# Chapter 1

# Introduction

Wavefield extrapolation in laterally inhomogeneous media has drawn increasing attention from geophysicists as they work on more complicated seismic problems, such as wave-equation velocity inversion, prestack migration, and migration of steep-dip reflections. Various numerical methods for dealing with this problem, such as finite-difference, finite-element, integral and Fourier methods, have been used in industry for many years. Unfortunately, while the conventional finite-difference methods are generally the least computationally expensive, they are unable to correctly migrate steep-dip reflections (reflections propagating at large angles).

Most of the finite-difference methods solve wavefield extrapolation problems in either Cartesian coordinates or time-retarded coordinates. By means of a special linear coordinate transformation introduced in this thesis, wavefields can be extrapolated nearly along the characteristic directions of wave propagation, so that the computations are more accurate and stable than those of conventional finite-difference methods. This new method is valid for reflections of any propagating angle and hence can be applied to solve problems of imaging steep-dip, near-vertical, or even overhanging salt-dome boundaries and thrust fault planes in petroleum exploration.

## § 1.1 PROBLEMS TO SOLVE

### Improving accuracy in imaging steep-dip reflections

Steeply dipping reflectors, such as salt-dome boundaries and thrust fault planes, are often indirect targets for oil exploration. They also play important roles in controlling the regional geology of prospect areas. Unfortunately, these steeply dipping reflectors are often poorly imaged in routine seismic data processing, being suppressed by both conventional common-midpoint (CMP) stacking and conventional post-stack migration. An accurate wavefield extrapolation method in variable velocity media should be able to migrate steep-dip reflections in well-stacked sections (e.g., dip-moveout stacked sections) to the proper reflector positions.

### Imaging overturned reflections

Imaging overhanging salt-dome boundaries is difficult in seismic data processing. Conventional techniques only process reflections from the topsides of salt-dome boundaries, while ignoring the underside reflections. However, the topside reflections are often too weak to be recorded by receivers, because they are diffracted and attenuated by complex geological structures, such as normal faults, developed over the tops of the salt domes. In contrast, seismograms are more likely to record reflections from the undersides of salt-dome boundaries (overturned reflections), because the ray paths of overturned reflections are usually outside the normal faulting system. These overturned reflections must be processed by techniques other than the conventional methods. Imaging overturned reflections requires a two-way wavefield extrapolation method.

### Prestack migration of seismic data

Conventional normal-moveout (NMO) and common-midpoint (CMP) stacking fail to stack steep-dip reflections into zero-offset sections. Because it averages reflectivities over a certain segment of a dipping reflector, the stacking of the reflections from the dipping reflector before migration obscures the details of of the underground structure. Therefore, migration before stacking, i.e., prestack migration, is necessary to image steep-dip reflections correctly. Because seismic reflections before stacking (in shot profiles or in CMP gathers) usually have much larger propagating angles than they are after stacking (in zero-offset sections), the accuracy of extrapolating steep-dip reflections becomes more important in prestack migration than in poststack migration. An accurate, efficient and stable prestack migration method in variable velocity media can also be applied to seismic inversions of elastic parameters of the earth .

## § 1.2 CONVENTIONAL PROCESSING

### Problems with conventional finite-difference migration

Conventional finite-difference migration methods employing approximate one-way wave equations have been used in industry for many years. However, these methods share the limitation of being unable to handle steep-dip reflections properly; their residual errors, caused by operator approximations, are proportional to the angle of wave propagation. These methods are, of course, unable to migrate overturned reflections, because overturned reflections have angles larger than 90 degrees at their turning points (Li et al., 1984).

Figure 1.1 shows a field data section migrated using a conventional finite-difference 15-degree equation method. The dataset contains reflections from the steeply dipping flanks of a salt dome in the Gulf of Mexico. The salt-dome boundaries are poorly imaged by the algorithm, because 15-degree wave-equation migration is unable to accurately extrapolate steep-dip reflections.

### Problems with conventional CMP stacking

Conventional CMP stacking is based on the assumption that travel-time curves of flat-bed reflections are hyperbolic; stacking sums along a hyperbola whose curvature is determined by the root-mean-square (RMS) velocity of the medium. The travel-time curves of reflections from dipping reflectors differ from those of flat-bed reflections because of a cosine factor in velocity correction (Levin, 1971; Judson et al., 1978; Hale, 1983). Before NMO and CMP stacking, a dip-moveout (DMO) correction for the cosine factor is sometimes used to stack reflections from dipping reflectors into zero-offset sections. As an alternative to DMO correction, prestack migration can be used to first migrate dipping-bed reflections and then stack images of these dipping-bed reflections.

CMP stacking fails for reflections greater than 90 degrees, because these overturned reflections manifest nonhyperbolic time-distance curves in CMP gathers. Ottolini pointed out that negative moveout must be applied before these overturned reflections can be stacked constructively (personal communication, 1984). Theoretically, overturned reflections can also be imaged using a two-way prestack migration process.

## § 1.3 WAVEFIELD EXTRAPOLATION USING THE LINEARLY TRANSFORMED WAVE EQUATION

Through a linear transformation, the 1-D constant-velocity scalar wave equation can be transformed into the 1-D characteristic wave equation. Extrapolating a wavefield along its characteristic direction is more stable and accurate than in any other direction. By applying a similar linear transformation to a 2-D scalar wave equation, we can obtain a LInearly Transformed Wave EQuation (LITWEQ) by which 2-D wavefield extrapolation can be made more stable and accurate than by conventional methods. Exactly transformed from the original wave equation, the LITWEQ will be valid for reflections of any propagating angle (reflections less than 90 degrees to greater than 90 degrees).

In figure 1.2 LITWEQ migration is applied to the same dataset, as used in the 15-degree migration method illustrated in Figure 1.1, using the same migration velocity. The salt-dome boundaries have been well resolved in Figure 1.2. The new method also images the overhanging side of the salt dome.

The LITWEQ method can also be used in seismic forward modeling, i.e., calculating the seismic response for a given earth model and a given source distribution. Specifying receiver locations, media velocities and source distributions as inputs to the LITWEQ modeling program, we will be able to simulate routine seismic experiments, such as surface seismic profiling and vertical (borehole) seismic profiling (VSP).

Besides being useful in migration of zero-offset sections (poststack migration), the LITWEQ method can also be applied to migration of nonzero-offset sections (prestack migration). Further applications of the LITWEQ method are expected in inversions of media velocities, migration of multiple reflections, and extrapolation of 3-D wavefields.

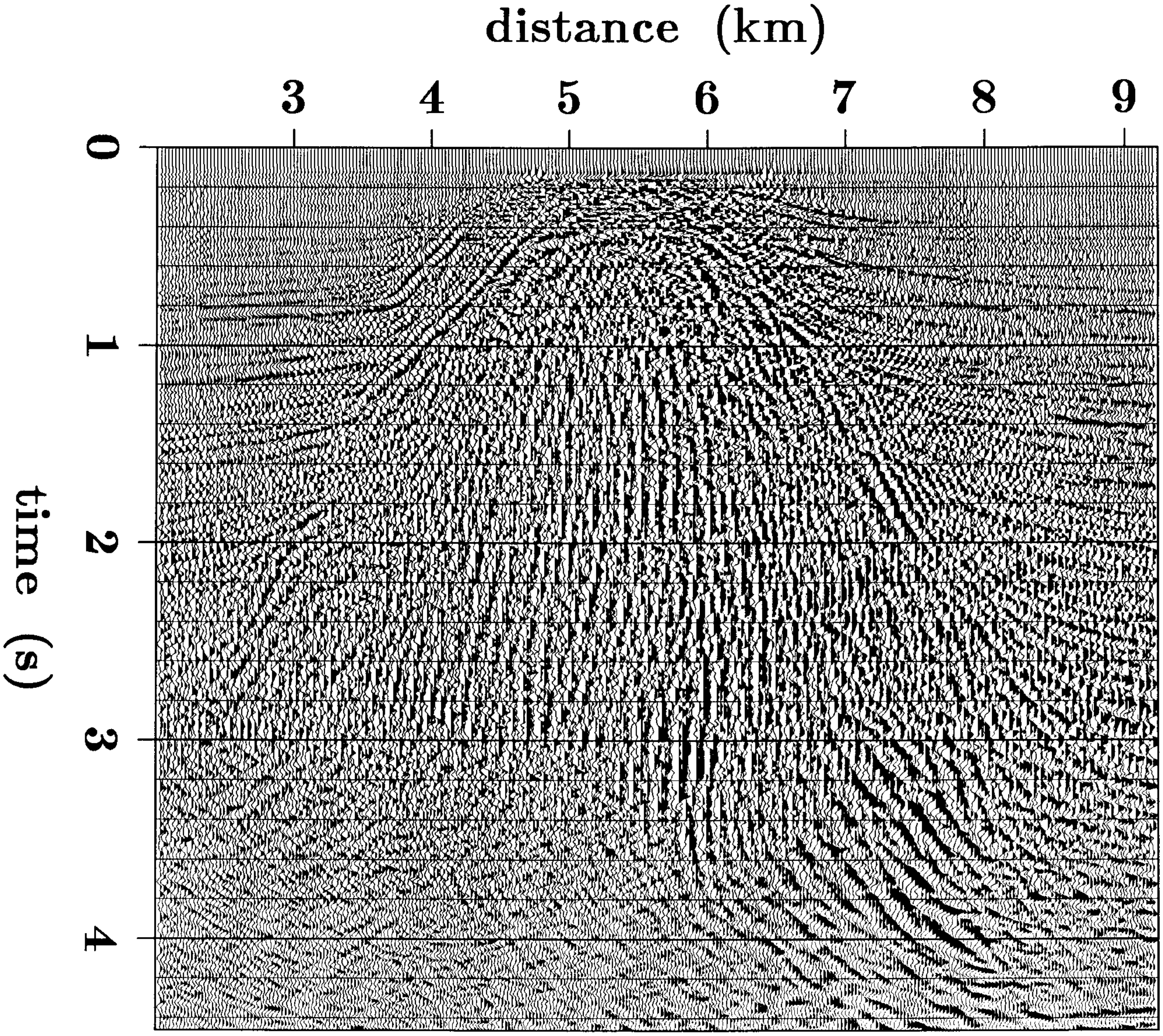

FIG. 1.1. Migrated section of reflections from a salt dome using conventional 15-degree wave-equation migration. The salt-dome boundary is poorly resolved, because the migration is unable to migrate steep-dip reflections generated from the steeply dipping salt dome boundary.

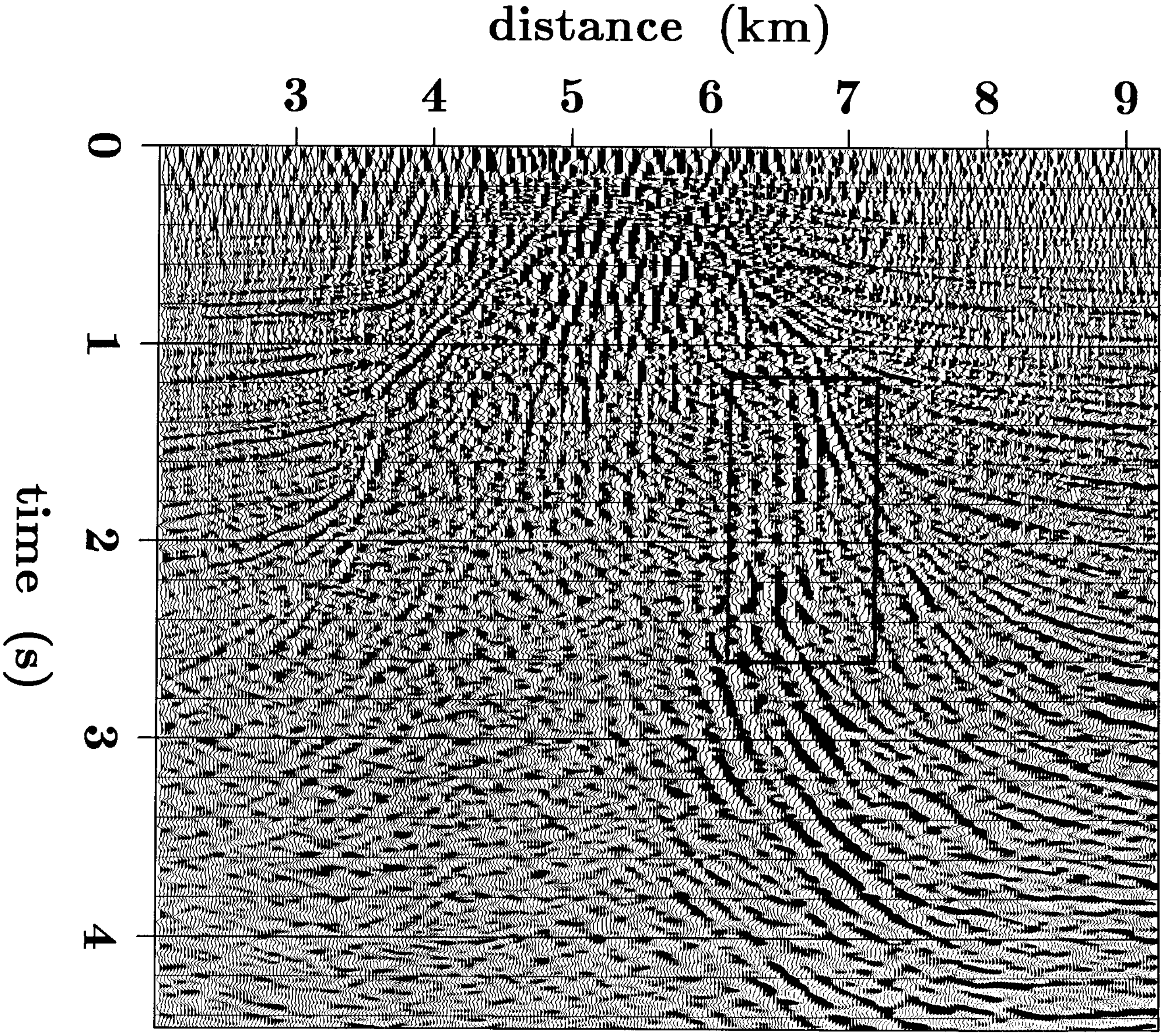

FIG. 1.2. Section of the salt-dome data migrated using the LITWEQ method, with the same migration velocity as in Figure 1.1. The boundary of the salt dome is well imaged. The overhanging side (enclosed in the rectangle) is also imaged by this two-way migration scheme (see chapter 3 for details).