A Soviet look at datum shift

Chuck Sword

ABSTRACT

In the previous paper in this report, Claerbout describes how velocity is more accurately determined if a datum shift is determined simultaneously. A Soviet paper shows that such an apparent datum shift can be caused by strong variations in velocity. These velocity variations, quantitatively described by the mean fourth power of velocity, can be determined from the parameters picked by Claerbout's *Overlay* program. The true (vertical-ray-path) root-mean-square velocity can also be determined from these parameters. This determination is accurate even when offsets two or three times greater than the reflector depth are used. The results of the Soviet paper may also have some application to Stolt stretch.

THEORY

In an article published eight years ago (Malovichko, 1979), a Soviet researcher studied the shape of traveltime curves from a flat-lying reflector in a vertically heterogeneous medium. His conclusions are relevant to a paper on datum shift that appears in this report (Claerbout, 1987).

Malovichko states, on the basis of a previous article (Malovichko, 1978), that traveltime curves in a layered medium are approximately described by the formula

$$T = t_0\left(1 - \frac{1}{S}\right) + \frac{1}{S}\sqrt{t_0^2 + S\frac{x^2}{v_{rms}^2}}, \tag{1}$$

where t_0 is the two-way vertical travel time, T is the traveltime at non-zero offset, x is the distance from shot to geophone, S is a measure of velocity inhomogeneity, and v_{rms} is the true vertical-raypath rms velocity. The rms velocity v_{rms} is defined according to:

$$v_{rms}^2 \equiv \frac{2}{t_0}\int_0^H v(z)dz, \tag{2}$$

where H is the depth of the reflector.

Furthermore, Malovichko states that

$$S = \frac{v_f^4}{v_{rms}^4}, \tag{3}$$

where

$$v_f^4 \equiv \frac{2}{t_0} \int_0^H v^3(z) dz. \tag{4}$$

That is, v_f is the mean fourth-power velocity (it may seem odd that v is raised only to the third power in equation (4), but recall that v is raised only to the first power in equation (2), the equation for the mean squared velocity). Thus, once S and v_{rms} are determined by fitting equation (1) to the measured traveltime curve, v_f is easily determined from equation (3).

Malovichko's equations can be transformed to fit equation (5) in Claerbout's paper (Claerbout, 1987); this equation is reproduced below:

$$(t-a)^2 = (\tau - a)^2 + x^2 s_0 \frac{\tau - a}{\tau} \tag{5}$$

(Claerbout's paper defines the notation). The reader can confirm that equation (1) is transformed into equation (5) by the substitutions

$$S = \frac{\tau}{\tau - a} \tag{6}$$

and

$$v_{rms} = \frac{1}{\sqrt{s_0}}, \tag{7}$$

along with the obvious substitutions $T = t$ and $\tau = t_0$.

The conclusions, then, are that

$$v_f^4 = \frac{\tau}{\tau - a} v_{rms}^4, \tag{8}$$

and that v_{rms}, as determined in equation (7), is the true vertical-ray rms velocity.

REMARKS

Malovichko's derivation of equation (1) was based on the assumption that the medium is vertically heterogeneous, and that the reflector is flat. Suppose that we have a medium made up of homogeneous areas separated by linear boundaries with differing dips. Equation (1) still holds for such a medium, but the interpretation of S and v_{rms} is more complicated than for the vertically stratified medium (Urupov and Malovichko, 1979).

Equation (1) is identical to an equation derived by Levin (1985) in connection with Stolt stretch. Stolt stretch is applied to post-stack seismic data before frequency-domain migration; this stretch allows Stolt (f–k) migration, which is based on the assumption of constant velocity, to be used for media with vertically varying velocities. After the stretch is applied to the data, a modified form of Stolt migration is used. Levin derived the equation for the time-domain summation path of this modified migration; it turns out to be identical to equation (1) in the present paper. Since a stretch is applied to the data before this summation path is used, the S in Levin's formula does not have the same physical meaning as the S in Malovichko's formula. But the correspondence between these two equations deserves further study.

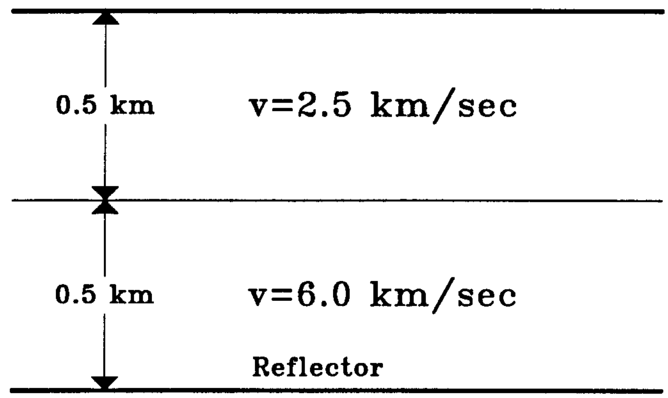

FIG. 1. Test model. This is the model used by Malovichko to produce the results in Figure 2. Note the large velocity contrast.

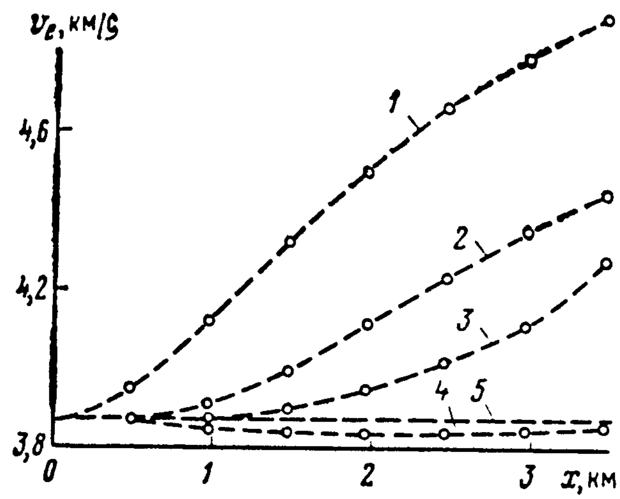

FIG. 2. Test results. This figure is reproduced from Malovichko (1979). It shows measured vertical-raypath rms velocity (vertical axis) versus total spread length (horizontal axis). Each curve represents a different measurement technique; see the text for details.

SYNTHETIC TEST

How well do these results hold for synthetic traveltime curves? Figure 1 shows a model that was used (by Malovichko) to generate such a synthetic curve. Notice the velocity contrast, which is greater than two-to-one. Figure 2 shows the results of different attempts to determine the true vertical-ray rms velocity. The vertical axis in this figure gives rms velocity (as measured by curve fitting), and the horizontal axis gives the total length of the spread used to determine this velocity. Curve 1 represents the results of trying to fit a simple hyperbolic moveout formula to the traveltime curve. It is accurate for near-offset data, but not for far-offset data. Curves 2 and 3 represent respectively attempts to fit three-term and four-term polynomial expansions of $t^2(x)$. Neither is too successful. Curve 4 shows the results of fitting equation (1) to the synthetic traveltime curve; curve 5 shows the correct answer. Note that curve 4 fits the data well even at an offset of 3 km, which is three times the depth to the reflector.

CONCLUSIONS

It is seen from the synthetic data that Malovichko's formula, equation (1), provides a curve that fits traveltime data from layered media, even at large offsets. Claerbout's formula, equation (5), is a transformation of Malovichko's formula. As a result, Claerbout's sloth s_0 is directly related to the true vertical-ray rms velocity, and his datum shift a and zero-offset traveltime τ are related, via equation (8), to the ratio of root-mean-square velocity to mean fourth-power velocity. Both s_0 and a can be determined using Claerbout's *Overlay* program.

There are some surprising correspondences between Malovichko's formula and the time-domain equivalent of migration after Stolt stretch. It is unknown whether Malovichko's results will prove useful in determining the s parameter that is used in post-Stolt-stretch migration.

REFERENCES

Claerbout, J.F., 1987, Datum shift and velocity estimation: SEP-51 (this volume).

Levin, S.A., 1985, Understanding Stolt stretch: SEP-42, 373–374.

Malovichko, A.A., 1978, A new representation of the traveltime curves of reflected waves in horizontally layered media: Applied Geophysics, **91**, 47–53 (in Russian: Маловичко А.А., 1978, Новое представление годографа отраженных волн для горизонтально-слоистых сред: Прикладная геофизика, вып. 91, с. 47–53).

Malovichko, A.A., 1979, Determination of the zero-offset effective velocity and the degree of velocity inhomogeneity from a single reflected-wave traveltime curve in the case of a horizontally layered medium: Applied Geophysics, **95**, 35–44 (in Russian: Маловичко А.А., 1979, Определение предельной эффективной скорости и степени скоростной неоднородности по одиночному годографу отраженных волн в случае вертикально-неоднородной среды: Прикладная геофизика, вып. 95, с. 35–44).

Urupov, A.K., and Malovichko, A.A., 1979, Representation of traveltime curves of reflected waves for layered media with dipping linear boundaries: Applied Geophysics, **95**, 12–26 (in Russian: Урупов А.К., Маловичко А.А., 1979, Представление годографов отраженных волн для слоистых сред с плоскими наклонными границами: Прикладная геофизика, вып. 95, с. 12–26).