Appendix

Inverse DMO

§ A.1 Prestack full migration

The presence of a single spike in the dataset, at travel time $t = t_h$, with shot at $(x, y, z) = (h, 0, 0)$ and receiver at $(-h, 0, 0)$, means that there is an ellipsoidal reflector at all points (x, y, z) such that the sum of the distance from a point (x, y, z) on the reflector to the source $\sqrt{(x-h)^2+y^2+z^2}$, plus the distance from that point to the receiver $\sqrt{(x+h)^2+y^2+z^2}$, equals the travel time times the velocity:

$$\sqrt{(x-h)^2+y^2+z^2} + \sqrt{(x+h)^2+y^2+z^2} = vt_h \ . \qquad \text{(A.1.1)}$$

The ellipsoid in equation (A.1.1) can be written as

$$\left(\frac{x}{a_x}\right)^2 + \left(\frac{y}{a_y}\right)^2 + \left(\frac{z}{a_z}\right)^2 = 1 \ . \qquad \text{(A.1.2)}$$

a_x can be found by setting $z = 0$ and $y = 0$ in equation (A.1.1):

$$a_x = vt_h /2 \ . \qquad \text{(A.1.3)}$$

Similarly,

$$a_y = a_z = \sqrt{(vt_h /2)^2 - h^2} = vt_n /2 \ , \qquad \text{(A.1.4)}$$

where t_n is the normal moveout (NMO) time:

$$t_n^2 = t_h^2 - (2h /v)^2 \ . \qquad \text{(A.1.5)}$$

Prestack migration of an impulse therefore produces the ellipsoid

$$\left(\frac{x}{vt_h /2}\right)^2 + \left(\frac{y}{vt_n /2}\right)^2 + \left(\frac{z}{vt_n /2}\right)^2 = 1 \ . \qquad \text{(A.1.6)}$$

Using equation (A.1.5) we have

$$\frac{x^2}{1 + (2h /vt_n)^2} + y^2 + z^2 = \left(vt_n /2\right)^2 \ . \qquad \text{(A.1.7)}$$

The change of variable

$$\chi^2 = \frac{x^2}{1 + (2h/vt_n)^2} \qquad \text{(A.1.8)}$$

compresses the ellipsoid (A.1.7) to the sphere

$$\chi^2 + y^2 + z^2 = (vt_n/2)^2 , \qquad \text{(A.1.9)}$$

which is the zero-offset migration of a spike at the NMO time t_n .

Zero-offset migration is described by the dispersion relation

$$\left(\frac{v}{2}\right)^2 \left(k_\chi^2 + k_y^2 + k_z^2\right) = \omega_n^2 . \qquad \text{(A.1.10)}$$

Inserting the Fourier transform of the change of variables (A.1.8)

$$k_\chi^2 = k_x^2\left[1 + (2h/vt_n)^2\right] , \qquad \text{(A.1.11)}$$

into equation (A.1.10) gives

$$\left(\frac{v}{2}\right)^2 \left(k_x^2 + k_y^2 + k_z^2\right) = \omega_n^2 - \left(\frac{hk_x}{t_n}\right)^2 . \qquad \text{(A.1.12)}$$

This is the dispersion relation for prestack, post-NMO, full migration.

§ A.2 Prestack partial migration

Prestack full migration (Figure A.2.1) can be done in three steps:

(1) NMO.

(2) Prestack partial migration (DMO): substitute

$$\omega_0^2 = \omega_n^2 - \left(\frac{hk_x}{t_n}\right)^2 \qquad \text{(A.2.1)}$$

on the right-hand side of equation (A.1.12). Equation (A.2.1) is time dependent: ω_n and t_n appear together. It is velocity independent (but constant velocity was assumed).

(3) Migration: using the dispersion relation:

$$\left(\frac{v}{2}\right)^2 \left(k_x^2 + k_y^2 + k_z^2\right) = \omega_0^2 . \qquad \text{(A.2.2)}$$

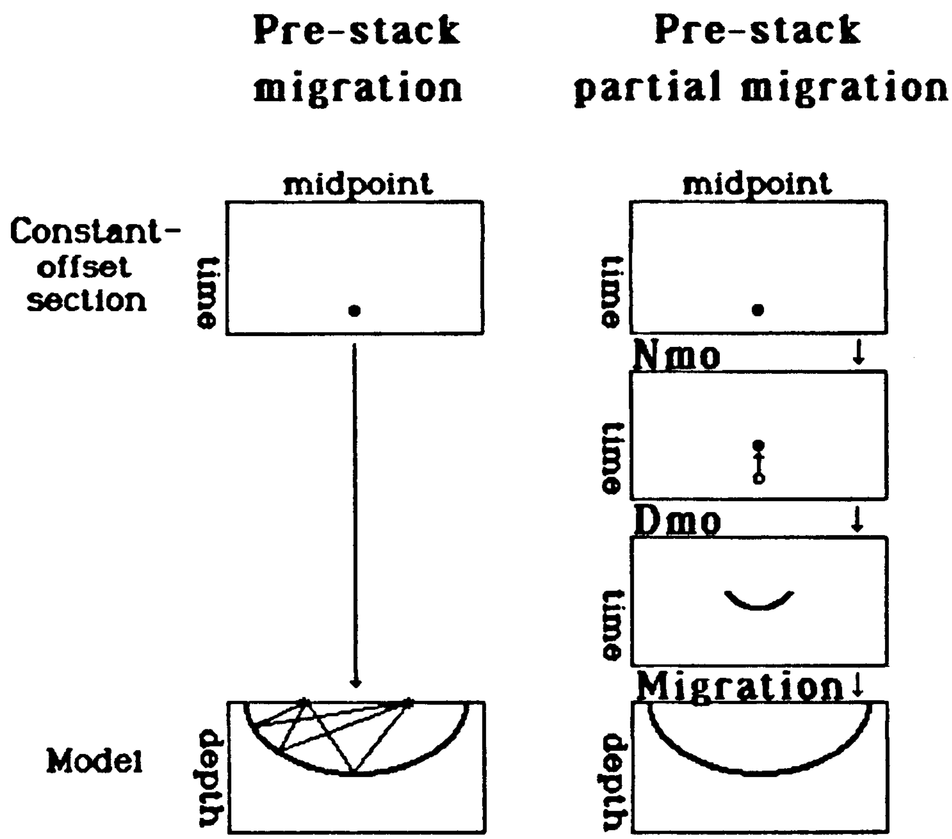

FIG. A.2.1. Decomposition of prestack full migration to NMO, DMO, and poststack migration.

§ A.3 Inverse DMO

The relation (A.2.1) can be used to derive the relation between the zero-offset section m and the common-offset section d.

Starting from the inverse Fourier transform

$$d(t_n, k_x, h) = \int d\omega_n \, e^{-i\omega_n t_n} \, d(\omega_n, k_x) \,, \tag{A.3.1}$$

and changing variables ω_n to ω_0, we have

$$= \int d\omega_0 \left| \frac{d\omega_n}{d\omega_0} \right| e^{-i\omega_n(\omega_0, t_n, h, k_x) t_n} \, d\left[\omega_n(\omega_0, t_n, h, k_x), k_x, h\right]$$

$$= \int d\omega_0 \left| \frac{d\omega_n}{d\omega_0} \right| e^{-i\omega_n(\omega_0, t_n, h, k_x) t_n} \, m(\omega_0, k_x) \,, \tag{A.3.2}$$

where $\omega_n(\omega_0, t_n, h, k_x)$ is given by equation (A.2.1). Defining

$$A = \sqrt{1 + (hk_x / \omega_0 t_n)^2}\ , \qquad (A.3.3)$$

we have

$$\omega_n = A\, \omega_0\ , \qquad (A.3.4)$$

and

$$\left| \frac{d\,\omega_n}{d\,\omega_0} \right| = \left| \frac{\omega_0}{\omega_n} \right| = A^{-1}\ . \qquad (A.3.5)$$

Substituting equations (A.3.4) and (A.3.5) into (A.3.2), we finally get

$$d\,(t_n, k_x, h) = \int d\,\omega_0\, A^{-1}\, e^{-i\,\omega_0 A t_n}\, m\,(\omega_0, k_x)\ . \qquad (A.3.6)$$

§ A.4 Impulse responses

Deregowski and Rocca (1981) showed that DMO will smear an impulse at $(x=0, t=t_n)$ to the ellipse

$$t_0^2\,(x) = t_n^2 \left(1 - \frac{x^2}{h^2} \right)\ . \qquad (A.4.1)$$

The inverse DMO will therefore smear an impulse at $(x=0, t=t_0)$ to the curve

$$t_n^2(x) = \frac{t_0^2}{\left(1 - \frac{x^2}{h^2} \right)}\ . \qquad (A.4.2)$$

The DMO and inverse DMO impulse responses are shown in Figure A.4.1. Applications of inverse DMO after DMO ($\mathbf{D}^+\mathbf{D} \approx \mathbf{I}$), and DMO after inverse DMO ($\mathbf{D}\mathbf{D}^+ \approx \mathbf{I}$), are shown in Figure A.4.2.

§ A.5 Inverse DMO in three dimensions

In equation (A.1.1), I made the assumption that the shot and receiver lie along the x axis. In general, however, they do not, and the half-offset is a vector

$$\mathbf{h} = \begin{pmatrix} h_x \\ h_y \end{pmatrix}\ . \qquad (A.5.1)$$

Let θ be the angle of rotation such that

$$\mathbf{h} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} h\ , \qquad h = \sqrt{h_x^2 + h_y^2}\ . \qquad (A.5.2)$$

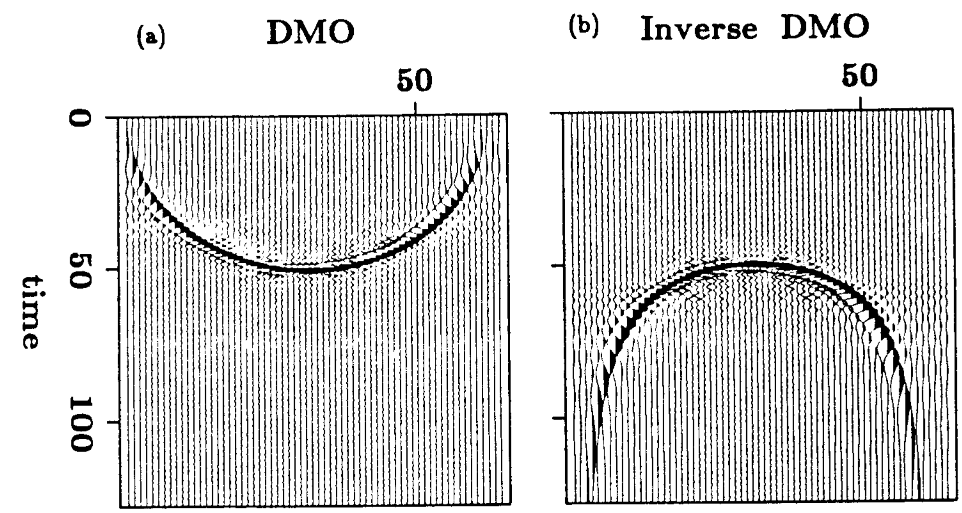

FIG. A.4.1. (a) DMO impulse response. (b) Inverse DMO impulse response.

We can rotate the (x, y) axes to (x', y') by

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} , \tag{A.5.3}$$

so that the shot and receiver lie along the x' axis, as in equation (A.1.1), and we can make the same development of equations (A.1.1) through (A.3.6) in (x', y') coordinates, obtaining

$$d(t_n, k_{x'}, k_{y'}, h) = \int d\omega_0 \, A^{-1} \, e^{-i\omega_0 A t_n} \, m(\omega_0, k_{x'}, k_{y'}) , \tag{A.5.4}$$

with

$$A = \sqrt{1 + (hk_{x'}/\omega_0 t_n)^2} . \tag{A.5.5}$$

To return to the original coordinates, recall that rotation at the space domain corresponds to a rotation in the same direction at the frequency domain:

$$\begin{pmatrix} k_{x'} \\ k_{y'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} k_x \\ k_y \end{pmatrix} . \tag{A.5.6}$$

So,

$$\begin{aligned} hk_{x'} &= h(\cos\theta \, k_x + \sin\theta \, k_y) \\ &= (h\cos\theta)k_x + (h\sin\theta)k_y \\ &= h_x k_x + h_y k_y \end{aligned} \tag{A.5.7}$$

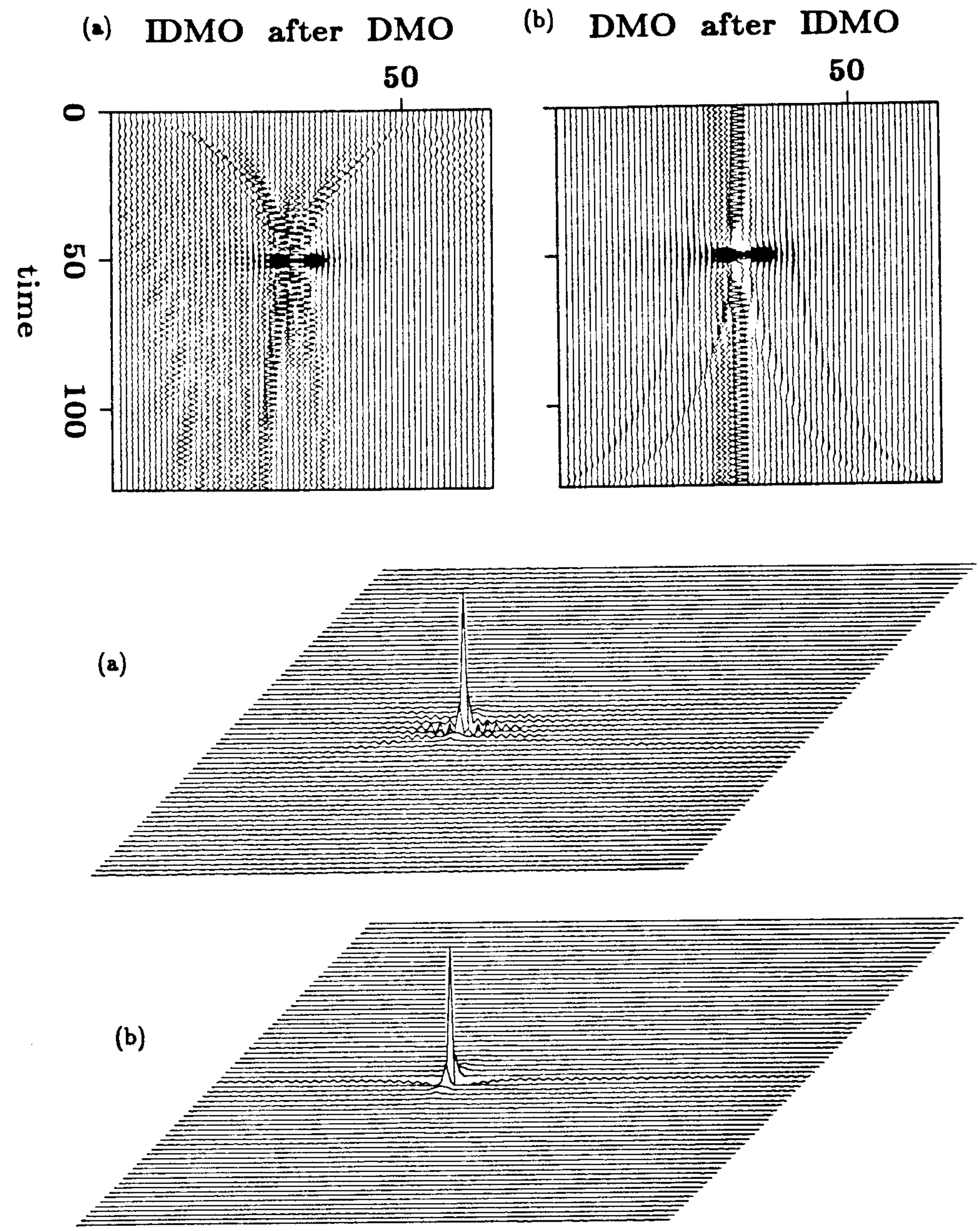

FIG. A.4.2. (a) Inverse DMO after DMO of an impulse. (b) DMO after inverse DMO of an impulse.

$$= \mathbf{h} \cdot \mathbf{k} \ ,$$

and equations (A.5.4) and (A.5.5) become

$$d\,(t_n\,,k_x\,,k_y\,,h\,) = \int d\,\omega_0\, A^{-1}\, e^{-i\,\omega_0 A t_n}\, m\,(\omega_0, k_x\,,k_y\,)\ , \tag{A.5.8}$$

where

$$A = \sqrt{1+(\mathbf{h}\cdot\mathbf{k}/\omega_0 t_n)^2}\ . \tag{A.5.9}$$

§ A.6 Summary

DMO and its pseudo-inverse are given by Table A.6.1.

DMO	$m(\omega) = \int dt\ A^{-1}\ e^{-i\omega A t}\ d(t,h)$
Inverse DMO	$d(t,h) = \int d\omega\ A^{-1}\ e^{i\omega A t}\ m(\omega)$

TABLE A.6.1. The DMO is a unitary-like operator since its pseudo-inverse is also its complex conjugate.

קשת יז כסלו תשמ"ו

KESHET Nov. 30, 1985.