image will move.

Rather than solve for the new depth of the reflector at fixed x by searching or ray tracing, it is useful to make an approximation. The dipping reflector segment *is tangent* to an ellipse that has foci not at source and receiver, but elsewhere on

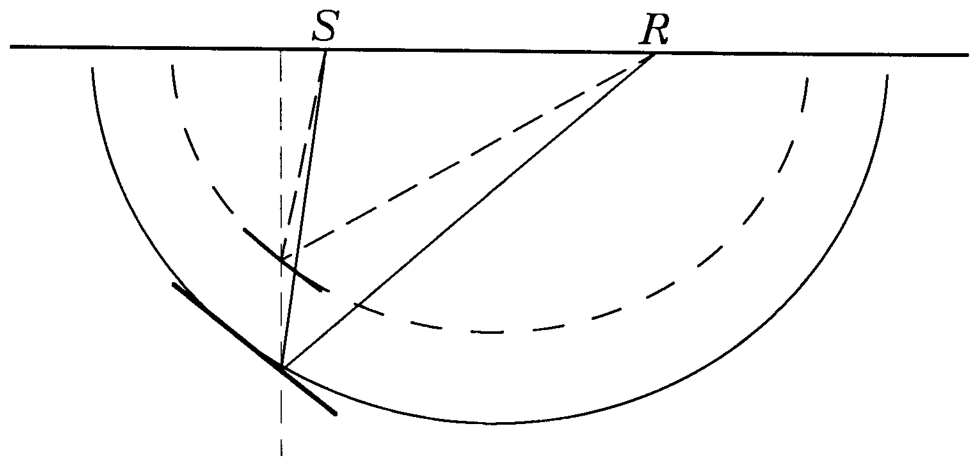

FIG. 3. Movement of the migrated image of a dipping reflector when the velocity is changed. In this case we fix our observation at x_r and observe a change in the migrated depth of the reflector.

the surface. These foci can be found by projecting up to the surface the straight line "rays" that have the same incidence angles at the reflector as the true rays as shown in Figure 6. The depth of the reflected event with velocity $v(x,z)$ can be written as

$$z_r = \sqrt{\frac{v_r^2 t^2}{4} - (x_r^2 + h^2) + \frac{4}{v_r^2 t^2} h^2 x_r^2} \ . \tag{5}$$

where v_r is no longer the velocity of the medium, but some function of overlying velocities and the geometry of the rays. Think of v_r as an effective constant velocity that describes the traveltime of the rays from the apparent source to reflector to apparent receiver. The traveltime becomes only a function of the opening angle of the rays, the constant velocity, and depth. The origin of coordinates has shifted to half-way between the apparent source and receiver where the straight line "rays" strike the surface; x_r and h refer to new distances defined by the straight "rays". Rather than follow the image of a single point on the reflector as is moves in both x and z, fix the observation at x_r and the image moves in migrated depth. Now, a similar equation could be written for z_n, the depth of the dipping reflector segment at velocity $cv(x,z)$ by substituting z_n and v_n in the appropriate places.

$$z_n = \sqrt{\frac{v_n^2 t^2}{4} - (x_r^2 + h^2) + \frac{4}{v_n^2 t^2} h^2 x_r^2} \tag{6}$$

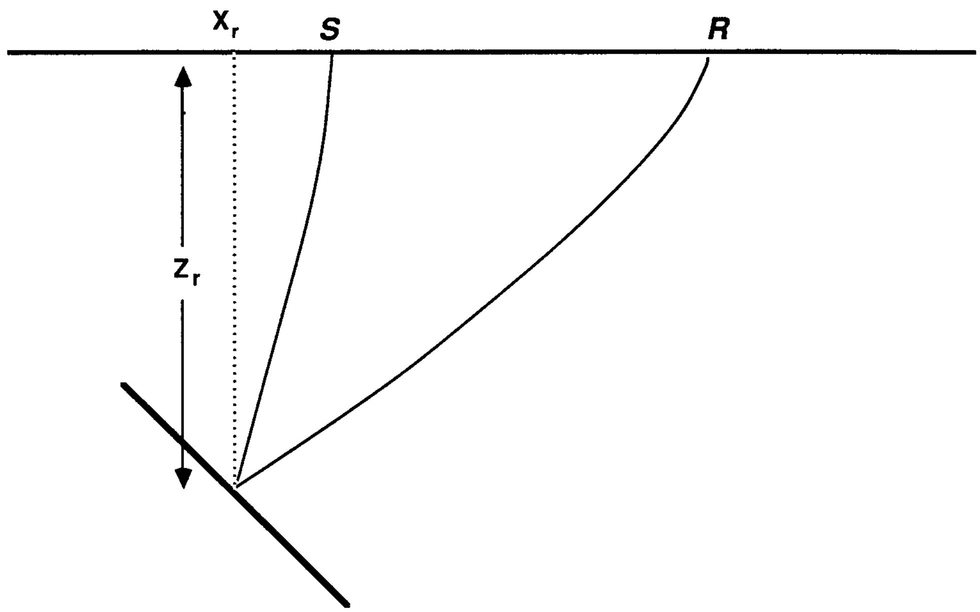

FIG. 4. Migrated image of a dipping reflector using non-constant velocity. The reflector segment is not tangent to an ellipse that has foci at the source and receiver.

The image of the reflector after migration with $cv(x,z)$ will also be tangent to an ellipse with apparent source and receiver where the straight line "rays" strike the surface as shown in Figure 7. Equation 6 can be combined with equation 5 to eliminate t and solved for z_n in terms of z_r, x_r, h, and $\gamma = v_n/v_r$. Since we are only interested in how z_n changes, we do not need to know v_r or v_n, only γ, the ratio of v_n/v_r. So far no approximations have been made, equations 5 and 6 give the ellipses to which the reflector segments are tangent. The approximation arises when $\gamma = v_n/v_r$ is equated to $cv(x,z)/v(x,z) = c$. The approximation is exact if the true rays are straight and approximate if they are not. Under this approximation write:

$$z_n = \sqrt{\gamma^2 \frac{X}{4} + (x_r^2 + h^2) + \frac{1}{\gamma^2}\frac{4h^2x_r^2}{X}} \quad , \tag{7}$$

Where

$$X = 2z_r^2 + 2(x_r^2 + h^2) + 2\sqrt{z_r^4 + 2z_r^2(h^2 + x_r^2) + (x_r^2 - h^2)^2} \quad .$$

Figure 8 shows the movement in depth of the image of the reflector at a fixed x_r.

The approximate change in migrated depth estimated by equation 7 may be biased if the true rays are either much shorter or much longer than the straight "rays" used in equation 7. In a constant velocity medium, γ can be related to the error in velocity. When velocity varies laterally, or even strongly vertically, it is more appropriate to use γ as a residual curvature parameter, and use a tomographic approach to relate the residual curvature to changes in the velocity model. The error

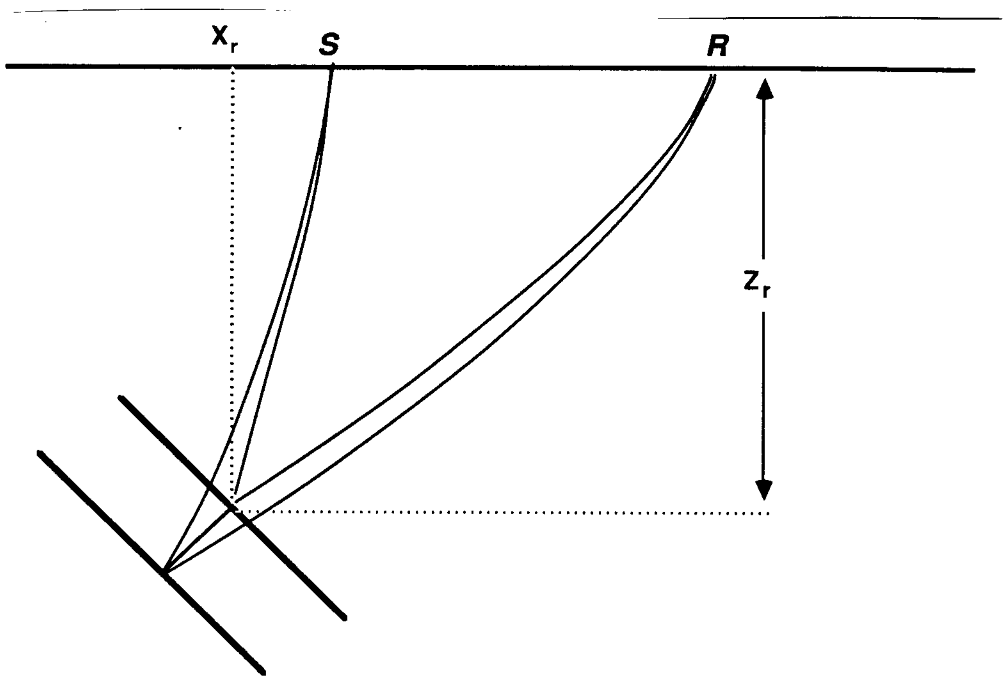

FIG. 5. Movement of the image of a dipping reflector migrated with non-constant velocity. The velocity is changed from $v(x,z)$ to $cv(x,z)$. The image moves normal to the reflector.

in the approximation will not be as important when γ is used as a residual curvature parameter, and not as a direct estimate of the change in velocity.

Equation 7 is a residual moveout equation; the RMO is applied to migrated shot profiles that are sorted into constant surface location gathers. The complicated nature of changes in migrated images at a reflection point due to changes in interval velocity models is replaced by residual moveout that is function of one parameter, depth, and the geometry of the rays at the reflection point. The ray geometry is a function of dip, so applying equation 7 requires estimating the local dip at the reflecting point. Dip estimates could be obtained from the staked image of all migrated profiles. Velocity analysis, only requires the stack semblance of the image at a given depth and a given CSL, so it will suffice to apply the RMO correction and calculate the semblance for a number of dip ranges and weight the result by the normalized dip spectrum at each point. RMO corrections can be applied to the images after migrating the data with the interval velocity model $v(x,z)$ for a range of γ values. The value of the stack semblance at each point for each dip and each γ is stored. From this data space, the value of semblance at any point for a velocity model $v(x,z) + \Delta v(x,z)$ near $v(x,z)$ can be obtained using the theory presented in the next section.

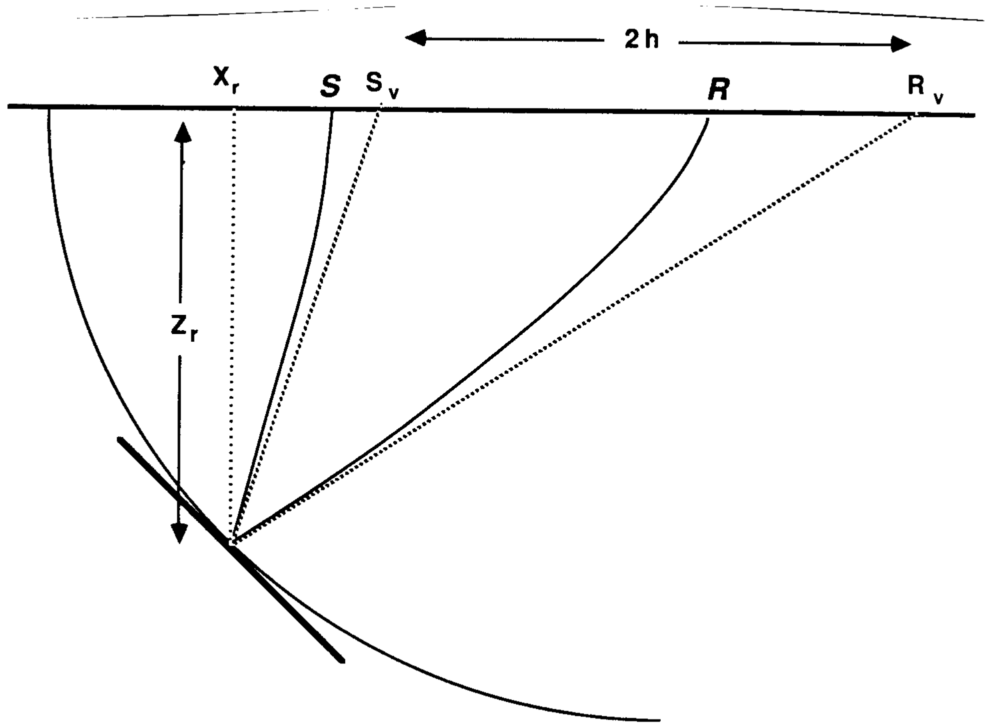

FIG. 6. The migrated image of a dipping reflector is not tangent to an ellipse that has foci at the source and receiver, but it is tangent to an ellipse with foci at a virtual source and receiver. The virtual source and receiver are found by projecting to the surface the straight line "rays" from the image of the reflector that have the same incidence angles as the true rays.

LINEAR THEORY

This section of the paper describes a linear relation between changes in the interval velocity model used for prestack depth migration and changes in the residual moveout governed by equation 7. A local perturbation to the interval velocity model will produce a change in travel time of any ray passing through the perturbation. Changes in the traveltime from source to reflector to receiver result in changes in the position of the image of the reflector in a migrated shot profile. If our observation is fixed at x_r, the movement of the image is a movement in depth. The changes in depth of the images of different shot profiles at a constant surface location lead to a change in the RMO curve. The change in the shape of the RMO curve is determined by a least squares fit of the changes of migrated depth, to changes in γ and z_r. A new value of the stack semblance can be obtained after correcting the images for the change in RMO given by the least squares fit. If the semblance is precomputed for a range of γ's then the appropriate value can be extracted from the precomputed data space.

Δt due to $\Delta w(x,z)$

If $v(x,z)$ is changed by $\Delta v(x,z)$, the traveltime of any ray passing through $\Delta v(x,z)$ will change. Write the traveltime along a ray from any subsurface point

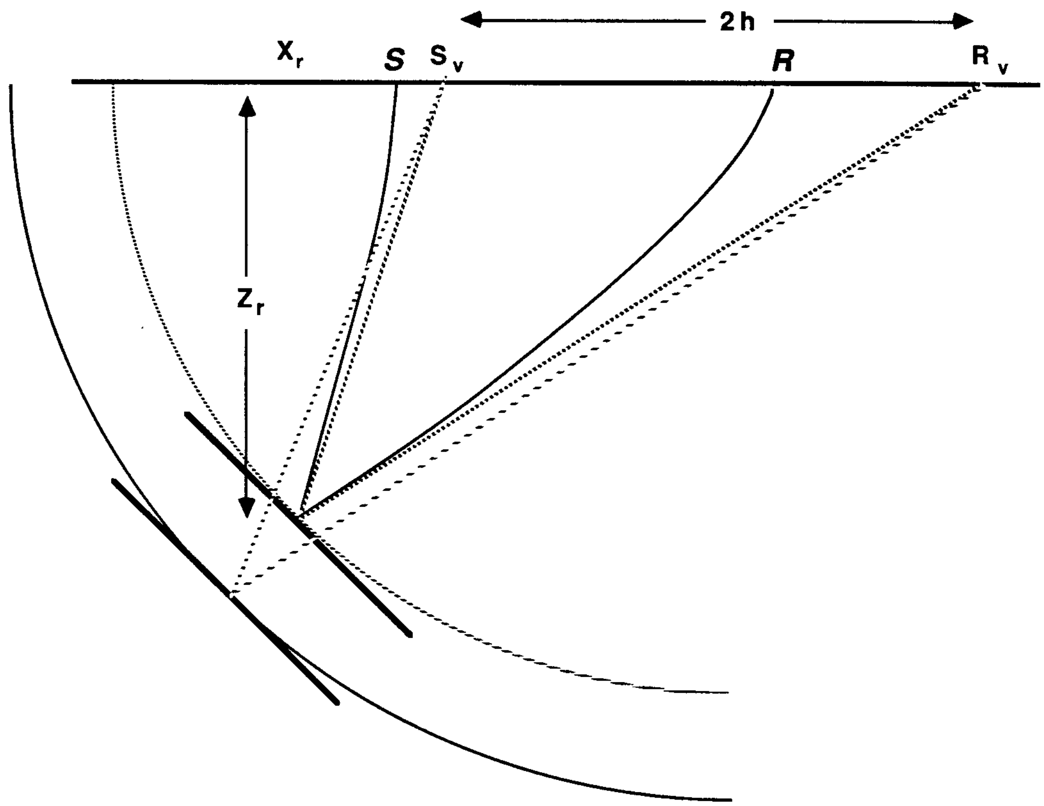

FIG. 7. Movement of the migrated image of a dipping reflector. The new image is also tangent to an effective constant velocity ellipse. The movement is normal to the bed.

to a surface point (a source or receiver) as

$$t_{ray} = \int_{ray} w(x,z)dS \tag{8}$$

Where $w(x,z) = 1/v(x,z)$ and is called the interval slowness. Although we commonly speak of velocity analysis, it is more convenient to do computations with the slowness. The raypath depends on $w(x,z)$, so calculating the change in travel time due to an arbitrary change of $w(x,z)$ requires computing the change in traveltime due to the change in slowness and the change in traveltime due to the change in the raypath. For small changes in the interval slowness model, $\Delta w(x,z)$, we can apply Fermat's principle which states that traveltime is stationary with respect to raypath. The change in traveltime due to a change in interval slowness can be calculated by considering the change of $w(x,z)$ along the original raypath since the change in traveltime due to changes in the raypath is zero to first order.

$$\Delta t = \int_{ray} \Delta w(x,z)dS \ . \tag{9}$$

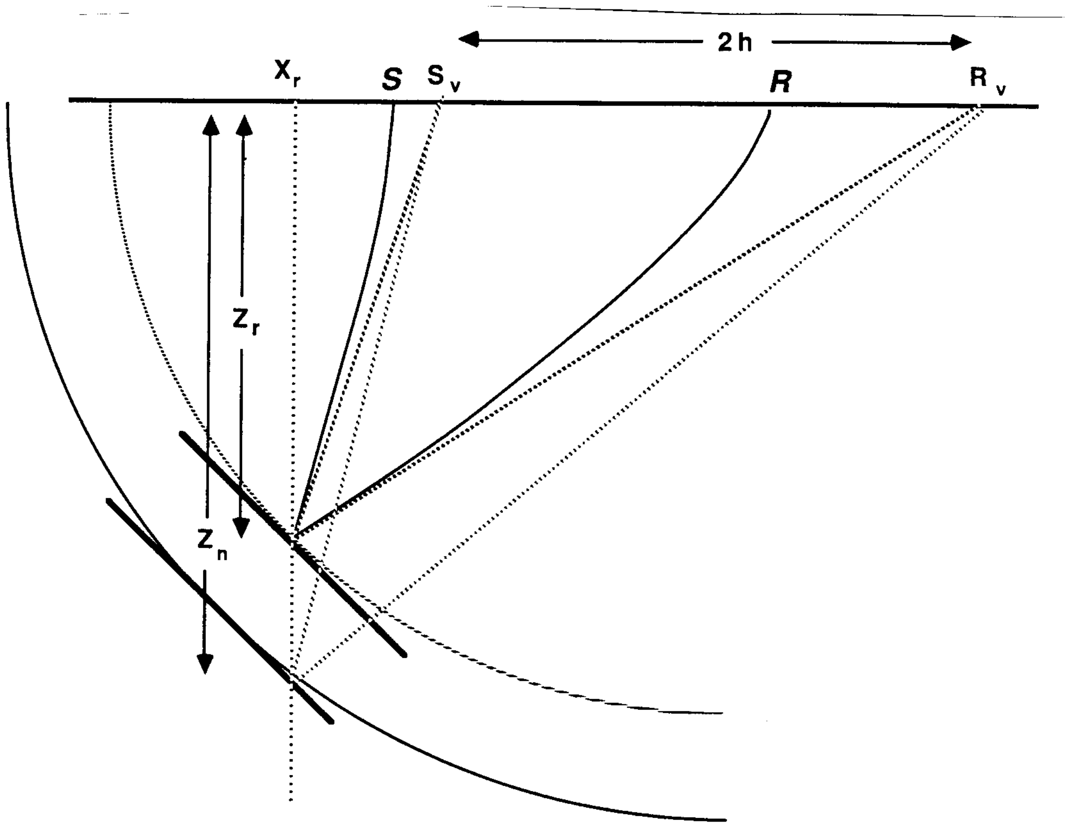

FIG. 8. Change in migrated depth at a fixed x_r due to a change in the imaging velocity. The change in depth is approximated by the movement of the tangent ellipses.

Δz_m due to Δt

For a given reflector point and a given dip θ there is a family of rays (the specular rays) connecting sources and receivers that obey Snell's law (angle of incidence = angle of reflection) at the reflector point as shown in Figure 9. These are the rays along which most reflected energy travels. A shot profile will only contain energy from a reflector with dip θ if there exists a specular ray from the source to the reflector to a receiver that falls within the geophone spread. If the specular ray does not reach the surface or falls outside the geophone spread, migrating the shot profile will not image a reflector with dip θ at a given point.

A change in traveltime along a specular ray caused by a change in interval slowness will lead to a change in the location of the image of the reflector. The reflector will move in the direction normal to the reflector. Although a given point on the reflector moves normal to the reflector, we observe the shift by holding x constant and noting the change in depth of the image of reflector. This presumes that the reflectors are locally continuous in the dip direction and the dip angle does not vary rapidly along the reflector over a small distance.

For a constant velocity medium, the derivative of the traveltime with respect to a change perpendicular to a bed of dip θ can be calculated using the chain rule.

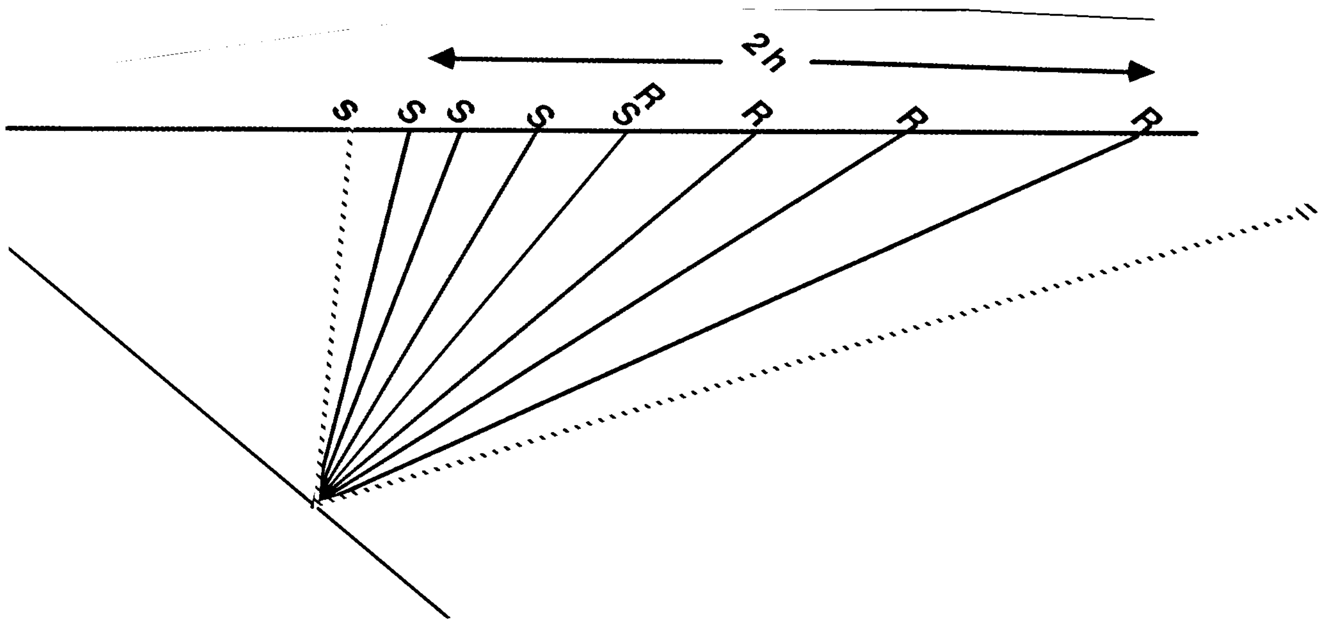

FIG. 9. Family of specular rays from sources to a given reflecting point to receivers. The dotted ray from the source to the reflector point does not end up in the receiver spread; it will not contribute to the image of a reflector with dip θ.

The arc length normal to the bed is s, $\Delta x_r = sin\theta \Delta s$, $\Delta z_r = cos\theta \Delta s$, and w is the slowness of the medium. Write the traveltime as

$$t = w\sqrt{z_r^2 + (x_r - h)^2} + w\sqrt{z_r^2 + (x_r + h)^2} \ ,$$

where x_r, z_r are the coordinates of the reflector point and w is the slowness of the medium. The derivative of traveltime with respect to movement perpendicular to the bed can be written as:

$$\frac{dt}{ds} = \frac{dt}{dx_r}\frac{dx_r}{ds} + \frac{dt}{dz_r}\frac{dz_r}{ds} \ ;$$

Where

$$\frac{dt}{dx_r}\frac{dx_r}{ds} = w\left(\frac{x_r - h}{\sqrt{z_r^2 + (x_r - h)^2}} + \frac{x_r + h}{\sqrt{z_r^2 + (x_r + h)}}\right) sin\theta \ ; \tag{10}$$

$$\frac{dt}{dz_r}\frac{dz_r}{ds} = w\left(\frac{z_r}{\sqrt{z_r^2 + (x_r - h)^2}} + \frac{z_r}{\sqrt{z_r^2 + (x_r + h)}}\right) cos\theta \ .$$

The derivative of t with respect to s does not depend on t, x, or z explicitly, but only on the dip and opening angles of the rays at the bed. Referring to Figure 10, recognizing the above quantities as functions of the ray angles, equation 10 can be rewritten as,

$$\frac{dt}{ds} = w(cos\psi cos\theta + sin\psi sin\theta + cos\phi cos\theta + sin\phi sin\theta) \ . \tag{11}$$

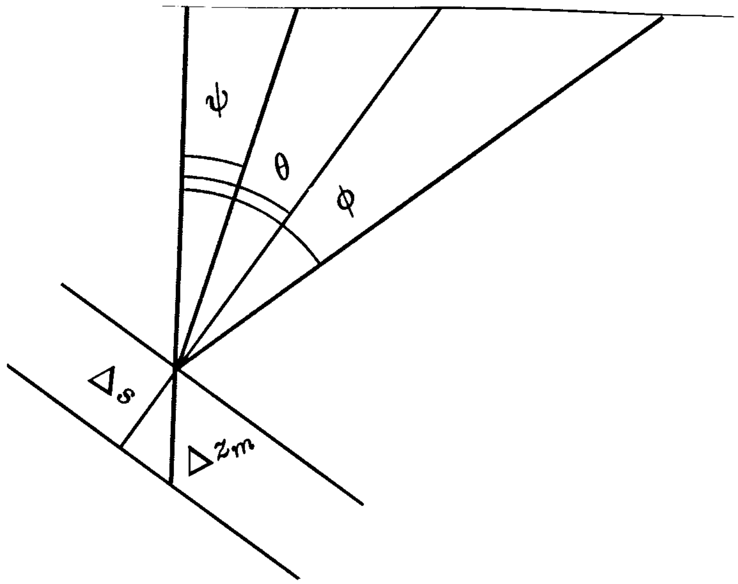

FIG. 10. Geometry of specular rays at a given reflector with dip θ. Near the reflector, the rays are approximately straight, and the change in depth of an image due to a change in traveltime along the rays only depends on the angles θ, ψ, and ϕ, and the local slowness $w(x_r, z_r)$.

The derivative of s, the distance normal to the reflector, with respect to a change in traveltime along the rays is just the inverse of equation 11. Although the derivation of the equations used the fact that velocity was constant, the derivative should be accurate even in non-constant slowness if w is taken to be the slowness at the reflector, $w(x, z)$, because the derivative only depends on the angles of the rays at the reflector.

$$\frac{ds}{dt} = \frac{1}{w(x_r, z_r)} \frac{1}{(cos\psi cos\theta + sin\psi sin\theta + cos\phi \cos\theta + sin\phi sin\theta)} \tag{12}$$

Equation 12 gives the derivative of the distance s, normal to the reflector with respect to traveltime along a specular ray from source to reflector to receiver, following the movement of the image in both x and z. To get the change in migrated depth, Δz_m, at fixed x_r, simply project the dipping reflector along the dip direction to find its new depth at x_r. As shown in Figure 10, this is accomplished by multiplying ds/dt by $sec\theta$. Write the derivative of the migrated depth z_m, with respect to traveltime along the specular ray, evaluated at a fixed surface location x_r as

$$\frac{dz_m}{dt} = \frac{1}{w(x, z)} \frac{1}{cos(\psi - \theta) + cos(\phi - \theta)} sec\theta \ . \tag{13}$$

Finally, write the change in migrated depth due to a change in traveltime along the specular ray from the shot to the reflector point to the receiver as

$$\Delta z_m = \frac{dz_m}{dt} \Delta t = \frac{1}{w(x, z)} \frac{1}{cos(\psi - \theta) + cos(\phi - \theta)} sec\theta \ \Delta t \ . \tag{14}$$

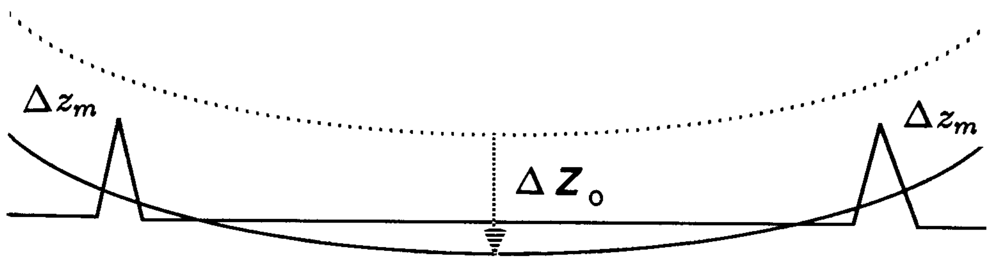

FIG. 11. A change in z_m of a given imaged shot profile, at a given depth and surface location may require both a change in γ and z_o of the residual moveout curve. The dotted curve is the RMO curve calculated for depth z_m with only a change in γ allowed, Δz_o was set to zero. To fit the distribution shown, the curve must shift down, that is z_o used to calculate z_m must change.

$\Delta\gamma$ and Δz_o due to Δz_m

The change in migrated depth Δz_m for different imaged shot profiles computed in the last two sections can be related to a change in the best fitting RMO curve generated by equation 7. The best fitting RMO curve will require not only changes in γ, but also changes in z_r the reference depth as illustrated in Figure 11. It is more appropriate to rewrite equation 7 and use z_o as the "depth" parameter for residual moveout; z_r will keep its meaning as the depth of a reflector. RMO and stacking is used as a n estimate of residual curvature of images and not as a direct velocity indicator.

$$z_n = \sqrt{\gamma^2 \frac{X}{4} + (x_r^2 + h^2) + \frac{1}{\gamma^2} \frac{4h^2 x_r^2}{X}} \quad , \tag{15}$$

where

$$X = 2z_o^2 + 2(x_r^2 + h^2) + 2\sqrt{z_o^4 + 2z_o^2(h^2 + x_r^2) + (x_r^2 - h^2)^2} \quad .$$

If either γ or z_o is changed, z_m will change. Finding the derivatives of z_m with respect to the parameters of the best fitting RMO curve linearizes equation 15 about a given value of γ and z_o. Immediately following prestack depth migration, before the slowness model is changed we can linearize about the γ and z_o that produce no residual moveout, $\gamma = 1$ and $z_o = z_r$. As the slowness model is changed, the reference values used for linearization will change. A change in z_m due to a change in γ and z_o can be written as

$$\Delta z_m = \frac{\partial z_m}{\partial \gamma} \Delta\gamma + \frac{\partial z_m}{\partial z_o} \Delta z_o \tag{16}$$

The expressions for the derivatives $\partial z_m/\partial\gamma$ and $\partial z_m/\partial z_o$ can be found in appendix A.

Rather than finding Δz_m from $\Delta\gamma$ and Δz_o, we need to do the opposite, find $\Delta\gamma$ and Δz_o from the different Δz_{m_i} at each imaged shot profile at a given depth, dip, and constant surface location. If we have used the results of the previous two sections to relate $\Delta w(x,z)$ to Δz_m for each imaged shot profile at a given surface location, $\Delta\gamma$ and Δz_o can be estimated by least squares. To find $\Delta\gamma$ and Δz_o the changes in the residual moveout curve that best explains the distribution of Δz_m over all migrated shot profiles, minimize

$$E = \sum_{shots} (\Delta z_m - \frac{\partial z_m}{\partial\gamma}\Delta\gamma - \frac{\partial z_m}{\partial z_o}\Delta z_o)^2 \quad . \tag{17}$$

The solution to this least squares problem for $\Delta\gamma$ and Δz_o, is the solution to the normal equations

$$\begin{pmatrix} \sum_{i=1}^{ns} A_i^2 & \sum_{i=1}^{ns} A_i B_i \\ \sum_{i=1}^{ns} A_i B_i & \sum_{i=1}^{ns} B_i^2 \end{pmatrix} \begin{pmatrix} \Delta\gamma \\ \Delta z_o \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{ns} A_i \Delta z_{m_i} \\ \sum_{i=1}^{ns} B_i \Delta z_{m_i} \end{pmatrix} \tag{18}$$

The sums run from 1 to ns the number of migrated shots in a CSL gather. A_i and B_i are $\partial z_{m_i}/\partial\gamma$ and $\partial z_{m_i}/\partial z_o$ respectively, at a fixed surface location, dip, and depth. It is important to realize that a change in the parameters describing the RMO curve is found, and not the value of the parameters themselves. The former is a linear problem while the latter is non-linear since z_m is not a linear function of γ and z_o. Equation 19, the solution to equation 18, relates changes in the interval slowness model to changes in the residual moveout that best fits the changes in depth caused by the changes in interval slowness.

$$\begin{pmatrix} \Delta\gamma \\ \Delta z_o \end{pmatrix} = \begin{pmatrix} \Gamma_\gamma \\ \Gamma_{z_o} \end{pmatrix} \Delta z_m \tag{19}$$

The expressions for Γ_γ and Γ_{z_o} can be found in appendix B.

Equation 19 can be combined with the operators described in the previous two sections to write one operator that relates changes in interval slowness to changes in the parameters that describe the RMO curve. We recognize $\partial t/\partial w$, as a tomography operator (Fowler, 1986), call it T. $\partial z_m/\partial t$ is a geometry factor which depends on depth, dip and opening angle of rays call it Θ. $\partial\gamma/\partial z_m$ and $\partial z_o/\partial z_m$ are really equation 19. In compact notation,

$$\begin{pmatrix} \Delta\gamma \\ \Delta z_o \end{pmatrix} = \begin{pmatrix} \Gamma_\gamma \\ \Gamma_{z_o} \end{pmatrix} \Theta\ T\ \Delta w \tag{20}$$

γ, z_o, $\Delta\gamma$, and Δz_o are functions of x_r, z_r, and also θ, because the part of $\Delta w(x,z)$ felt by the ray from shot to reflector to receiver depends on the dip θ; and the geometry factors that relate Δt to Δz_m depend on θ.

Gradient of the objective function

Residual moveout corrections for a range of γ and for all dips in the migrated and stacked image can be applied to the common surface location gathers after prestack depth migration. The resulting hyper-cube serves as a data space for estimating corrections to the interval slowness model. Changes in interval slowness $\Delta w(x,z)$ are mapped successively into changes in traveltime, changes in depth and changes in γ and z_o. We are interested in finding a better estimate of the interval slowness model, so the gradient of the objective function with respect to the interval slowness model is needed. The objective function Q is the total stack semblance of the migrated (and residual moveout corrected) data.

$$Q(\gamma, z_o) = \sum_{x_r} \sum_{z_r} \sum_{\theta} W(\theta, x_r, z_r) S(\gamma, z_o, x_r, z_r, \theta) \tag{21}$$

Where S is the stack semblance at a given depth, surface location, dip, and γ. $W(\theta, x_r, z_r)$ is the weight of the normalized dip spectrum at x_r, z_r. Assuming that the slowness model is known on a discrete grid (or parametrized by basis functions), the derivative of the objective function with respect to the interval slowness $w_i(x,z)$ can be found by applying the chain rule.

$$\frac{\partial Q}{\partial w_i} = \frac{\partial Q}{\partial \gamma} \frac{\partial \gamma}{\partial z_m} \frac{\partial z_m}{\partial t} \frac{\partial t}{\partial w_i} + \frac{\partial Q}{\partial z_o} \frac{\partial z_o}{\partial z_m} \frac{\partial z_m}{\partial t} \frac{\partial t}{\partial w_i} \tag{22}$$

If we have precomputed values of S for each x_r, z_r, θ, and γ, the derivative of the objective function with respect to γ and z_o can be calculated by finite differences.

$$\nabla_\gamma Q = \sum_{x_r} \sum_{z_r} \sum_{\theta} W(x_r, z_r, \theta) \frac{1}{\delta\gamma} \left[S(\gamma + \delta\gamma, z_o, x_r, z_r, \theta) - S(\gamma, z_o, x_r, z_r, \theta) \right] \tag{23}$$

$$\nabla_{z_o} Q = \sum_{x_r} \sum_{x_r} \sum_{\theta} W(x_r, z_r, \theta) \frac{1}{\delta z_o} \left[S(\gamma, z_o + \delta z_o, x_r, z_r, \theta) - S(\gamma, z_o, x_r, z_r, \theta) \right] \tag{24}$$

Substituting the expression for Q in terms of the semblance at each reflector, the *ith* component of the gradient of Q with respect to the interval slowness model is

$$\nabla_{w_i} Q = \sum_{x_r} \sum_{z_r} \sum_{\theta} W(\theta, x_r, z_r) \left[\frac{\partial S}{\partial \gamma} \frac{\partial \gamma}{\partial z_m} \frac{\partial z_m}{\partial t} \frac{\partial t}{\partial w_i} + \frac{\partial S}{\partial z_o} \frac{\partial z_o}{\partial z_m} \frac{\partial z_m}{\partial t} \frac{\partial t}{\partial w_i} \right] . \tag{25}$$

$\partial W / \partial \gamma$ is explicitly zero since the dip spectrum is estimated at $\gamma = 1$ and held constant for all other γ. I also assume that $\partial W / \partial z_o$ is small, that is that the dip spectrum does not change rapidly with depth. Recognizing the derivatives in equation 25 to be the transpose of the linear operator derived in the previous section, write

$$\nabla_w Q = T^T \Theta^T \Gamma_\gamma^T \nabla_\gamma Q + T^T \Theta^T \Gamma_{z_o}^T \nabla_{z_o} Q \ . \tag{26}$$

OPTIMIZATION

Optimizing Q, the objective function, will find the interval slowness model that best explains the residual moveout of the events on all the CSL gathers. Gradient ascent methods (steepest ascent or conjugate gradient) can be applied to optimize Q iteratively. The data space for the optimization is prepared as follows: i) perform prestack depth migration using $w(x,z)$, an estimate of the interval slowness model. ii) Apply a range of residual moveout corrections to the images of the migrated shot profiles at each common surface location. iii) Calculate and store the stacking semblance for each moveout correction at each CSL gather and each depth. This space serves as the data space for an optimization problem that estimates corrections to the interval velocity model using RMO. iv) Ray trace the current interval slowness model and compute the linear operator to relate changes in $w(x,z)$ to changes in parameters of the residual moveout equation. Once the data space is prepared, the goal is to find $\Delta w(x,z)$ that best removes the residual curvature from the images of the shot profiles in CSL gathers. The algorithm to accomplish this can be described as follows:

$$Prestack\ depth\ migrate\ D(s,g,t) \Rightarrow I(x_r,z_r,s)$$
$$Estimate\ W(x_r,z_r,\theta)$$
$$Apply\ residual\ moveout\ for\ a\ range\ of\ \gamma\ and\ \theta\ ; I(x_r,z_r,s) \Rightarrow R(x_r,z_r,\gamma,\theta)$$
$$Initialize\ \gamma(x_r,z_r,\theta) = 1\ ;\ z_o(x_r,z_r,\theta) = z_r$$
$$Ray\ trace\ current\ model\ of\ interval\ slowness\ w(x,z)$$

Loop on interations

$$\left[\begin{array}{l} Compute\ \nabla_\gamma Q,\ \nabla_{z_o} Q\ by\ finite-differences \\ Compute\ \nabla_w Q = T^T\ \Theta^T\ \left[\Gamma_\gamma^T \nabla_\gamma Q + \Gamma_{z_o}^T \nabla_{z_o} Q\right] \\ Line\ search\ for\ \alpha\ that\ maximizes\ Q(w(x,z) + \alpha\nabla_w Q) \\ Update\ model,\ \gamma(x_r,z_r,\theta),\ and\ z_o(x_r,z_r,\theta) \\ \quad w(x,z) = w(x,z) + \alpha\nabla_w Q \\ \quad \gamma(x_r,z_r,\theta) = \gamma(x_r,x_r,\theta)\ +\ \Gamma_\gamma\ \Theta\ T\ \alpha\nabla_w Q \\ \quad z_o(x_r,z_r,\theta) = z_o(x_r,z_r,\theta)\ +\ \Gamma_{z_o}\ \Theta\ T\ \alpha\nabla_w Q \\ Check\ for\ convergence \end{array}\right.$$

If the change in the interval slowness model is small, then prestack depth migration with interval slowness $w(x,z) + \Delta w(x,z)$ is well approximated by prestack depth migration followed by appropriate residual moveout. If the interval slowness model changes a large amount, then prestack depth migration plus RMO is not a valid approximation to prestack migration with the new velocity model. At present, I do not know how large the region of validity is for residual moveout after prestack depth migration. Since the method is designed to find laterally varying interval velocities, I will anticipate that the interval slowness will need to change more than residual moveout will accurately handle. In this case, it is necessary to remigrate the data. Although the optimization by RMO will not give the same $\Delta w(x,z)$ as using prestack depth migration for all computations, the $\Delta w(x,z)$ estimated by the algorithm above improve the stack semblance and reduce the distortion of the images in constant surface location gathers. Rather than abandoning $\Delta w(x,z)$ given by RMO, we can line search using prestack depth migration the direction $w(x,z)+\beta\Delta w(x,z)$ for the best β, where $\Delta w(x,z)$ is the result of optimizing the stack semblance with RMO. Then a new residual moveout space can be initialized at $w(x,z)+\beta\Delta w(x,z)$. Another optimization can then be carried out using residual moveout. The process can be repeated as many times as necessary to find the optimal prestack depth migrated image of the data. We can describe the complete velocity analysis method with a sketch of the algorithm.

$Set\ w_k(x,z) = \ initial\ model;\ k=1$

Loop on outer iterations

- $Prestack\ depth\ migrate\ with\ w_k(x,z); D(s,g,t) \Rightarrow I(x_r,z_r,s)$
- $Estimate\ W(x_r,z_r,\theta)$
- $Apply\ residual\ moveout\ for\ a\ range\ of\ \gamma\ and\ \theta\ ; I(x_r,z_r,s) \Rightarrow R(x_r,z_r,\gamma,\theta)$
- $Initialize\ \gamma(x_r,z_r,\theta)=1\ ;\ z_o(x_r,z_r,\theta)=z_r$
- $Raytrace\ current\ model\ of\ interval\ slowness\ w(x,z)$
- $i=1\ ;\ w_i(x,z)=w_k(x,z)$
- **Loop on inner interations**
  - $Compute\ \nabla_\gamma Q,\ \nabla_{z_o} Q\ by\ finite-differences$
  - $Compute\ \nabla_w Q = T^T\ \Theta^T\ \left[\Gamma_\gamma^T \nabla_\gamma Q + \Gamma_{z_o}^T \nabla_{z_o} Q\right]$
  - $Line\ search\ for\ \alpha\ that\ maximizes\ Q(w(x,z)+\alpha\nabla_w Q)$
  - $Update\ model,\ \gamma(x_r,z_r,\theta),\ and\ z_o(x_r,z_r,\theta)$
    - $w_i(x,z) = w_i(x,z) + \alpha\nabla_w Q$
    - $\gamma(x_r,z_r,\theta) = \gamma(x_r,x_r,\theta)\ +\ \Gamma_\gamma\ \Theta\ T\ \alpha\nabla_w Q$
    - $z_o(x_r,z_r,\theta) = z_o(x_r,z_r,\theta)\ +\ \Gamma_{z_o}\ \Theta\ T\ \alpha\nabla_w Q$
  - $\Delta w_k = w_{i+1} - w_k$
  - $If\ |\Delta w(x,z)| > bound,\ exit\ inner\ loop$
    $else\ increment\ i$
- $Line\ search\ for\ \beta\ that\ maximizes\ Q(w_k+\beta\Delta w_k)$
  $using\ prestack\ depth\ migration$
- $Update\ w_{k+1}(x,z) = w_k(x,z) + \beta\Delta w(x,z)$
- $Check\ for\ convergence$
- $increment\ k$

The method presented should be able to remove the residual curvature of migrated events in all CSL gathers of a data set, while estimating a model of the interval slowness. The method will be less effective on distortions of the CSL gathers that are not well explained by residual curvature. In this case it might be worthwhile to forego the RMO correction and try to estimate the depth corrections directly by shifting the images of different shot profiles independently, much like residual statics corrections. Doing this increases the effective dimensionality of the problem however, so it is wise to try the theory presented here, and first remove any distortions from the data that are well explained by residual curvature.

CONCLUSIONS

Velocity analysis by prestack depth migration requires knowing how the image changes when the velocity changes. There is no exact theory that describes the movement of migrated events with a change to a general velocity model. Under the ray theory approximation, it is possible to derive a linear operator to relate changes in the interval slowness to changes in the depth of images of migrated shot profiles. Furthermore, it is possible to relate changes in depth of images to changes in a best fitting residual moveout curve using linear least squares. A range of residual moveout can be applied to the data and the resulting stack semblance computed and stored. The linear operator can be used to find the stack semblance of the migrated image that would be obtained by prestack depth migration at any velocity model near the velocity model used for migration by extracting the appropriate value from the precomputed space of semblances. The linear operator also allows efficient computation of the gradient of the total stack semblance by back-projection.

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APPENDIX A

The derivatives of $z_m(\gamma, z_o)$ at fixed x_r, z_r with respect to γ and z_o for each migrated shot are used to linearize equation 15 of the text. Restating Equation 15, write

$$z_m = \sqrt{\gamma^2 \frac{X}{4} + (x_r^2 + h^2) + \frac{1}{\gamma^2}\frac{4h^2x_r^2}{X}} \;, \tag{A.1}$$

where

$$X = 2z_o^2 + 2(x_r^2 + h^2) + 2\sqrt{z_o^4 + 2z_o^2(h^2 + x_r^2) + (x_r^2 - h^2)^2} \;.$$

Differentiating with respect to γ:

$$\frac{\partial z_m}{\partial \gamma} = \frac{1}{2}\Big[\gamma^2\frac{X}{4} + (x_r^2 + h^2) + \frac{1}{\gamma^2}\frac{4h^2x_r^2}{X}\Big]^{-\frac{1}{2}}\Big(\frac{\gamma X}{2} - \frac{2}{\gamma^3}\frac{4h^2x_r^2}{X}\Big) \;, \tag{A.2}$$

where X has the same meaning as in equation A.1. Also differentiate with respect to z_o.

$$\frac{\partial z_m}{\partial z_o} = \frac{1}{2}\Big[\gamma^2\frac{X}{4} + (x_r^2 + h^2) + \frac{1}{\gamma^2}\frac{4h^2x_r^2}{X}\Big]^{-\frac{1}{2}}\Big[\frac{\gamma^2}{4} - 4\frac{h^2x^2}{\gamma^2X^2}\Big]\frac{\partial X}{\partial z_o} \tag{A.3}$$

where

$$\frac{\partial X}{\partial z_o} = 4\bigg[z_o + \Big[z_o^4 + 2z_o^2(h_2 + x^2) + (x^2 - h^2)^2\Big]^{-\frac{1}{2}}\Big[z_o^3 + z_o(h^2 + x^2)\Big]\bigg]$$

and X is the same as in equation A.1.

APPENDIX B

Write the normal equations that solve for $\Delta\gamma$ and Δz_o, the change in the parameters of the best fitting RMO curve as

$$\begin{pmatrix} \sum_{i=1}^{ns} A_i^2 & \sum_{i=1}^{ns} A_iB_i \\ \sum_{i=1}^{ns} A_iB_i & \sum_{i=1}^{ns} B_i^2 \end{pmatrix} \begin{pmatrix} \Delta\gamma \\ \Delta z_o \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{ns} A_i \Delta z_{m_i} \\ \sum_{i=1}^{ns} B_i \Delta z_{m_i} \end{pmatrix} \;, \tag{B.1}$$

where $A_i = \partial z_{m_i}/\partial\gamma$ and $B_i = \partial z_{m_i}/\partial z_o$.

The solution to the normal equations is

$$\begin{pmatrix} \Delta\gamma \\ \Delta z_o \end{pmatrix} =$$

$$\frac{1}{\sum_{i=1}^{ns} A_i^2 \sum_{i=1}^{ns} B_i^2 - (\sum_{i=1}^{ns} A_i B_i)^2} \begin{pmatrix} \sum_{i=1}^{ns} B_i^2 & -\sum_{i=1}^{ns} A_i B_i \\ -\sum_{i=1}^{ns} A_i B_i & \sum_{i=1}^{ns} B_i^2 \end{pmatrix} \begin{pmatrix} \sum_{i=1}^{ns} A_i \Delta z_{m_i} \\ \sum_{i=1}^{ns} B_i \Delta z_{m_i} \end{pmatrix} \quad . \qquad (B.2)$$

Writing these expressions in terms of the operators in the text,

$$\Gamma_\gamma = \frac{1}{\sum_{i=1}^{ns} A_i^2 \sum_{i=1}^{ns} B_i^2 - (\sum_{i=1}^{ns} A_i B_i)^2} \left[\sum_{i=1}^{ns} B_i^2 \sum_{i=1}^{ns} A_i \Delta z_{m_i} - \sum_{i=1}^{ns} A_i B_i \sum_{i=1}^{ns} B_i \Delta z_{m_i} \right] , \qquad (B.3)$$

$$\Gamma_{z_o} = \frac{1}{\sum_{i=1}^{ns} A_i^2 \sum_{i=1}^{ns} B_i^2 - (\sum_{i=1}^{ns} A_i B_i)^2} \left[\sum_{i=1}^{ns} A_i^2 \sum_{i=1}^{ns} B_i \Delta z_{m_i} - \sum_{i=1}^{ns} A_i B_i \sum_{i=1}^{ns} A_i \Delta z_{m_i} \right] . \qquad (B.4)$$