Processing converted-wave data using the method of controlled directional reception

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INTRODUCTION

In this paper I will explain how the method of controlled directional reception can be used to process converted-wave reflection seismic data. I shall assume that the reader is already familiar with two other papers by the author: one on the kinematics of converted-wave data (Sword, 1984a), and one on the method of controlled directional reception (Sword, 1984b).

The CDR method can be applied to converted-wave data in two possible ways. It can be applied accurately, using exact travel times for the converted waves, but this is relatively expensive, since most of the equations involved can only be solved iteratively. The alternative is to approximate the converted-wave travel times, which gives a slightly less exact result, but which gives equations that can be solved analytically (and thus cheaply). In this paper I will derive the exact equations and show how they can be approximated, and I will present the results of processing synthetic data using these approximate equations.

Note that I will not give any explicit references to my previous two papers; I rely so much on both of them that continual references would become tedious.

NOTATION

It is useful to establish a consistent system of notation. Figure 1 shows a typical recording geometry; the notation should be familiar from my other two papers.

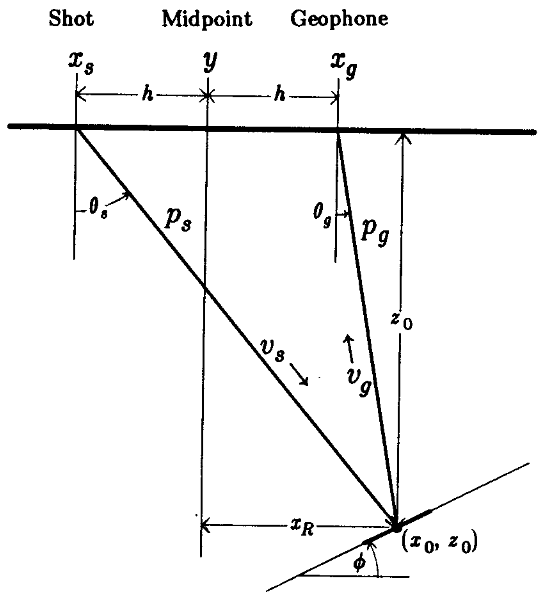

FIG. 1. Typical recording geometry. The figure is drawn in such a way that all parameters shown on it are positive.

The notation used is:

x_s	– Shot position
x_g	– Geophone position
h	– Half offset: $(x_g - x_s)/2$
y	– Midpoint: $(x_g + x_s)/2$
x_0, z_0	– Reflector position
x_R	– Horizontal distance from midpoint y to the reflector
t	– Travel time
v_s	– Velocity from shot to reflector (velocity of the downgoing ray)
v_g	– Velocity from reflector to geophone (velocity of the upcoming ray)
γ	– Ratio of velocities: v_g / v_s
p_s	– Ray parameter of the downgoing ray ($p_s = -dt/dx_s$)
p_g	– Ray parameter of the upcoming ray ($p_g = -dt/dx_g$)
θ_s	– Angle (from vertical) of the downgoing ray
θ_g	– Angle (from vertical) of the upcoming ray
ϕ	– Angle (from horizontal) of the reflector

MIGRATION IF v_s AND v_g ARE ALREADY KNOWN

I will assume throughout the rest of the paper that v_s and v_g are constant, and that γ is known. I will make the additional assumption in this section that v_s is both constant and known. I will not discuss the question of how p_s, p_g, and t are picked, but will immediately begin discussing how to migrate picked converted-wave data using the method of controlled directional reception.

Recall the travel-time equation for converted waves:

$$t = \frac{1}{v_s}\sqrt{(x_R + h)^2 + z_0^2} + \frac{1}{\gamma v_s}\sqrt{(x_R - h)^2 + z_0^2} \ . \tag{1}$$

In addition, recall the three transformations used in approximating the kinematics of converted waves:

$$x_R' \equiv x_R + h \cdot \frac{1 - \frac{1}{\gamma}}{1 + \frac{1}{\gamma}} \ , \tag{2}$$

$$h' \equiv h \cdot \frac{2}{\sqrt{\gamma}\left(1 + \frac{1}{\gamma}\right)} \ , \tag{3}$$

and

$$v \equiv \frac{2v_s}{1 + \frac{1}{\gamma}} \ . \tag{4}$$

If we solve equation (1) for z_0, and replace x_R with x_R' in accordance with equation (2), we obtain

$$z_0^2 = \frac{1}{\left(1 - \frac{1}{\gamma^2}\right)^2}\left[-\left(1 - \frac{1}{\gamma^2}\right)\left\{ x_R'^2\left(1 - \frac{1}{\gamma^2}\right) + \frac{4hx_R'}{\gamma}\right\} + \right. \tag{5}$$

$$\left. + \left(1 + \frac{1}{\gamma^2}\right) v_s^2 t^2 - \frac{2v_s^2 t^2}{\gamma}\sqrt{1 - \frac{4h}{v_s^2 t^2} x_R'\left(1 - \frac{1}{\gamma^2}\right) + \frac{4h^2}{v_s^2 t^2}\left(1 - \frac{1}{\gamma}\right)^2} \right] .$$

If we assume that h , t , γ, and v_s are known, then this large and unpleasant equation can be used to find z_0 as a function of x_R'. We know that the dip bar must lie somewhere in the x_R'-z plane along this line. To find its exact location, however, we need to know its slope; this can be found by means of the following equation:

$$\tan\phi = \frac{v_s(p_s + p_g)}{\sqrt{1 - v_s^2 p_s^2} + \sqrt{\frac{1}{\gamma^2} - v_s^2 p_g^2}} \ . \tag{6}$$

Once the slope is known, we can differentiate equation (5) with respect to x_R' and use the identity $dz_0/dx_R' = -\tan\phi$ to obtain:

$$-\tan\phi = \frac{1}{z_0}\cdot\frac{1}{1 - \frac{1}{\gamma^2}}\left[-x_R'\left(1 - \frac{1}{\gamma^2}\right) - \frac{2h}{\gamma} + \right. \tag{7}$$

$$\left. + \frac{2h}{\gamma}\sqrt{1 - \frac{4h}{v_s^2 t^2} x_R'\left(1 - \frac{1}{\gamma^2}\right) + \frac{4h^2}{v_s^2 t^2}\left(1 - \frac{1}{\gamma}\right)^2} \right] .$$

Note that there is a z_0 term in this equation; equation (5) can be used to replace this term with an expression involving x_R'. Once z_0 has been eliminated, we have an equation in only one unknown, x_R' (recall that ϕ can be determined by means of equation (6)). This expression is too complicated to be solved analytically for x_R'; an iterative method must be used. Once x_R' has been determined, z_0 can be found by means of equation (5). Thus it is possible, with some difficulty, to find the exact values of x_R and z_0 once h, p_s, p_g, t, γ, and v_s are known.

This method is rather expensive, since it involves iterations of relatively complicated formulas. It is possible to approximate these formulas, however, and obtain formulas which will prove much easier to solve. Recall that a square root can be approximated by means of the Taylor-series expansion

$$\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8} . \tag{8}$$

The square-root term in equation (5) can be expanded according to this approximation, and after some algebra equation (5) can be rewritten in the form

$$z_0^2 \approx \left(1 - \frac{4h^2}{\gamma {v_s}^2 t^2} \right) \left(\frac{{v_s}^2 t^2}{\left(1 + \frac{1}{\gamma}\right)^2} - x_R'^2 \right) . \tag{9}$$

This equation can be differentiated with respect to x_R', and we already know that $dz_0/dx_R' = -\tan\phi$. Combining this information, and solving the resulting equation for x_R', we find that

$$x_R' \approx \frac{\left(\dfrac{v_s t}{1 + \frac{1}{\gamma}} \right) \cdot \tan\phi}{\sqrt{1 - \dfrac{4h^2}{\gamma {v_s}^2 t^2} + \tan^2\phi}} . \tag{10}$$

By combining equations (9) and (10) we can find a similar formula for z_0:

$$z_0 \approx \frac{\left(\dfrac{v_s t}{1 + \frac{1}{\gamma}} \right) \left(1 - \dfrac{4h^2}{\gamma {v_s}^2 t^2} \right)}{\sqrt{1 - \dfrac{4h^2}{\gamma {v_s}^2 t^2} + \tan^2\phi}} . \tag{11}$$

Thus, we have seen that if h, t, p_s, p_g, γ, and v_s are known, it is possible to obtain explicit, albeit approximate, equations for x_R' and z_0. And once x_R' is known, x_R is easily found by means of equation (2); x_0 then can be found according to the formula

$x_0 = x_R + y$. If it is necessary to create a time section, t_0 (migrated travel time) can be found through a simple depth-to-time conversion:

$$t_0 = \frac{z_0}{v_s(1+\gamma)} . \tag{12}$$

It should be noted that of the three transformations equations, equations (2), (3), and (4), I have used only equation (2). It is not necessary to find the transformed values h' and v.

DETERMINING v_s

It has been shown that x_0 and z_0 (and t_0 as well) can be found either exactly or approximately, as long as h, y, p_s, p_g, γ, and v_s are known. It is not always the case, however, that v_s is known. And even if an overall value for v_s is known, we still want to determine a value for each dip bar so that we can discriminate between dip bars that represent signal (and have a nearly-correct value of v_s) and those that are merely the result of noise. We shall see, as in the previous section, that we can use either an expensive exact solution or an inexpensive approximate solution to find v_s.

Let us recall the previous convention of adopting a separate symbol to represent the value of v_s associated with a particular dip bar; an appropriate symbol is v_{sCDR}. It can be shown that

$$t = \frac{2h}{\gamma v_{sCDR}} \cdot \frac{\cos\theta_s + \gamma\cos\theta_g}{\sin\theta_s\cos\theta_g - \cos\theta_s\sin\theta_g} , \tag{13}$$

where $\sin\theta_s \equiv p_s\, v_{sCDR}$ and $\sin\theta_g \equiv p_g\, v_{gCDR} = \gamma p_g\, v_{sCDR}$. The only unknown quantity in this equation is v_{sCDR}, but since v_{sCDR} appears in so many places (notice how it is embedded in the sine and cosine terms), that there is no way to solve equation (13) analytically. It can be solved iteratively, however, and thus equation (13) represents our exact solution for the determination of v_{sCDR}.

Recall that once we make the transformations in equations (2), (3), and (4), we can find corresponding transformations that define other variables, including p_s' and p_g'. Thus, it is possible to show that

$$p_s' = \frac{1}{2}\left[\left(1+\frac{1}{\sqrt{\gamma}}\right)p_s + \left(1-\sqrt{\gamma}\right)p_g\right] \tag{14a}$$

and

$$p_g' = \frac{1}{2}\left[\left(1-\frac{1}{\sqrt{\gamma}}\right)p_s + \left(1+\sqrt{\gamma}\right)p_g\right] . \tag{14b}$$

Now we use the property that the kinematics of converted waves in the new (primed) system of coordinates is very similar to the behavior of conventional waves in the old coordinate system. Once we have made this approximation, we can use the conventional formula for determining velocity:

$$v_{CDR}'^2 \approx \frac{1 - \frac{h'}{t}(p_s' - p_g')}{(p_s' - p_g')\frac{t}{4h} + p_s' p_g'} \; . \qquad (15)$$

An equation based on equation (4) can be used to determine v_{sCDR}, once v_{CDR}' has been found:

$$v_{sCDR} = \frac{v_{CDR}'}{2} \cdot \left(1 + \frac{1}{\gamma}\right) \; . \qquad (16)$$

Thus, it is possible to determine v_{sCDR} for each dip bar, and, as in the previous section, this can be done either approximately and cheaply or exactly and expensively.

So far I have assumed that v_s is constant. As I noted in a previous paper (Sword, 1984b), this assumption is not usually correct in practice; I won't repeat here the discussion of how this limitation can be dealt with.

PROCESSING SYNTHETIC DATA

When I previously processed my synthetic data with CDR I determined a smoothed velocity function that I called $\overline{v}$. This function represented the P-wave velocity. I again used this velocity function when processing the converted-wave part of the data, so I did not need to pick converted-wave velocities. Figure 2 shows the result of processing the data for SP-wave reflections ($\gamma = 1.732$); processing was performed using the approximate equations developed in this paper. No attempt was made to use the exact equations. Figure 3 shows the result (from a previous paper) of processing for SP-wave reflections by the method of coordinate transformation.

Some of converted-wave reflectors are more visible on the CDR plot than they are on the other plot. Others, such as reflector 1, have disappeared. In addition, the reflectors on the CDR plot do not extend as far to the right as those on the other plot.

CONCLUSIONS

It has been shown that converted-wave data can be processed by means of controlled directional reception, once the proper equations have been developed. Processing by this method has the advantages and limitations that are always inherent in the use of

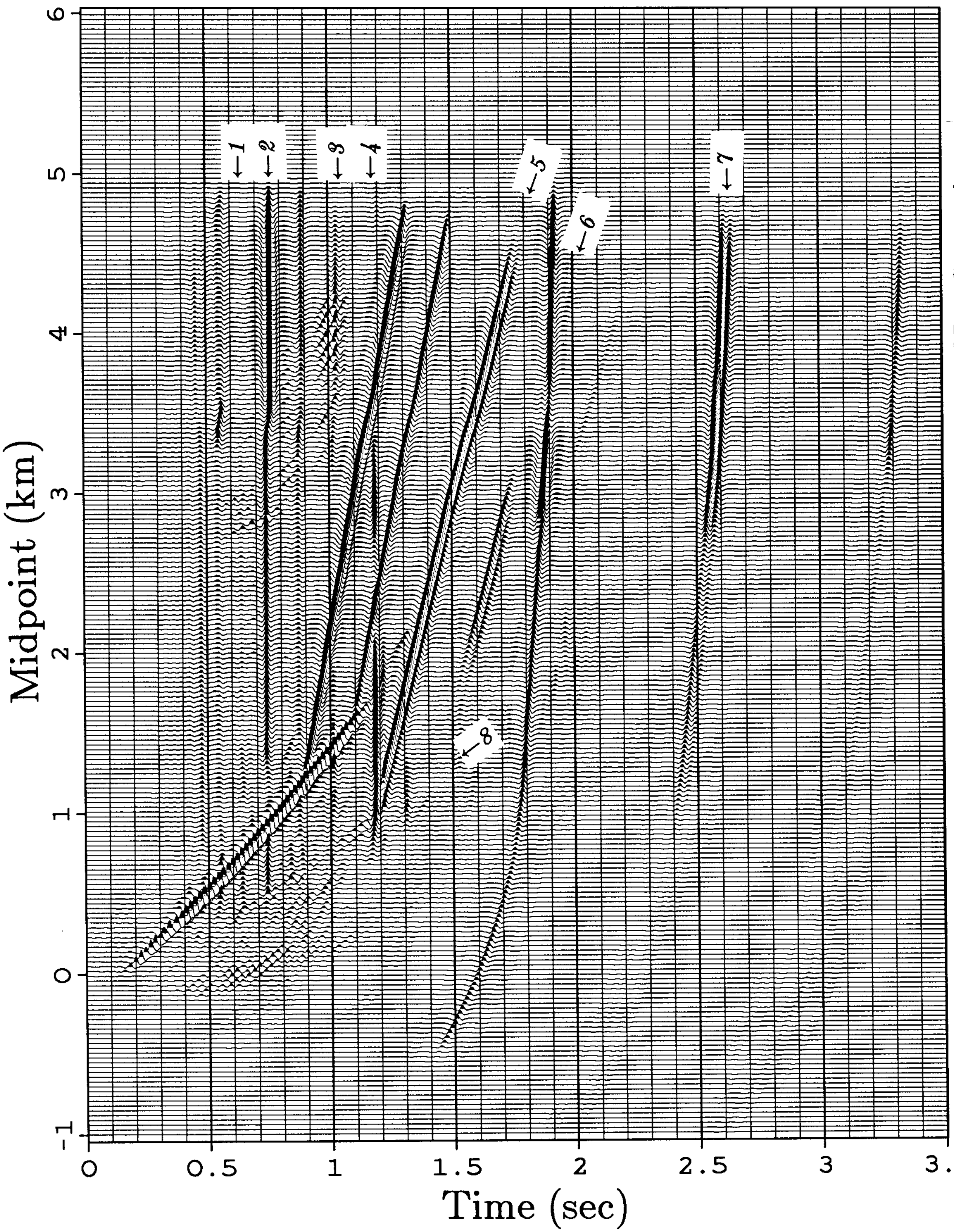

FIG. 2. Result of processing converted-wave data with CDR. Converted-wave reflections are marked by numbered arrows; the number corresponds to the number of the reflector.

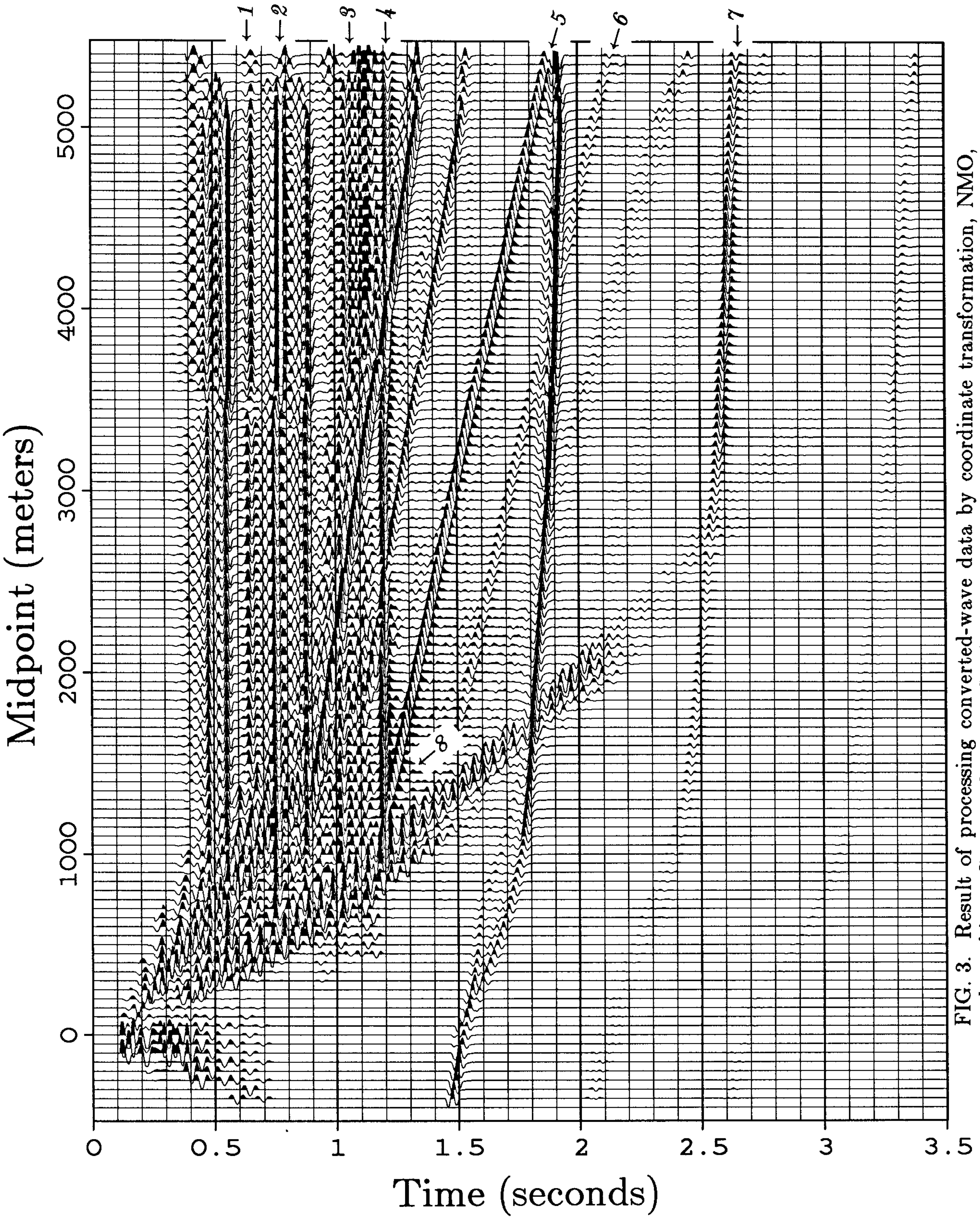

FIG. 3. Result of processing converted-wave data by coordinate transformation, NMO, and stacking. Converted-wave reflections are marked by numbered arrows; the number corresponds to the number of the reflector.

CDR. I did not attempt to use the exact converted-wave equations to process my synthetic data; the approximate equations seem to have given satisfactory results.

ACKNOWLEDGMENTS

Many of the ideas in this paper were developed during my 9-month stay at the Gubkin Institute of Petrochemical and Gas Production in Moscow. This stay was made possible by the International Research and Exchange Board, which oversees, among other programs, an exchange of graduate students and young faculty members between the US and the USSR. I should also mention Boris Zavalishin of the Gubkin Institute, who helped make my stay possible, and Dr. G. Matveenko of VNII Geofizika, who provided the synthetic data set used in this paper. Evgeniy Varov explained to me the details of the CDR method, and wrote the programs that picked ray parameters from the synthetic data set. And of course I thank Jon Claerbout and the sponsors of the Stanford Exploration Project.

REFERENCES

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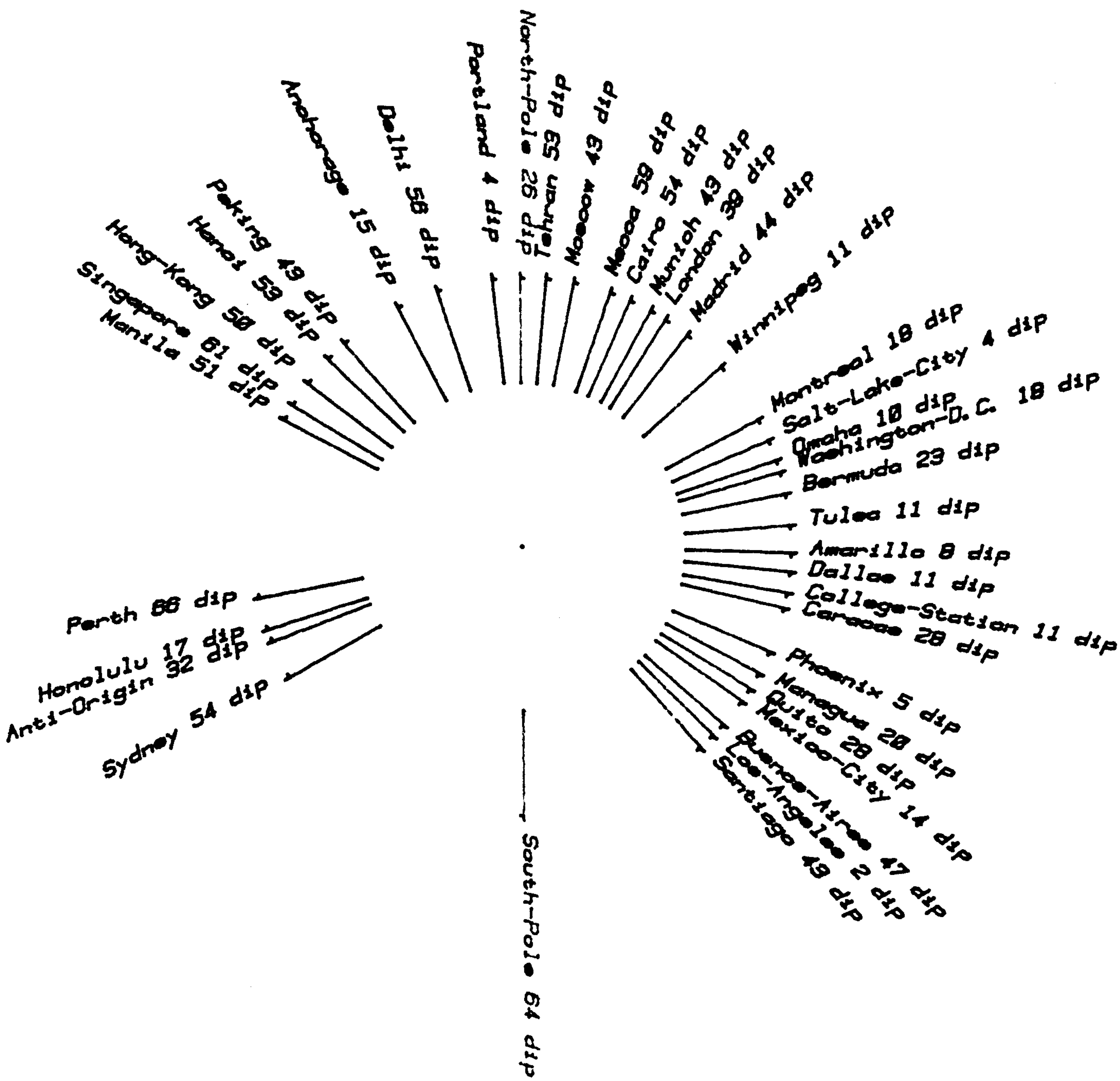

Plot for San-Francisco lat. 37.45 long. -122.21.