

**IMAGING STEEP-DIP REFLECTIONS BY THE
LINEARLY TRANSFORMED WAVE EQUATION METHOD**

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ABSTRACT

The migration, or imaging, of steep-dip reflections in variable-velocity media has been an important problem in seismic exploration for many years. Claerbout suggested a two-pass phase shift method to image both the topsides and undersides of steep-dip reflectors. Synthetic data calculations confirmed that Claerbout's method improves resolution in imaging steeply dipping reflectors in media where velocities vary vertically. Solving the problem of imaging both the topsides and undersides of steeply dipping reflectors in laterally inhomogeneous media led me to a new wavefield extrapolation method which I call the *linearly transformed wave equation* (LITWEQ) method.

The LITWEQ method involves transforming a two-dimensional wave equation over both the time and depth axes, analogously to the characteristic method used for solving a one-dimensional wave equation. Applications of LITWEQ migration show that it is more accurate in imaging steep-dip reflections than conventional finite-difference methods of both one-way wave-equation migration (such as 15-degree wave equation migration) and two-way wave-equation migration (such as second-order reverse time migration). Since it can handle lateral inhomogeneity, the LITWEQ method also has advantages over Fourier-domain wide-angle methods, such as phase shift and Stolt, when applied to wavefield extrapolations in media where velocities vary laterally.

In strong laterally inhomogeneous media, conventional stacking techniques fail to stack both flat-bed and dipping-bed reflections correctly into common-midpoint (CMP) stacked sections. Even in laterally homogeneous media, normal-moveout (NMO) and

CMP stacking techniques still suppress dipping reflections. In order to preserve and image steep-dip reflections, prestack migrations must be applied. Because of its accuracy in imaging steep-dip reflections and its flexibility in handling lateral inhomogeneity, LITWEQ prestack migration gives satisfactory results when applied to both synthetic and field data.

Approved for publication:

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