Appendix A

Adequacy of finite differences

A.1 Introduction and overview

A.1.1 Finite difference scheme

The inversion algorithm presented in chapter 3 requires a method to solve the elastic wave equation. I use the method of finite differences (Mora, 1986) because it can handle realistic 2D distributions of elastic parameters. It numerically solves the wave equation in an inhomogeneous medium using an explicit finite difference scheme by approximating spatial derivatives with convolutional operators and time derivatives with finite difference formulas. The algorithm is based on Kosloff's method (Kosloff et al., 1984) but uses eight-point convolutional operators rather than Fourier transforms to do spatial derivatives. On the CONVEX C-1, my eight-point scheme is much faster than Kosloff's and represents an optimal scheme in terms of memory and speed.

One consequence of using finite differences is that the medium must be discretized onto a square grid so the wavenumber spectrum for elastic parameters contains an upper cutoff (i.e. layer boundaries have limited sharpness). Also, recall that I have used an absorbing boundary condition rather than a more realistic free surface boundary condition at the Earth's surface in the inversion examples.

A.1.2 Questions of adequacy for inversion

Some questions arise concerning the finite difference modeling. Is the accuracy of the finite difference solution adequate for inversions? What are the implications of the upper cutoff in the wavenumber spectrum? What is the effect of not including the free surface boundary condition? These questions have particular relevance concerning P-P and P-S events on which I have focussed attention throughout the examples in the thesis (because P-P and P-S are the main events that resolve the P- and S-wave velocity models in typical reflection seismic data).

The above questions are addressed by studying two specific questions.

(1) How does the finite difference solution compare with the solution obtained with another method. I have chosen to make the comparison with the solution obtained by the Haskell-Thompson method because this is a well known and accurate method.

(2) Does the free surface play an important role on the amplitudes of P-P and P-S reflections.

A.1.3 Answers

Results from my elastic finite difference program (Mora, 1986) are compared with results from "solid", a Haskell-Thompson program (due to John Sherwood). The seismograms calculated using the two methods look qualitatively alike. The Rayleigh wave generated by the finite difference program does not compare well with the Haskell-Thompson Rayleigh wave. However, P-P and P-S reflection amplitudes are comparable except when the incidence angles are above 45 degrees. The differences between the two sets of results can be attributed to the upper cutoff in finite differences (described above) which means that high angle waves propagating on a finite difference grid see horizontal interfaces as smooth transition zones.

That the results from the two methods compare well below 45 degrees suggests a correct finite difference implementation and that it has an adequate accuracy for inversion (i.e. P-P and P-S reflections are accurately modeled). Use of finite differences implies that the best-fit wavenumber-limited model will be located in the inversion (i.e. best-fit model for the given discretization).

A.2 Finite differences compared to Haskell-Thompson

A.2.1 Differences in the assumptions

The essential differences between my elastic finite difference program and Sherwood's Haskell-Thompson program that I expect to cause differences in the results are:

(1) The finite difference program cannot handle interfaces sharper than the grid discretization while the Sherwood's program only handles homogeneous layers separated by sharp interfaces.

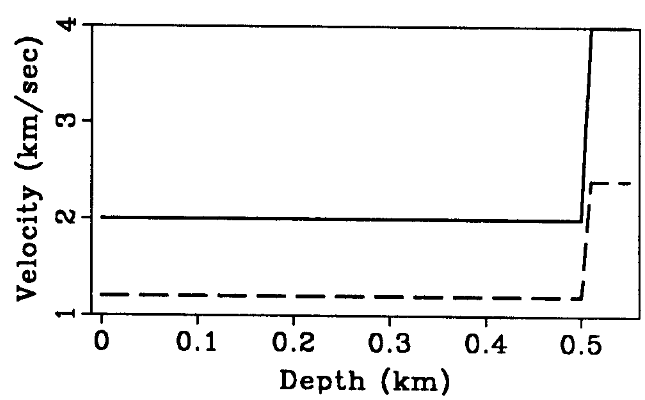
(2) My sources are not point sources. I use a bell shaped spatial distribution to avoid dispersive numerical noise which would occur if high spatial frequencies were introduced.

(3) The version of Sherwood's program I used could only generate explosive sources while mine can do vertical, horizontal or shear sources as well.

A.2.2 Comparison of results

Figure A.1 shows a 1D velocity model used to generate some synthetic data. The velocity contrast (2:1) and offset-depth ratio (6:1 $\Rightarrow \theta_{inc} = 70°$) are unusually large to highlight differences in the two methods. Note that the shear to compressional velocity ratio (β/α) is 0.6 which implies a zero offset P-S to P-P traveltime ratio (t_{P-S}/t_{P-P}) of 1.33.

Figure A.1: The velocity model.



Figures A.2 and A.3 show vertical component shot profiles due to a compressional source respectively computed using my finite difference program and Sherwood's Haskell-Thompson program. A free surface boundary condition was used in the finite difference computations while an air-solid interface was used in the Haskell-Thompson calculations.

Qualitatively, results seem alike although the Rayleigh wave and high angle reflections have significant differences. Are the differences due to different assumptions in the methods or inaccuracies in the finite difference scheme? How will the inversions be affected? The next sections show some examples to try to answer these questions.

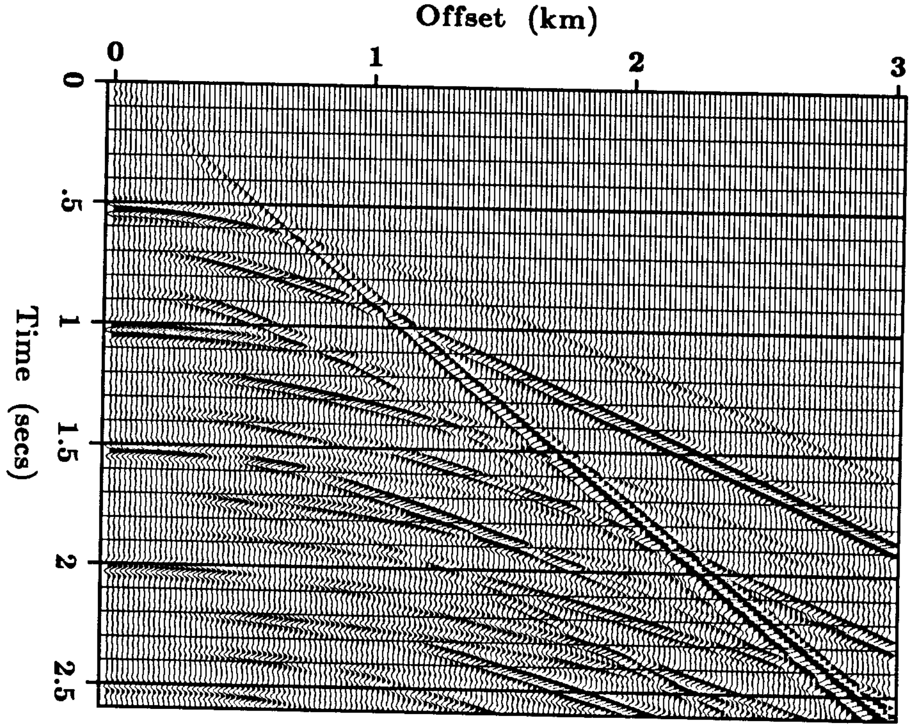


Figure A.2: The pressure source - vertical receiver shot gather via FD (c.f. Figure A.3).

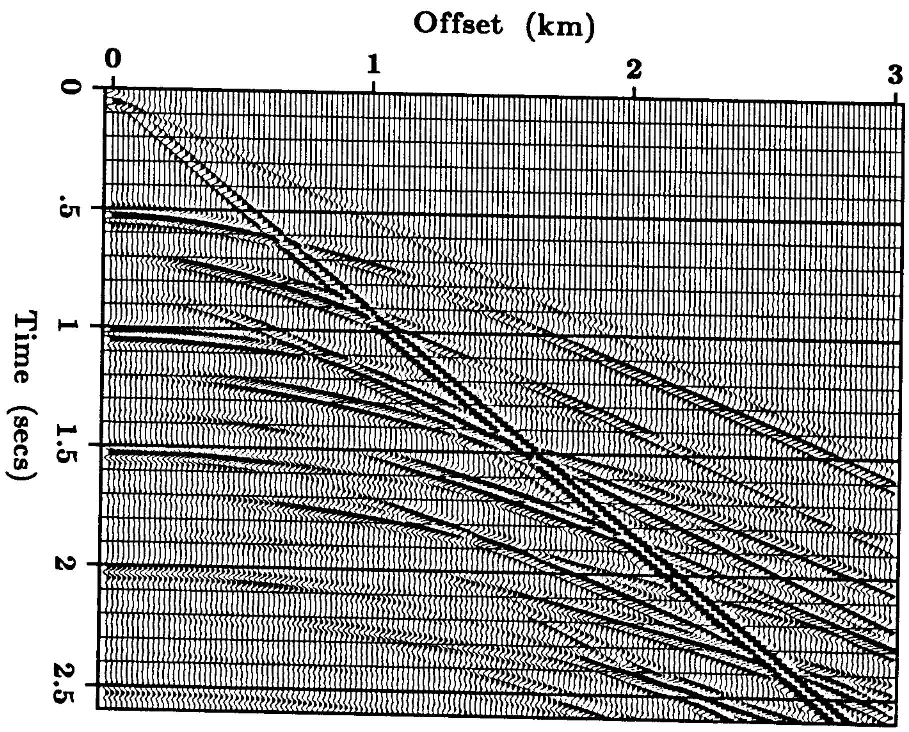
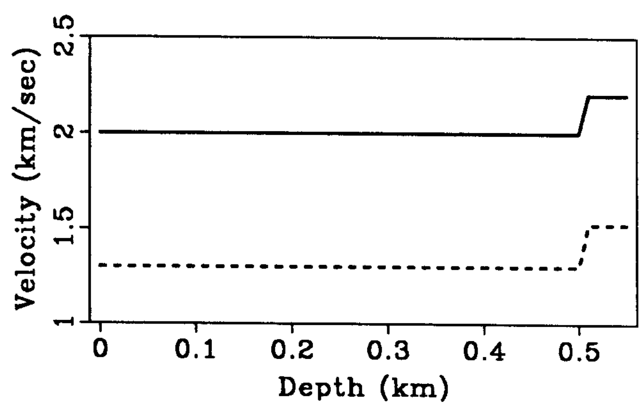


Figure A.3: The pressure source - vertical receiver shot gather via Sherwood (c.f. Figure A.2).

A.2.3 Comparison of P-P and P-S reflections

Figure A.4 shows the 1D velocity model I will use in the following tests. The reflector depth is .5 km and offset ranges used in these tests is from 0.0 km to 2.0 km. Therefore, the incidence angles range from 0 degrees at the inner offset to about 65 degrees at the highest offset. This is beyond the typical range in seismic exploration. For example, the field data in my thesis has a maximum incidence angle of about 45 degrees at the depth of interest.

Figure A.4: The velocity model.



Figures A.5 and A.6 show vertical component shot profiles due to a compressional source respectively computed using my finite difference program and Sherwood's Haskell-Thompson program. Qualitatively, the amplitudes of the P-P and P-S reflections of Figures A.4 and A.5 are in close agreement except at incidence angles above about 45 degrees (at offsets larger than 1.0 km).

The difference between Figures A.5 and A.6 is plotted in Figure A.7 at the same scale. This indicates that the two programs quantitatively yield similar results except at incidence angles beyond 45 degrees. The differences are probably due to the non sharp interface felt by waves propagating on the finite difference grid. At high incidence angles, this effect is more pronounced.

A.2.4 Comparison of results using a free surface B.C.

Figures A.8 and A.9 show a comparison between vertical force - vertical receiver shot gathers computed by the two methods. Note that when a free surface is present, Sherwood's

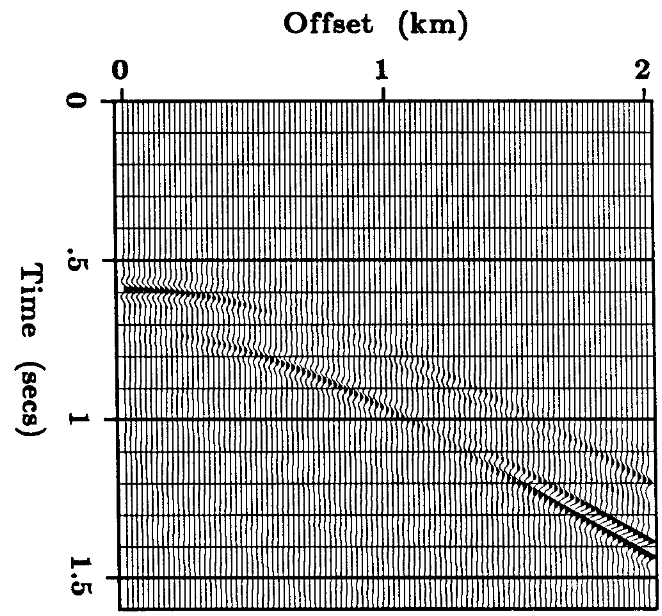


Figure A.5: The pressure source - vertical receiver shot gather via FD (c.f. Figure A.6).

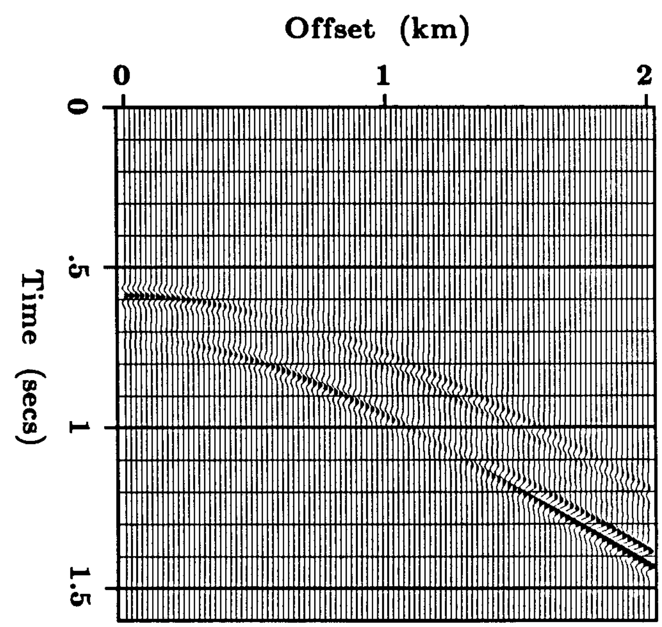


Figure A.6: The pressure source - vertical receiver shot gather via Sherwood (c.f. Figure A.5).

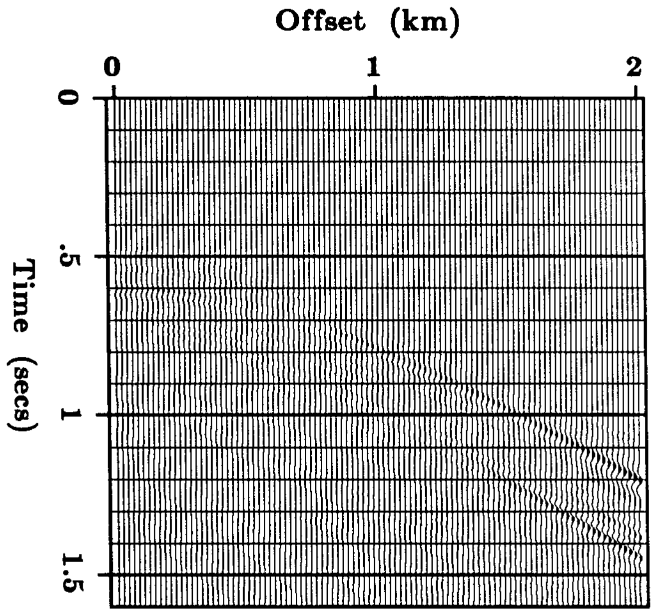


Figure A.7: The difference between Figures A.5 and Figure A.6 (Figure A.5 - FigureA.6).

program can be used to simulate a vertical source by placing a P source just above the air-solid interface instead of just below it (Sherwood, pers. comm.).

Qualitatively, the synthetic data generated with the two methods seems to be in reasonable agreement. The main differences are the presence of an air wave on Sherwood's result, the Rayleigh waves have a different character, and there is a weak direct P-wave on the finite difference result.

The air wave on Sherwood's result occurs because his program models the Earth's surface as a contact between air and the Earth. By comparison, the finite difference program models the Earth's surface as a free surface using a zero stress boundary condition. An air layer cannot be used because the grid spacing needed to avoid numerical grid dispersion would be so small that the model would no longer fit onto the computer.

From the results of Figures A.8 and A.9, it is clear that Rayleigh waves are not well modeled by the finite difference method. At a sharp free surface interface, the Rayleigh waves are expected to be non-dispersive and their waveform should not change (see Sherwood's result in Figure A.9). However, the finite difference Rayleigh wave suffers a phase rotation as it propagates perhaps due to the non sharp nature of the free surface felt by the waves propagating in the finite difference grid.

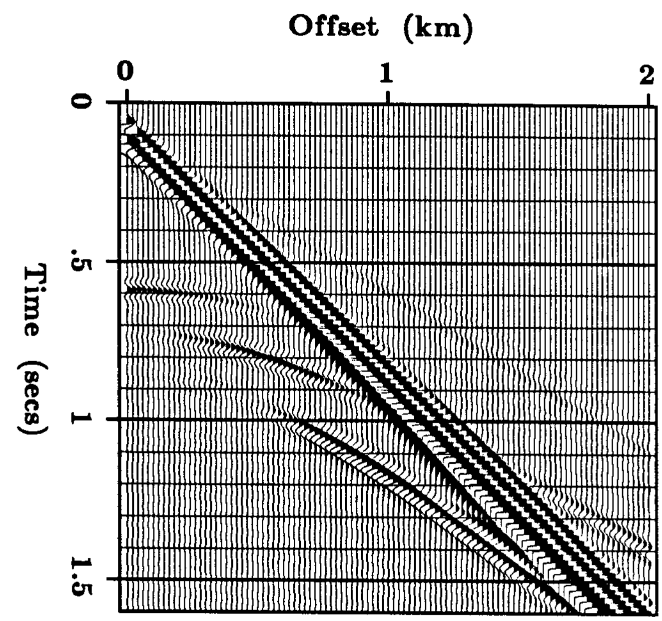


Figure A.8: The vertical source - vertical receiver shot gather via FD (c.f. Figure A.9) computed using a free surface boundary condition.

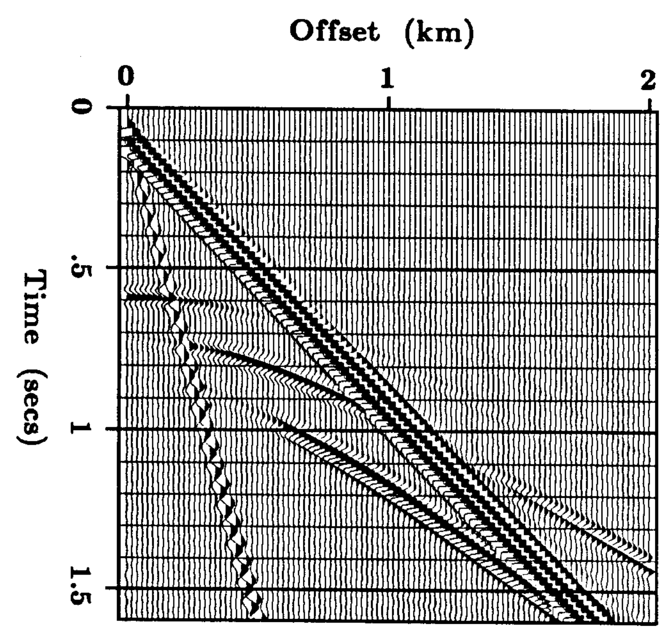


Figure A.9: The vertical source - vertical receiver shot gather via Sherwood (c.f. Figure A.8) computed using an air-solid surface boundary condition.

The differences between the two sets of results are shown quantitatively by subtracting Sherwood from Mora (Figure A.10).

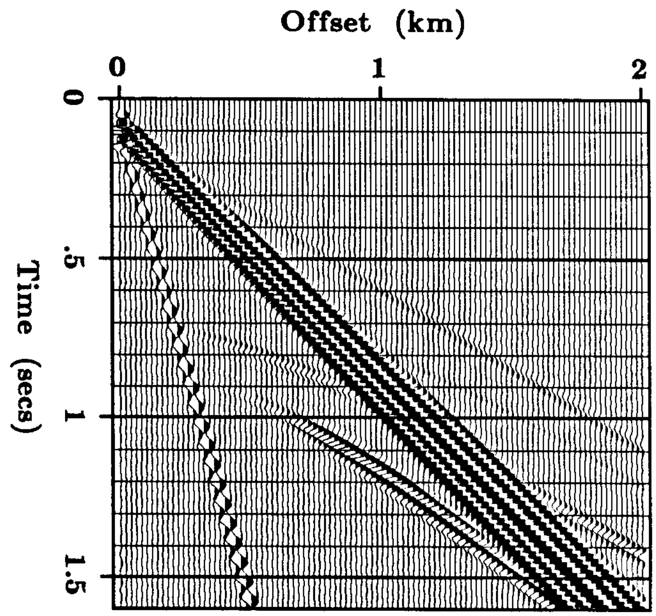


Figure A.10: The difference between Figures A.8 and Figure A.9 (Figure A.8 - Figure A.9).

Figure A.10 emphasizes that Rayleigh waves are different, the air wave cannot be modeled by FD and that the P-P and P-S reflections are about the same until about 45 degrees. While the S-S reflection in the FD and Haskell-Thompson appeared to be qualitatively alike, Figure A.10 shows that there is a large quantitative difference especially at high incidence angles. The reason for this difference in the S-S amplitude at larger incidence angles is probably the same as the reason for the Rayleigh wave phase rotation (i.e. non-sharpness of the free surface as represented on a finite difference grid).

A.3 Are free surfaces needed for P-P and P-S amplitudes?

In this section, I will examine the effect of the free surface on the P-P and P-S reflections to evaluate whether it plays an important role in the inversion examples of this thesis. To make a valid comparison I must use the same modeling program. I chose my FD program because it can model vertical sources with an absorbing boundary condition (Sherwood's can't) and I will make a comparison with the results in the previous section.

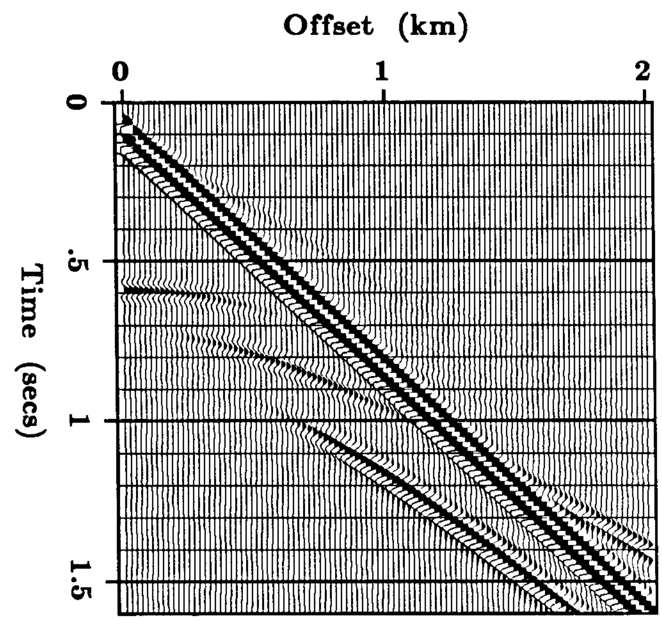


Figure A.11: The vertical source - vertical receiver shot gather via FD (c.f. Figure A.8) computed using an absorbing surface boundary condition.

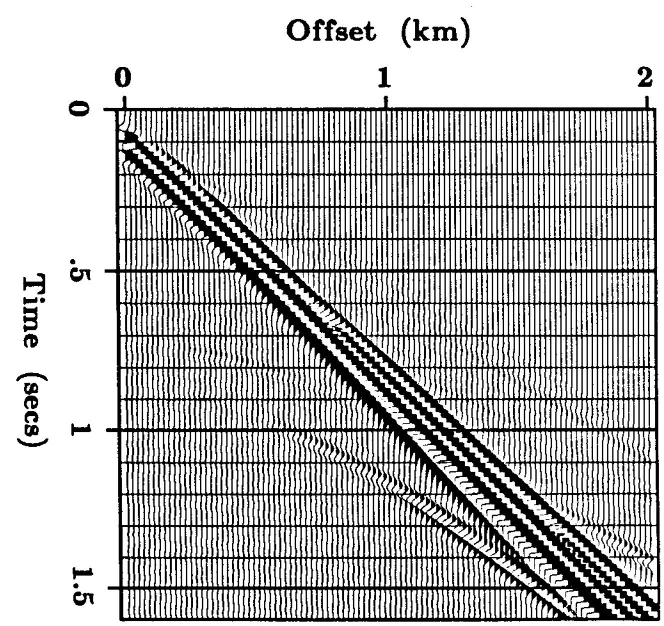


Figure A.12: The difference between Figures A.11 and Figure A.8 (Figure A.11 - Figure A.8) to see the effect of the free surface on the reflections. The difference is small except beyond 45 degrees.

Figure A.11 shows the same vertical source - vertical receiver gather shown in Figure A.8 but computed with an absorbing boundary. Except for the Rayleigh wave and the direct wave, the results are qualitatively almost identical. This is demonstrated by plotting the difference between Figure A.11 and Figure A.8 (see Figure A.12). The difference in the P-P and P-S reflections is small for offsets less than 1.0 km (45 degrees) but is significant at offsets around 2.0 km (65 degrees). There is also a significant difference in the S-S reflection amplitude beyond 45 degrees.

A.4 Conclusions

My elastic finite difference program and Sherwood's Haskell-Thompson programs generate P-P and P-S reflection amplitudes that are not significantly different for incidence angles less than 45 degrees. Beyond 45 degrees there are differences especially in S-S waves if present. Sherwood's program can model air waves and better model Rayleigh waves (at a sharp Earth-air surface).

The free surface does not greatly affect the amplitudes of the P-P and P-S reflections for incidence angles less than about 45 degrees.