Appendix B

Velocity Sampling for Prestack Migration Velocity Analysis

In this appendix, I derive an expression to estimate the necessary velocity sampling for prestack migration velocity analysis by constant velocity migration.

We want to sample the velocity in such a way that an energy packet does not move more than a given upper bound from one velocity panel to the next. This upper bound is governed by the seismic experiment. Half the wavelength is a reasonable upper bound on the vertical shift; half the width of the Fresnel zone is a reasonable upper bound on the horizontal shift.

Let's see what happens if we migrate a profile using Kirchoff's method as shown in Figure B.1. If we migrate the diffraction curve with velocity v, then the point (x_g, t_g) will be mapped to the point (x_0, z_0). If instead we use velocity $\hat{v}$, then it will be mapped to the point $(\hat{x}, \hat{z})$.

When we use the velocity v, the traveltime curve for the diffraction in Figure B.1a is given by

$$t = \frac{1}{v}\left[\sqrt{(x_g - x_0)^2 + z_0^2} + \sqrt{(x_s - x_0)^2 + z_0^2}\,\right] , \tag{B.1}$$

From which we obtain

$$(x_g - x_0)^2 = \left(tv - \sqrt{(x_s - x_0)^2 + z_0^2}\,\right)^2 - z_0^2 . \tag{B.2}$$

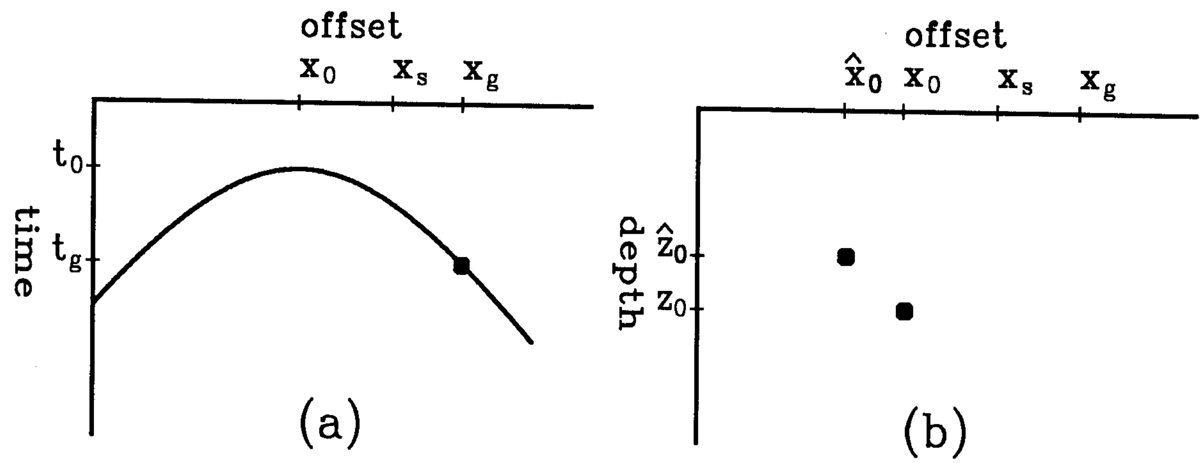

FIG. B.1. (a) A diffraction curve in a field profile. (b) The energy at (x_g, t_g) is migrated to (x_0, z_0) when velocity v is used, and to $(\hat{x}_0, \hat{z}_0)$ when velocity $\hat{v}$ is used.

When the velocity v is replaced by $\hat{v}$, equation (B.2) becomes

$$(x_g - \hat{x}_0)^2 = \left(t\hat{v} - \sqrt{(x_s - \hat{x}_0)^2 + \hat{z}_0^2}\right)^2 - \hat{z}_0^2 \, . \tag{B.3}$$

Subtracting (B.3) from (B.2), we obtain

$$\begin{aligned}(x_g - x_0)^2 - (x_g - \hat{x}_0)^2 &= t^2(v^2 - \hat{v}^2) + (x_s - x_0)^2 - 2tv\sqrt{(x_s - x_0)^2 + z_0^2} \\ &\quad -(x_s - \hat{x}_0)^2 - 2t\hat{v}\sqrt{(x_s - \hat{x}_0)^2 + \hat{z}_0^2} \, .\end{aligned} \tag{B.4}$$

Letting $\Delta x_0 = \hat{x}_0 - x_0$ and $\gamma = \hat{v}/v$, equation (B.4) becomes

$$2x_g\Delta x_0 = t^2v^2(1-\gamma^2) + 2x_s\Delta x_0 - 2tv\left[\sqrt{(x_s - x_0)^2 + z_0^2} - \gamma\sqrt{(x_s - \hat{x}_0)^2 + \hat{z}_0^2}\right] . \tag{B.5}$$

Now,

$$\begin{aligned}\gamma &= \frac{\hat{v}}{v} = \frac{v + \Delta v}{v} \\ &= 1 + \frac{\Delta v}{v} \, ,\end{aligned} \tag{B.6}$$

and

$$\gamma^2 \approx 1 + \frac{2\Delta v}{v} . \tag{B.7}$$

From (B.5), (B.6) and (B.7),

$$\frac{\Delta v}{v} \approx \frac{\Delta x_0(x_s - x_g) + tv\left[\sqrt{(x_s - \hat{x}_0)^2 + \hat{z}_0^2} - \sqrt{(x_s - x_0)^2 + z_0^2}\right]}{t^2v^2 - tv\sqrt{(x_s - \hat{x}_0)^2 + \hat{z}_0^2}} . \tag{B.8}$$

Note that

$$\hat{x}_0^2 = (x_0 + \Delta x_0)^2 \approx x_0^2 + 2x_0\Delta x_0 ,$$

and

$$\hat{z}_0^2 = (z_0 + \Delta z_0)^2 \approx z_0^2 + 2z_0\Delta z_0 .$$

Therefore,

$$(x_s - \hat{x}_0)^2 + \hat{z}_0 \approx (x_s - x_0)^2 - 2\Delta x_0(x_s - x_0) + z_0^2 + 2z_0\Delta z_0 . \tag{B.9}$$

Letting X to be equal to the right-hand side of (B.9), equation (B.8) becomes

$$\frac{\Delta v}{v} \approx \frac{\Delta x_0(x_s - x_g) + tv\left[\sqrt{X} - \sqrt{(x_s - x_0)^2 + z_0^2}\right]}{t^2v^2 - tv\sqrt{X}} . \tag{B.10}$$

For x_s=.5 km, x_g=1 km, x_0=0 km, z_0=1 km, v=2 km/sec, and $\Delta x_0 = \Delta z_0 = 50$ m, equation (B.10) evaluates Δv to be about 18 m/sec, which is a conservative estimate. Experience shows that a coarser sampling is possible. Figure B.2 shows how $\Delta v/v$ varies with Δx_0 and Δz_0. As expected, the larger we allow Δx_0 and Δz_0 to be, the larger $\Delta v/v$ becomes. That is, if we allow the difference between adjacent velocity panels to be large, then we can sample the velocity more coarsely than if we restrict it to be small.

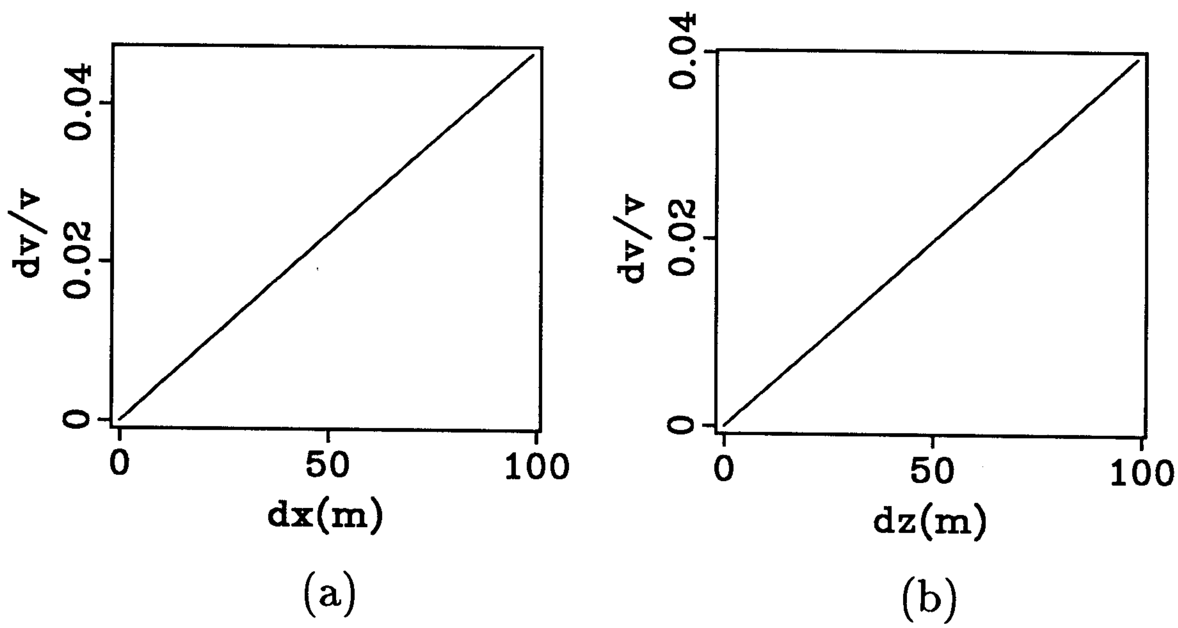

FIG. B.2. $\Delta v/v$ as a function of dx and dz. For both plots: x_s=.5 km, x_g=1 km, x_0=0 km, z_0=1 km, v=2 km/sec. (a) Making dz=0 and varying dx. (b) Making dx=0 and varying dz.