Chapter 1

Dreaming of inversion

1.1 The seismologists' dream

The goal of seismology is to use seismic data observations and a thorough understanding of seismic wave propagation to gain knowledge of the Earth. The physical and mathematical foundations to achieve this goal were laid down by Newton and Leibnitz who invented calculus and Newton who discovered the equations of motion. Using the analytical calculus and mechanics of Euler and Lagrange, the French school, including Cauchy, Poisson, and Lamé, developed the basis for the theory of elasticity. Now, the stage was set for the detailed analysis of waves in elastic media; noteworthy contributions were made by Love and Rayleigh. Modern theoretical seismology is based on these works and approximates seismic waves as elastic waves.

That seismic waves can be modeled as elastic waves has proven valid in the fields of earthquake seismology, rock physics and modern reflection seismic prospecting. This is because elastic wave theory predicts the observed traveltimes and amplitudes of the two seismic wave types, compressional waves and shear waves, called Primary and Secondary waves or P- and S-waves for historical reasons. Other wave types such as Rayleigh and Love waves are manifestations of these two fundamental wave types at the Earth's surface.

The goal of gaining knowledge about the Earth can be achieved through an understanding of how the P- and S-waves are influenced by the Earth's properties. The most important physical properties are the compressional and shear wavespeeds. There are two components to each of these two properties, namely the high- and low-wavenumber components. The high wavenumbers are sharp variations that cause reflections while the low wavenumbers are gross features that control the overall traveltime of waves.

The seismologists' dream is to be able to supply seismic data observations to a computer and simply wait for the corresponding model of the Earth to pop out. This model should contain at least the four most resolvable (and hence important) parameters mentioned above, namely the high- and low-wavenumber components of the P- and S-wave velocity.

1.2 Conventional wisdom fails

Decades of developmental work have yielded many proven methods for the processing of reflection seismic data, to obtain useful information about the Earth. Unfortunately, because of the high cost of recording multi-component data and wide offsets in the old days, waves that travel both down to a reflector and back up to the Earth's surface as compressional waves, called P-P reflection events, dominated seismic records. Hence, the processing theory was developed around the acoustic wave equation. Today, multi-component data and wide offsets are more routine and shear waves are frequently recorded. The main shear events are the different primary reflections. These are mode-converted waves that travel down to a reflector as one wave type and are reflected back as the other (P-S and S-P) and shear wave primaries (S-S). What can be done to account for these waves? How can they be used to extract additional information from the data?

One possibility is to bend the conventional methods such as velocity analysis and migration to process the P-S, S-P and S-S reflections. However, this approach fails to use the relationship between the different modes (implied by the the elastic wave equation) to help to constrain the model of the Earth. Furthermore, standard methods frequently make assumptions that are not valid for wide offsets and mode converted waves. For example, velocity analysis assumes that raypaths are straight and that both downgoing and upgoing waves travel at the same speed.

Aside from such shortcomings, there are some other tradeoffs in applying standard processing methods. For instance, tomography can account for ray bending but only at the expense of including interpretive steps to identify reflectors and pick their traveltimes.

A few standard methods and their potential pitfalls are listed below.

1. Methods to obtain interval velocity.
    - (a) Conventional NMO velocity analysis: theoretically valid only when the Earth is laterally homogeneous and when rays are straight and hence offsets are small.
    - (b) Reflection tomography: needs assumptions about the reflectors (e.g. reflector depths or locations).
    - (c) Transmission tomography: needs sources and receivers surrounding the region of interest for good results.
    - (d) Migration velocity analysis: always assumes that waves are acoustic and frequently that either the Earth is smoothly varying in the lateral direction or the waves are more or less vertically propagating (although a wide range of propagation angles is needed to resolve interval velocity).
2. Methods to obtain reflectivities.
    - (a) Migration: is frequently applicable only to acoustic waves traveling more or less vertically; handles elastic wave amplitudes incorrectly; and often has difficulty in laterally variable media.
    - (b) Standard inversion: does not obtain the low vertical wavenumbers in the velocity model.
3. Most seismic processing methods to obtain Earth parameters.
    - (a) Most methods: cannot handle a wide variety of data types. For example, VSP data, well-to-well, surface seismics are usually handled independently although these data sets all record seismic waves governed by the same equation.
    - (b) Most methods: treat Rayleigh waves as noise instead of using them to help determine the near-surface velocity variations and thus solve the statics problem.
    - (c) Most methods: do not handle mode conversions. Thus, they must separate the P-P, P-S, S-P and S-S events and process them separately. Even then, amplitudes and traveltimes of the different events, particularly the mode-converted waves, are usually handled incorrectly.

Clearly something is wrong: there are multitudes of methods, each with different limitations and ranges of applicability, yet waves are governed by one equation; information is ignored and treated as noise; and information cannot be regained accurately about S-wave velocity.

1.3 Inversion to the rescue

Why not have one method that is theoretically without bad assumptions or tradeoffs for all problems? This can be realized by assuming the correct physical equation for seismic wave propagation and finding a model that predicts synthetic data that best matches the seismic observations. This is inversion. The results of inversion will be as good as the assumed modeling theory and as the algorithm will be as efficient as the inversion method is smart. A smart method will recognize how different components of the Earth are resolved from seismic data and use this knowledge to gain efficiency without sacrificing correctness and accuracy.

In this thesis, I derive and test a method of inversion based on finding a model of the Earth corresponding to synthetic data that best matches the observed seismic wavefield. The forward modeling theory assumes that seismic waves are 3-D waves propagating in a 3-D anisotropic elastic solid. I will specialize this theory to the isotropic case and solve for the P- and S-wave velocities and density, which are the most important properties because they determine the amplitudes and traveltime curves of the seismic waves to first order. This inversion technique theoretically overcomes the pitfalls of standard processing methods mentioned above, particularly in that different modes are related through the elastic wave equation.

For practical reasons I will make some assumptions in the tests that do affect the main point, namely that elastic wavefield inversion resolves the high- and low-wavenumber components of the P- and S-wave velocity. For instance, I assume the Earth is 2-D because of excessive computer time required to do the 3-D calculations. I also assume that the seismic source wavelet is known to and that the non-wave-like effects on amplitudes, such as geophone couplings and source antenna effects, are negligible or have been removed.