Chapter 4

Forward modeling using the LITWEQ method

Chapter 3 has shown that the LITWEQ can be applied to the migration of seismic data. Because it is highly accurate in extrapolating events of any propagating angle, the LITWEQ method can give better results in modeling the seismic response of a given earth model than can methods using small-angle approximations of the wave-equation or conventional finite-difference two-way wave equation modeling. The flexibility of the LITWEQ method in handling velocity changes is an advantage over Fourier wavefield extrapolation methods (e.g. phase-shift and Stolt methods). When the velocities of the media change both vertically and laterally, special interpolation, or stretching, processes are necessary in the Fourier methods (Gazdag and Sguazzero, 1984 and Stolt, 1978).

This chapter will focus on how to use the LITWEQ in the modeling of (1) zero-offset sections of primary reflections, and (2) nonzero-offset sections, such as common-shot profiles, common-midpoint gathers and vertical seismic profiles (VSPs). By using two-way LITWEQ modeling, we will be able to compute not only primary reflections and diffractions, but also direct arrivals, refractions, and multiple reflections in the nonzero-offset sections.

§ 4.1 ZERO-OFFSET MODELING OF REFLECTIONS

A zero-offset seismogram can be calculated by a reversal of the steps of the LITWEQ migration, using new boundary conditions for modeling.

First let us define *pseudo-depth* τ as

$$\tau = \int_0^z \frac{d\varsigma}{v_r(x, \varsigma)} , \tag{4.1}$$

where $v_r(x, z)$ is the transformed velocity defined in section 3.5. Depth z should be converted to τ using equation (4.1) before applying the LITWEQ modeling program. Converting τ to z after modeling is simple (see section 3.5).

Zero-offset modeling of exploding reflectors usually uses the following boundary conditions: (1) no waves exist before time $t = 0$ (causality in time); (2) no waves are recorded above the surface (perfect reflecting surface at depth $z = 0$, or $\tau = 0$); (3) zero-value, or zero-slope, or absorbing (Engquist and Majda, 1977), boundary conditions exist at both sides of the computation grid (ends of the x axis).

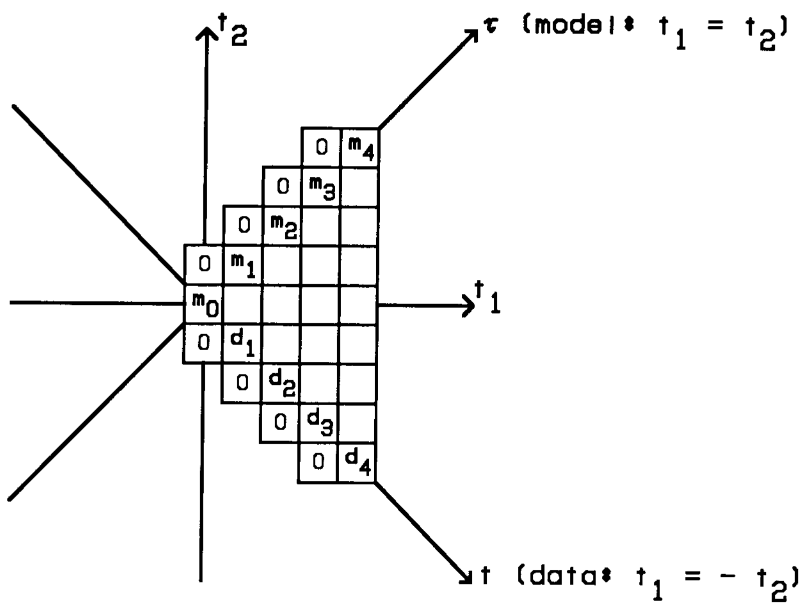

FIG. 4.1. Finite-differencing grid used in zero-offset LITWEQ modeling. The structure of the model is given on the $(x', t_1 = \tau/\sqrt{2}, t_2 = \tau/\sqrt{2})$ plane, and the data is recorded on the $(x', t_1 = t/\sqrt{2}, t_2 = -t/\sqrt{2})$ plane. The boundary and the initial conditions are denoted by 0's along the grid boundaries. The velocity used in the zero-offset modeling is half of the earth velocity. Notice also the structure of the earth model must be treated as the source (or force) term in the computation, instead of the boundary conditions.

The modeling uses an implicit scheme along the x axis, though an explicit scheme can be used if the stability condition is satisfied (Mitchell and Griffiths, 1980). Because we are only interested in modeling primary reflections, the reverberation term described in section 3.5 must be ignored. The LITWEQ modeling handles lateral velocity variations correctly. Figure 4.1 shows finite-differencing grids on the (t_1, t_2) plane for zero-offset modeling. The boundary conditions are

$$\begin{cases} P(x\,,\,t=-\Delta t\,,\,\tau) &= 0\ ,\ \text{and} \\ P(x\,,\,t\,,\,\tau=-\Delta\tau) &= 0\ . \end{cases} \tag{4.2}$$

The calculation starts from the plane $t = 0$ where the structure of the model is specified (as the source term in the scaler wave equation), and ends at the plane $\tau = 0$ where the zero-offset record is received.

Figure 4.2a shows a model of a section that has both vertical and lateral velocity variations. Figure 4.2b is the corresponding zero-offset seismogram calculated by the LITWEQ zero-offset modeling algorithm. The reflections and the diffractions have been calculated and shown in the figure.

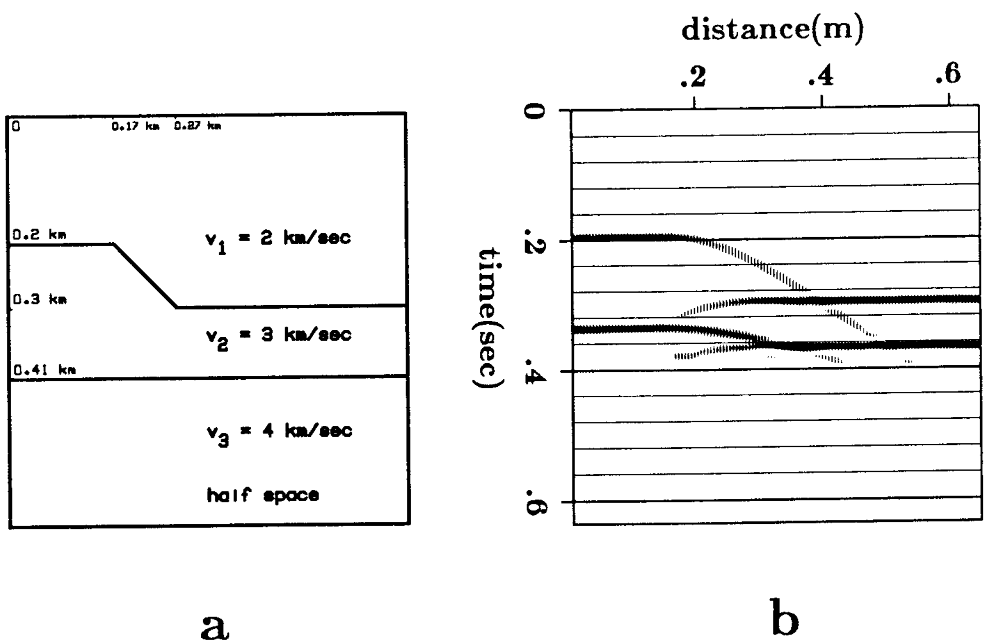

FIG. 4.2. **a.** Model with vertical and lateral changes in velocity. **b.** Calculated zero-offset seismogram calculated by the LITWEQ method.

LITWEQ zero-offset modeling can be used to calculate both reflections and diffractions of a given earth model. To compute other seismic events, such as direct arrivals, multiple reflections, and refractions, for any recording geometry, we have to use a two-way modeling algorithm, which will be discussed in the next section.

§ 4.2 TWO-WAY LITWEQ MODELING

Expanded grids for modeling both downgoing and upcoming waves

Figure 4.1 shows the triangular region $0 \leq (t+\tau) \leq t_{\max}$ on the (t, τ) plane, used for zero-offset modeling. Since two-way wave modeling calculates all possible seismic waves characterized by the wave equation for any source location (x_s, τ_s) and any receiver location (x_g, τ_g) at any time t, it requires extension of the triangular region to a rectangular region (Figure 4.3), $0 \leq t \leq t_{\max}$ and $0 \leq \tau \leq \tau_{\max}=(\tau_s + t_{\max})$ on the (t, τ) plane, where $t_{\max}$ is the maximum time of observation, τ_s is the pseudo-depth of the source, and τ_g is the pseudo-depth of the receiver. The two boundary conditions $P(x, t<0, \tau) = 0$ and $P(x, t, \tau<0) = 0$ are still valid for two-way modeling. However, a third condition, $P(x, t<t_{\max}, \tau>\tau_{\max}) = 0$, must be added, signifying that no waves can reach the depth of $\tau_{\max}$ when $t < t_{\max}$.

To begin two-way modeling, an impulse is placed on each shot position specified on the plane $(x, t=0, \tau)$, while the interfaces, or structures, are specified by discontinuities in the velocity model. The result of LITWEQ two-way modeling is a two-dimensional wavefield $P(x, t, \tau)$.

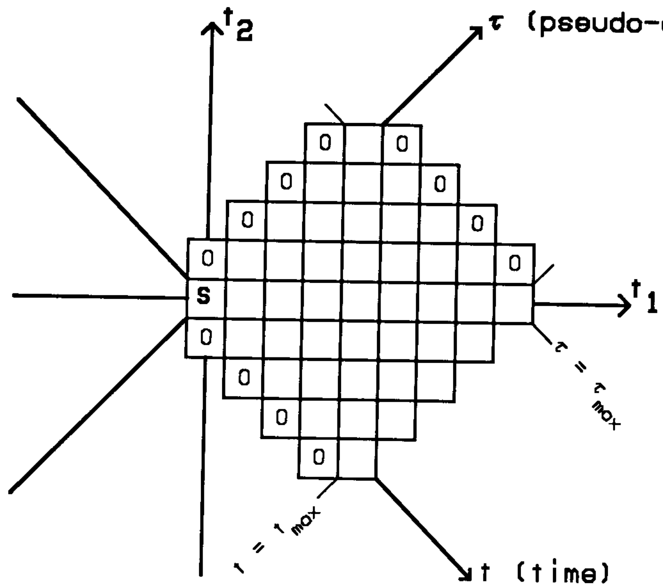

FIG. 4.3. Finite-differencing grid used in two-way wave modeling by the LITWEQ method. The seismic source is denoted by S on the $(x', t_1{=}\tau/\sqrt{2}, t_2{=}\tau/\sqrt{2})$ plane. An impulse (as a source term) is placed at the source location S at $t=0$. The initial values of the wavefield at non-source positions on this plane are zero. The zero-value boundary conditions are specified by 0's along the boundary. The wavefield is calculated for any position on the (x', t_1, t_2) plane.

Two-way modeling

Figure 4.4 shows a cube where the wavefield $P(x, t, \tau)$ is calculated and displayed. The source is located at the origin ($x=0$, $t=0$, $\tau=0$). There are two flat reflectors in the model. We compute a wavefield for every point in the (x, τ) plane for every time t. The front frame of the cube shows a snapshot of the wavefield, i.e., wavefronts on the (x, τ) plane at time t; the top frame shows a common-shot profile, i.e., the wavefield observed along the x axis; the side frame shows a VSP (vertical seismic profile), i.e., the wavefield observed along a vertical line $x = x_g$.

LITWEQ calculation of the wavefield in (x', t_1, t_2) can provide seismograms for any arrangement of sources and receivers. For example, defining a source at ($x_s=0$, $\tau_s=0$) and receivers at (x_g, $\tau_g=0$) will give us a common-shot profile. Defining

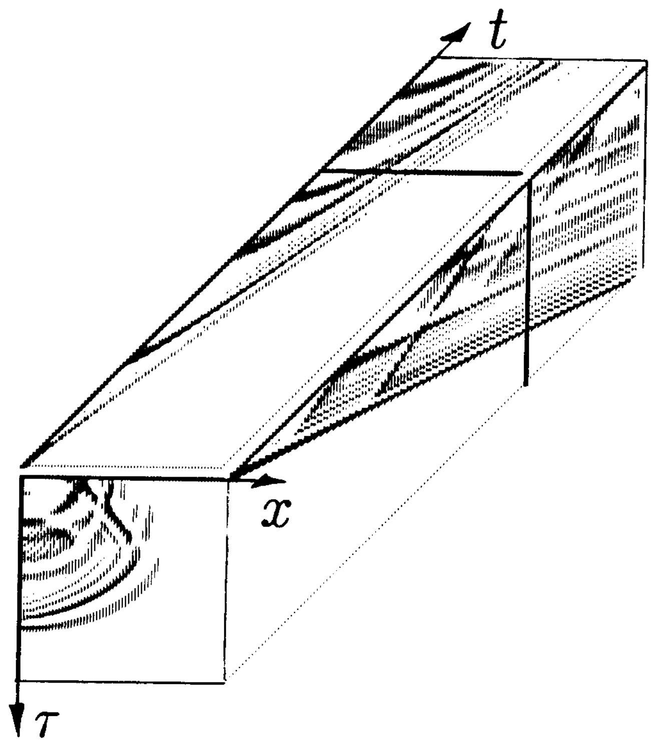

FIG. 4.4. A cubic display of the two-dimensional wavefield $P(x, t, \tau)$. t is the time axis; x is the horizontal axis; τ is the pseudo-depth axis.

source positions and receiver positions symmetrically about one point on the surface will give us a common-midpoint gather. Specifying the positions of the receivers along a vertical line and the position of the source on the surface will give us a VSP.

Figure 4.5b is the common-shot profile generated from a model of three layers shown in Figure 4.5a. Notice that the direct arrival, refraction, reflections, and multiples are all present on this profile. The first arrival at near traces is the direct arrival, while the first arrival at the far offset traces is the refraction (head wave) from the first interface. At the near traces, the second event is the reflection from the first interface; the third and the fifth events are the multiples of the first reflection; the fourth event is the reflection from the second interface; and the last event is the peg leg multiple of the reflection of the second interface. Figure 4.5c is the common-midpoint gather from the model.

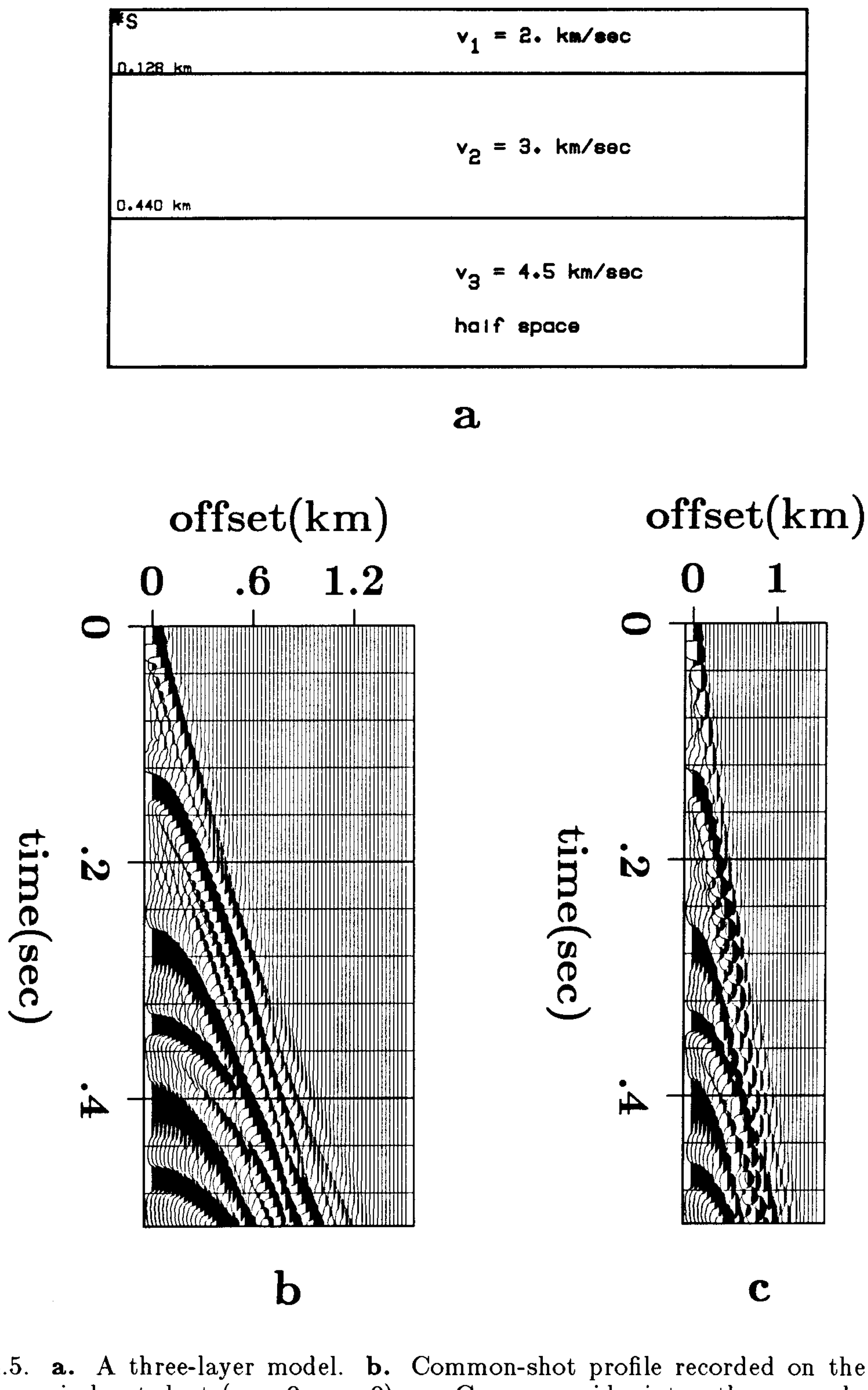

FIG. 4.5. **a.** A three-layer model. **b.** Common-shot profile recorded on the surface; the source is located at (x_s =0, τ_s =0). **c.** Common-midpoint gather recorded on the surface; the sources and the receivers are located symmetrically about one point on the surface.

Dispersion and aliasing effects can be seen in these synthetic seismograms, especially on the far offsets, because large grid sizes were used in this finite-difference calculation of the seismograms. Reducing the grid size will suppress these effects.

Figure 4.6 is a VSP with the source offset 120 m from the well (where the receivers are placed). The depth model in Figure 4.5a is used here. Again we can see the reflections, direct arrivals, and multiples.

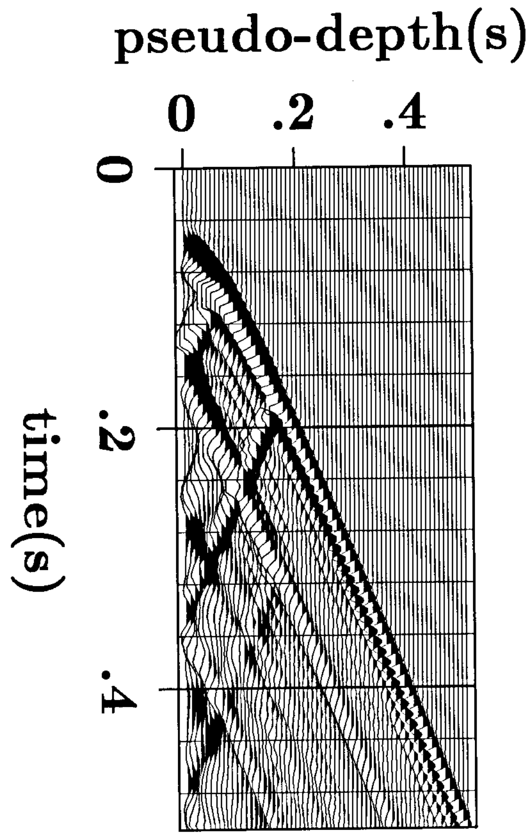

FIG. 4.6. VSP recorded in a vertical well located 120 m away from the source.

The modeling just discussed calculates only the impulse response of a given earth model, because the source wavelet is assumed to be an impulse. Convolving the impulse response $P(x, t, \tau)$ with a given source wavelet $S(t)$ along the time axis will give a seismogram with the given source wavelet (assuming no attenuation).

Other boundary conditions

The zero-value boundary condition at $\tau = -\Delta\tau$ can be changed to a zero-slope [$P(x, t, \tau{=}-\Delta\tau) = P(x, t, \tau{=}0)$], absorbing, or partially transmitting boundary condition, depending on the reflectivity of the surface. The partially transmitting boundary condition (waves are partially transmitted into the space above the upper boundary) is obtained on the upper boundary $\tau = 0$, when the transmission coefficient of the surface is given. Imposing the absorbing side boundary conditions at $x = x_{\min}$ and $x = x_{\max}$, we can suppress the artifacts of side boundary reflections generated by the zero-slope, or zero-value, boundary conditions.

§ 4.3 SUMMARY

The LITWEQ method can be used to do two-way modeling of the seismic response of a given earth model. The algorithm calculates not only reflections but also direct arrivals, refractions, diffractions, and multiples, etc.. Therefore, it can be used to simulate the field data for any given recording geometry, after some amplitude corrections (geometric spreading and attenuation). The amplitude correction is necessary because the modeling of seismograms is two-dimensional while the earth is three-dimensional.

Two-way modeling can be used in seismic inversion problems where the calculation of seismograms for a given model of velocity, or impedance, is needed. The combination of LITWEQ modeling and LITWEQ migration can be used to do prestack migration.