Some Anisotropic Modeling Examples

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INTRODUCTION

Amplitude variation with offset can be caused by reflection from an interface. It can also be caused by anisotropy. As an example, we will show calculated impulse responses for Greenhorn Shale. For S_v waves, the amplitude can vary by as much as a factor of 5.

A SIMPLE MODELING SCHEME

An impulse can be represented as a sum of impulsive plane waves, uniformly distributed in direction. To make a snapshot of the impulse response, we shift these plane waves away from the source a distance proportional to their velocity.

In the Fourier domain, we set up a two dimensional grid. Each grid point represents a monochromatic plane wave propagating in a certain direction. Knowing the direction, we know the distance the wave has travelled, and thus we know the phase shift. Doing this for each grid point in the Fourier domain, we have an array which can be inverse Fourier transformed giving the desired time-domain representation of a snapshot of the impulse response.

A subtle problem

There is an apparent problem with this scheme, in that it is a scalar one. In reality, associated with each plane wave is also a particle motion direction. The plane waves should be summed vectorially.

However, summing the waves as scalars does have meaning. The Christoffel equation is the Fourier-transformed wave equation:

$$(\mathbf{M}\,\mathbf{C}\,\mathbf{M}^* + \rho v^2 \mathbf{I})\,\mathbf{V} = 0, \tag{1}$$

where $\mathbf{M}$ is a 6 by 3 matrix of direction cosines, $\mathbf{C}$ is a 6 by 6 matrix of stiffness constants, $\mathbf{V}$ is a column vector giving the particle velocity direction, v is the velocity of the wave, and $\mathbf{I}$ is a 3 by 3 identity matrix. Note that this is an eigenvalue problem, and so **V** only determines a direction, and not a magnitude. The magnitude of the plane wave is determined by the source.

$\mathbf{M\,C\,M^*}$ is a 3 by 3 real symmetric matrix. Thus, it can be diagonalized. Let the matrix **U** have the property that $\mathbf{U^*U = I}$, and $\mathbf{U\,M\,C\,M^*\,U^*}$ is diagonal. Premultiplying equation 1 by **U**, and utilizing the fact that $\mathbf{U^*U = I}$, we get

$$\mathbf{U}(\mathbf{M\,C\,M^*} + \rho v^2 \mathbf{I})\mathbf{U^*U\,V} = 0,$$

which then simplifies to

$$(\mathbf{D} + \rho v^2 \mathbf{I})(\mathbf{UV}) = 0, \tag{2}$$

where **D** is a diagonal matrix of eigenvalues.

If we now consider $\mathbf{V'} = \mathbf{UV}$, we have 3 separate equations, one for each wave type. The eigenvalues of this transformed equation (and hence the associated velocities) are the same as those in equation 1. This new equation gives meaning to our modeling scheme. By summing plane waves as scalars, we are constructing a time-domain representation of $\mathbf{V'}$, not **V**. Although not precisely the same as the true amplitude field, this scalar field is the more useful mathematically.

A SEQUENCE OF IMPULSE RESPONSES

In order to display both the P and S_V impulse responses together, I have summed the resulting modeled impulse responses. This is not a problem because the two impulse responses are well separated in the snapshots presented.

Figure 1 on the next page shows three different modeled impulse responses, the only difference between the three being in the value of C_{13}. The middle one is "Greenhorn shale", which is discussed in "Kinematics of Axisymmetric Anisotropy" (SEP – 42) and elsewhere in this report.

Triplication and the Hankel tail

These models were constructed using a two-dimensional scheme, and as a result the resulting wave field has a Hankel tail. This is useful, because the tail shows in which direction the waves are traveling, information not usually available in a "snapshot". The tails are quite small and are not evident on the "3-D" plots shown in figure 1 (except where the wave field is cut at the edge of the plot).

The "tails" are made visible in figure 2.

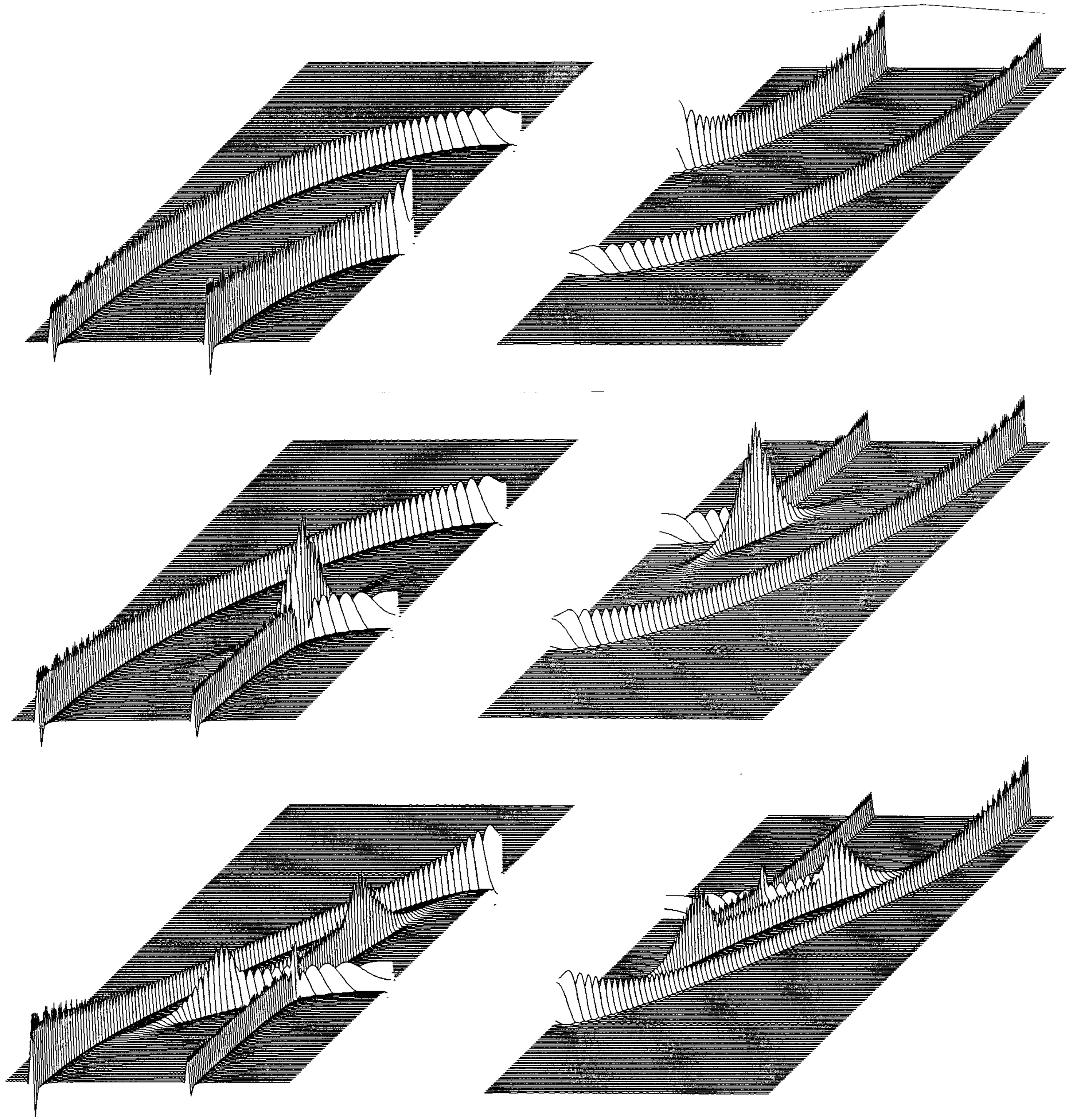

FIG. 1: Impulse responses for three different sets of elastic constants. The middle set corresponds to "Greenhorn shale". The upper set has a higher value for C_{13}, and the lower set has a lower value for C_{13}. Each model is shown from two different angles. On the left, the source is in the lower right hand corner, and on the right the source is in the upper left hand corner.

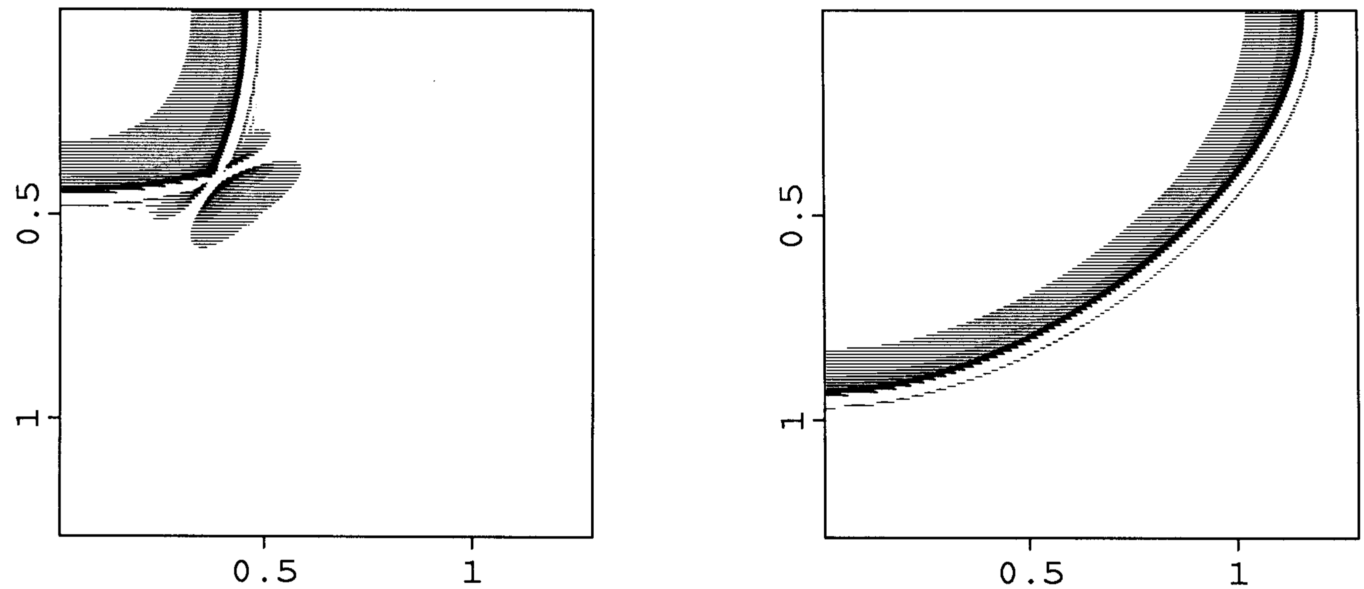

FIG. 2: Another view of Greenhorn shale. The impulse is in the upper left hand corner. S_V and P have been plotted separately, with S_V on the left and P on the right. In these plots the gain has been set very low to exaggerate the tail. Since the tail has a negative polarity, the sign has also been reversed so that black becomes negative. Note that the tail obediently trails behind the wavefront, *except* in the region of the triplication, where it extends in *both* directions. Since the outer (P) surface cannot triplicate, causality is preserved. The tail from the inner (S_V) surface also appears to always decay to zero before reaching the outer surface.