# Surface-consistent deconvolution

*Stewart A. Levin*

## INTRODUCTION

Deconvolution methods are based, by and large, on a single trace model: a random white reflectivity sequence is convolved with a stationary, or slowly varying, waveform and contaminated with noise and measurement errors. The object of the method is to recover, as far as possible, the original reflectivity. In many cases more data than just a single trace are needed to stabilize in the presence of noise; spatial averaging over nearby traces having, presumably, the same shot waveform or multiple reverberation pattern is used to form an averaged signature. Several examples of such averaging have been reported in the literature (Otis and Smith, 1977; Sicking, 1982.)

Recently Taner and Koehler (1981), Morley (1982) and Pollet, et al. (1982) employed surface-consistent spatial averaging in their deconvolutions. In this model the reflectivity is assumed to have been convolved with three successive filters - the shot waveform, a multiple reverberation pattern, and a last recording and instrumentation filter. These workers (staticsticians?) applied the techniques of surface-consistent statics to decompose the data in the log spectral (cepstral) domain, the primary difference being that Pollet, et al.'s homomorphic deconvolution used phase unwrapping to generate the imaginary part of the log spectrum while the others, assuming minimum phase filters, used the Hilbert transform to create the imaginary part. Sword (1983) attempted direct surface-consistent decomposition of phase-unwrapped data treating the phase ambiguity as a linear programming problem with disappointing results. Rothman (private communication) believes that Monte-Carlo methods he applied to the statics problem could be successfully adapted to Sword's approach. Phase unwrapping is not really the issue, Tribolet's adaptive method or any one of three other methods described in Oppenheim and Schafer (1975) will do a good job. The problem is the large influence of noise and measurement errors on the phase spectrum.

The widespread and successful use of surface-consistent solutions to the statics problem is a powerful reason to suppose surface-consistent deconvolution has a similar broad range of application. Indeed a static correction is a simple one parameter time-shifting filter and so surface-consistent statics is a special case of surface-consistent deconvolution. Donoho's discussion of minimum entropy deconvolution (1981) gives additional reason to expect surface-consistent spatial averaging to produce superior results. There he proves that whenever the original white reflectivity is non-gaussian it is possible to identify (up to a time shift and a scale factor) any wavelet that has been convolved with it. Simply averaging spectra over, say, common shot gathers is not enough; we cannot generally expect a good waveform estimate when the underlying traces are still convolved with unknown, spatially variable wavelets. Using surface-consistency allows more confidence that the underlying reflectivities we estimate are truly white.

## DISCUSSION

Surface-consistency is an algebraic condition. Invariably this reduces to overdetermined and underconstrained parameter variations. For the statics problem, the solution is only determined up to arbitrary constant time shifts that may be added to, say, the geophone component and simultaneously subtracted from the shot component. Sword noted that the same applies at each frequency of his frequency dependent statics approach - an arbitrary (complex) constant may be added to the log spectrum of one component and subtracted from another. That is, surface-consistency by itself only determines the separable components up to some constant filter. Estimating the surface-consistent effects could be accomplished as with residual statics; relative trace-to-trace shaping filters would be decomposed into surface-consistent components to balance near-surface weathering effects. This process I'll term residual deconvolution although it doesn't rely on any statistical assumptions or measured wavelets for the data.

Separate residual deconvolution may offer some advantages over full surface-consistent deconvolution. First, it deals with a smaller amount of data, a short shaping filter replaces each trace in the input. Second, the output of the process is now "surface-inconsistent" making sensible the goal of constraining either pre- or post-stack deconvolution operators to be slowly varying functions of midpoint.

This latter idea seems not to have appeared in the literature yet, but is the most direct extension of residual statics where trace-to-trace differences are what are decomposed. Instead of time shifts determined by trace to trace correlation, trace to trace shaping filters are computed. These, presumably short, filters would then be decomposed in a surface-consistent manner.

## PROPOSED STUDY

In this project I will study the behavior of a number of different deconvolution algorithms generalized to the surface consistent setting. My goals are to:

1) Develop iterative surface-consistent algorithms without resort to the cepstral domain with its need to make specific phase assumptions and poor control over the time duration of the resulting wavelets. From a statistical viewpoint, treating each frequency independently introduces very many free parameters in the problem to (poorly) estimate. Constraining the time duration of the wavelets to be estimated usually makes physical sense as well. Surface-consistent static solutions are computed iteratively by successive spatial averaging over source, geophone, and offset gathers until convergence is attained. Following Claerbout (1982b), I will apply this same approach to surface-consistent deconvolution. As he points out, this treats the problem in its multiplicative form where phase unwrapping is not at issue. This is both straightforward and natural. The difficulties will be to find a good measure of convergence, methods of under and overrelaxation to improve convergence, and to decide whether the answer converged to is correct or at least an improvement over no processing at all.

2) Investigate preprocessing by temporal and spatial prewhitening (divergence correction; gain; NMO; trace balancing; prefiltering and smoothing). Classically these are applied to make a gather's statistics more stationary. Claerbout (1984a) argues that spatial whitening can improve temporal deconvolution by strengthening the contributions of lower amplitude primary reflections and diffractions to wavelet estimation.

3) Judge on theoretical and empirical grounds stability, convergence, and sensitivity to noise/static errors/measurement errors. Here the log linear model provides a starting point. Marcoux (1981) gave a Fourier analysis of the surface-consistent linear model justifying, among other things, the sinusoidal eigenfunctions that Wiggins, et. al. (1976) found empirically. Exponentiating such sinusoidal summands gives quite a variety of periodic multiplicative filter components as illustrated in Figure 1. We see the exponentiation of the amplitudes sharpens peaks and troughs and flattens the areas inbetween. While these provide some feel for what surface-consistent deconvolution can produce, they might also be misleading as the statics solution, with its regular sinusoidal variations, estimates spatial variation of a one-parameter

(time-shifting) filter. The present problem generalizes this to multi-parameter filters and so I expect analogous variations to appear in its solutions even if they are not eigenmodes.

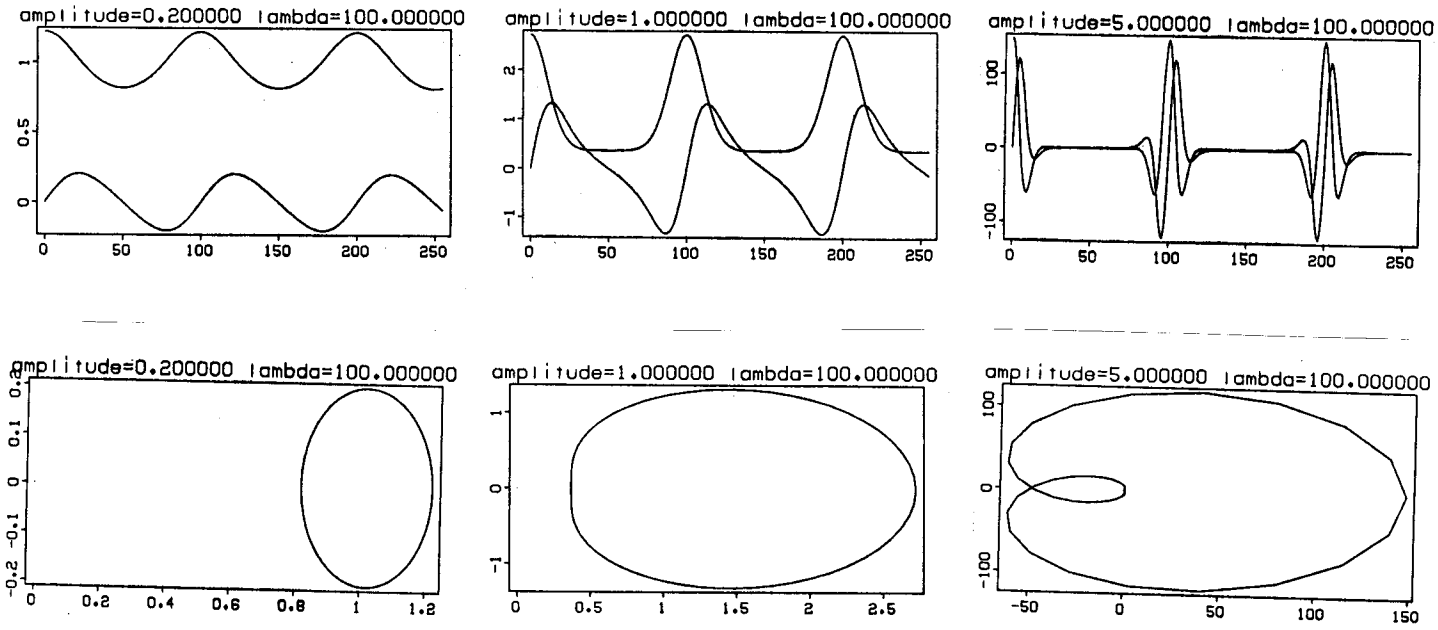


FIG. 1. $\exp\{ A \exp(2\pi i x / \lambda) \}$ for selected values of amplitude $A$. The upper row displays the real and imaginary parts and the lower the corresponding trajectory in the complex plane.

4) Study the effects of breakdown of the underlying model e.g. antenna patterns and interbed multiples, non-white earth impulse response. Donoho (personal communication) believes that estimates of the closeness of fit of the resulting model to the data gives us independent evidence to test the validity of the convolutional model of the seismic trace.

## PROGRESS REPORT

The framework I've devised for surface-consistent deconvolution tests is simple. A survey is read and placed on disk and the coordinates of each trace stored in core. At each iteration I specify a gather type (shot, receiver, cmp or

common offset) and a command string. I also specify an optional plot command and frequency for quality control plots. The procedure then sorts consecutive gathers off to a temporary disk file and invokes the specified command on that set of traces, replacing them with the output. All the difficult work is moved outside this program to devising suitable deconvolution programs to invoke with the command string.

So far I have worked with three different deconvolution methods: Wiener-Levinson prediction error filtering, Burg's maximum entropy deconvolution and Wiggin's minimum entropy method. For the first I adapted a single trace program written by Dave Hale to process gathers by averaging the autocorrelations of the individual traces, normalized to have zero lag 1, and then increasing the zero lag of the average to $1 + \epsilon$ for stability. (As Francis Muir pointed out, the average of autocorrelation functions is not necessarily itself an autocorrelation.) The prediction error for each window is then renormalized to the input amplitude level for convenient comparison. Options provide for either power of $t$ gain and/or sliding windows to handle nonstationarity in the input data. For the second method I employed a time-variant Burg algorithm. This averages leaky integration of the forward and backwards prediction errors to produce time variant filter reflection coefficient estimates. The results are averaged across each gather. Woodward and Dellinger, following a suggestion of Muir's, have recent results in the present volume on a robust method using medians to average reflection coefficients, rather than traces or filters, across a gather. The third method, minimum entropy deconvolution, I resurrected from Will Gray's varimax files. This program was already multichannel as experience had shown multi-trace averaging crucial to stabilizing this algorithm. The only time variance is provided by a gain option.

A major problem that I encountered right off the bat is illustrated in Figures 2 through 10 which show stacks of a line from the Central Valley of California during processing with iterative surface-consistent deconvolution. Deconvolution algorithms are just not designed with repeated application in mind. In an ideal world (Claerbout 1984b), a decon should produce its idea of the best possible result in one run. Subsequent deconvolution should not change that result. In these first surface consistent deconvolutions I ran the results of one pass over shots looked very similar to the input. Iterating then over receivers produces a noisier, less appealing image and after a couple more passes over the survey the stack was simply awful. Changing stacking velocities was not the

cause, I repicked velocities a couple of times without improving the results. To understand this problem better I went back to single trace deconvolution.

I generated 28 10 fold gathers by replicating each trace in the 28 trace CMP profile of Figure 11. This artifice allowed me to test single trace decon using my arsenal of multitrace programs. Figures 12 through 14 show the results of 11 iterations of the three algorithms I've used for a reasonable set of parameters I chose. The side panels show some selected trace as a function of iteration number. This illustrates that both Burg and prediction error filtering (Pef) change noticeably for the first three or four iterations and then, at least for this gather, converged to a final answer. Minimum entropy deconvolution changed the data much more slowly than Burg or Pef through 11 iterations. All of the algorithms failed the "ideal" one iteration decon test. They did however converge better than the surface-consistent iterations of Figures 2-10 which became noisier and noiser without really improving resolution.

These single channel tests also brought to light a problem in iterative time-variant deconvolution. Figure 15 shows repeated application of Pef with sliding 375 msec windows. The misfeature, of course, is the banding which coincides with the edges of each of the overlapping windows. This may perhaps be seen under close scrutiny in the first iteration but is not readily apparent until the second or third. It appeared to be related to what was happening at the end of each window. I observed that at the bottom of each trace a high frequency (Nyquist) end effect the crept higher into the trace with each iteration. When these are repeatedly blended with data from overlapping windows you might well expected trouble.

Actually the problems arose not from the ends of each window but from the beginnings! I found this by displaying individual windows before blending. The beginning of each window had quite high amplitude after inverse filtering when compared to values further down in the window. Explanation? The start of all windows but the first are overlain by the tail ends of wavelets from the previous window. This causes large prediction errors to appear at the start of the deconvolved window which are initially attenuated by linear ramping when blending overlapping windows. The attenuation is not total, however, and these artifacts build up as iteration proceeds. To get around this I now discard a number of samples equal to the filter length minus one from the beginning of each filtered window. Alternatively I could precede each input window with a number of samples equal to the filter length before convolution with the

prediction error operator. As for the end of the trace I concluded it was an artifact of previous processing or recording.

Another problem of judging the deconvolution results of Figures 2-10 is frequency content. Deconvolution wants, usually, to increase the frequency content of a trace, occasionally to an unreasonable degree. Cosmetically, we'd like to compare apples to apples by reshaping the output to a reference spectrum with frequency content similar to the input data's. This may well be incorrect - nonlinear estimation can correctly predict sharp spikes with frequency content outside the range of the input data. A reasonable and simple method of comparison is to apply a bandpass filter. When it's applied makes quite a difference. The accompanying videotape shows iterations of the form $(Pef + Bandpass)^N$ vs $Pef^N + Bandpass$. The former is characterized in profile by reflectors moving continuously towards the surface of the earth, the latter by reflectors staying still and their coda shrinking upwards to meet them. For these examples I used zero-phase bandpass filtering; minimum phase bandpass filtering would have reduced these differences at least for the minimum phase algorithms I employed. Time has not permitted me to rerun these comparisons yet.

At this stage I've found single trace deconvolution, either before or after stack, to be superior to surface consistent decon for these data. A lot of the difference is, I think, due to the iterative approach I'm employing. One of my next steps will be to compare it to Morley's approach for the Flemish Cap data he used in his work.

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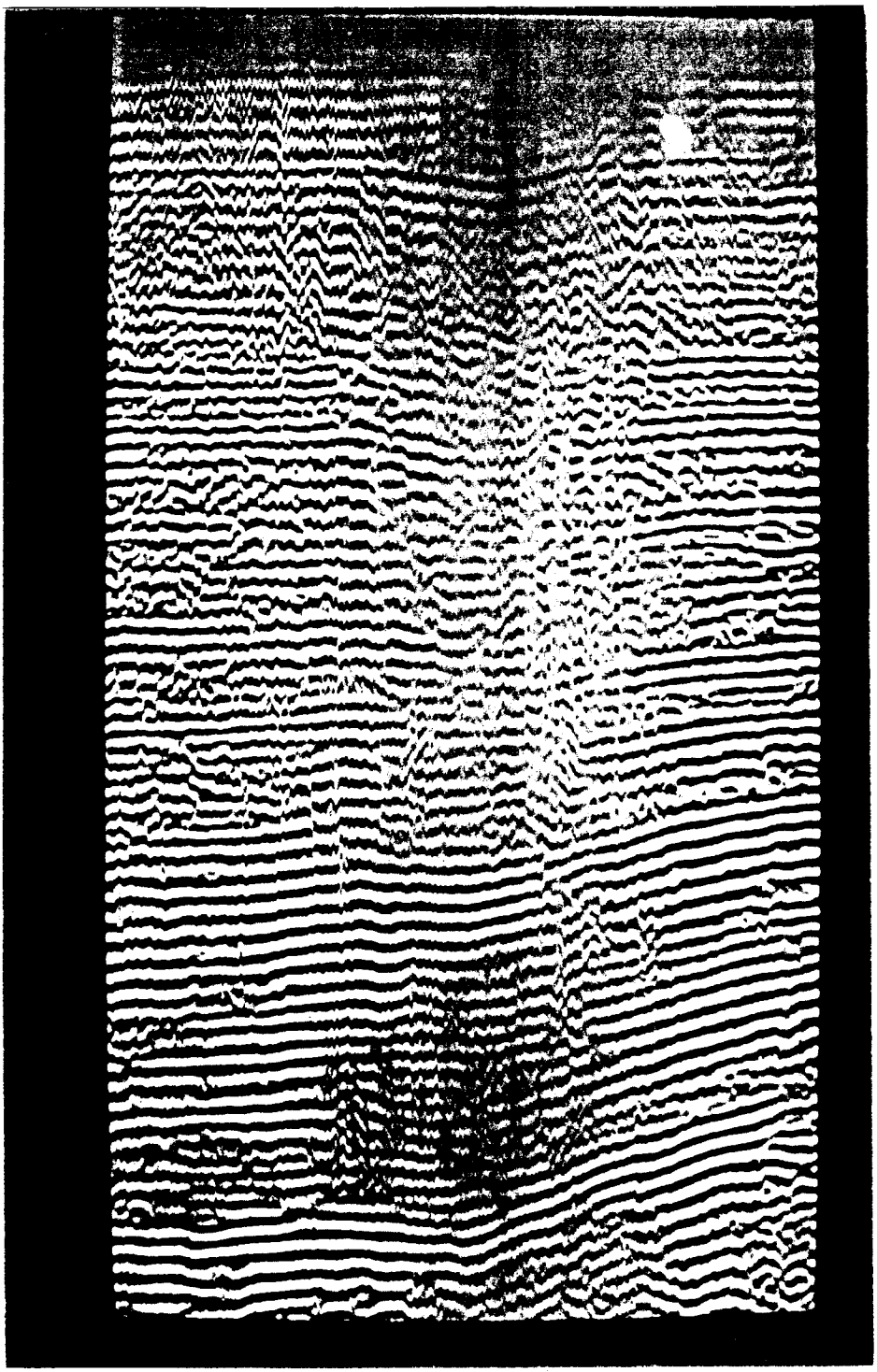
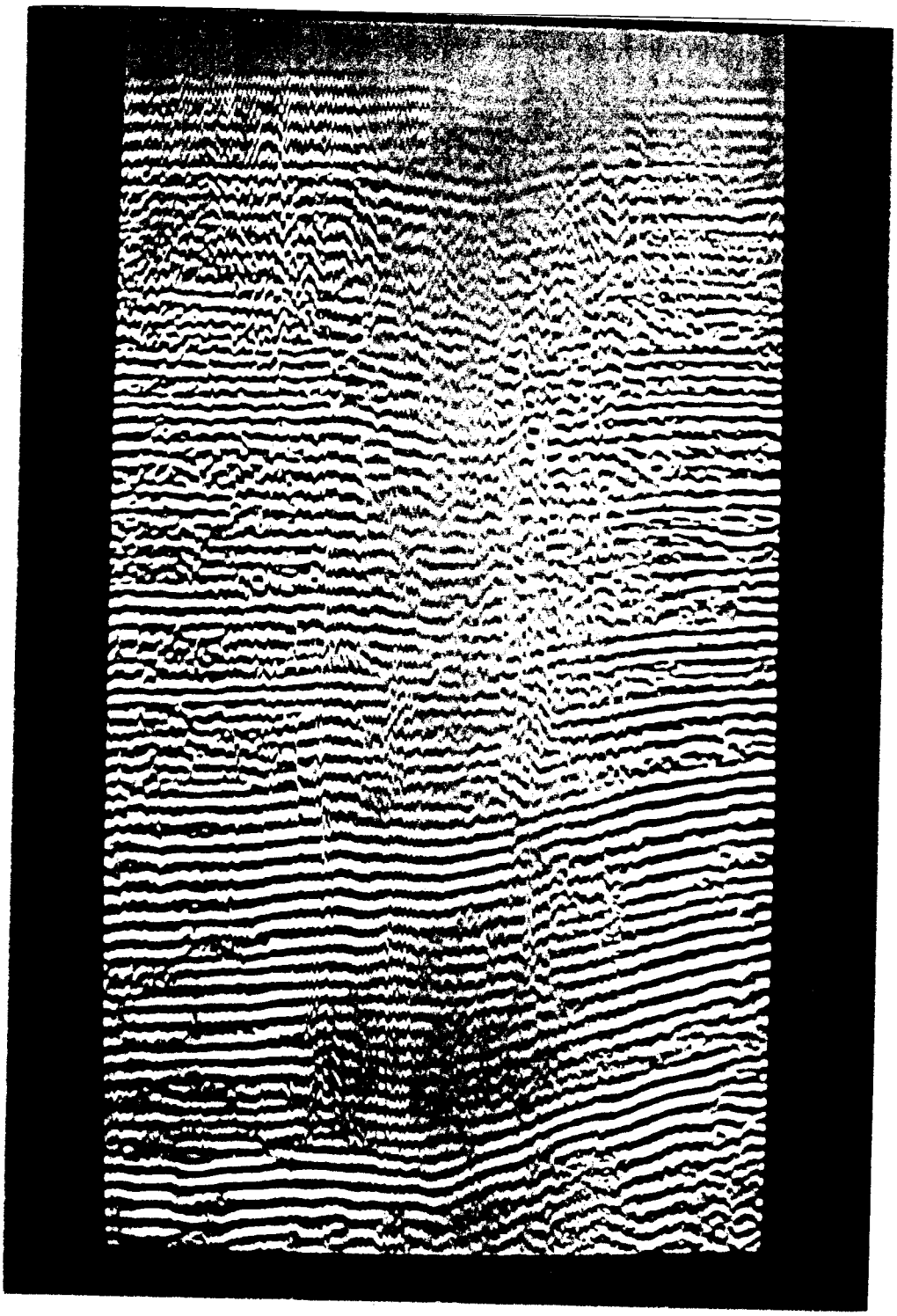


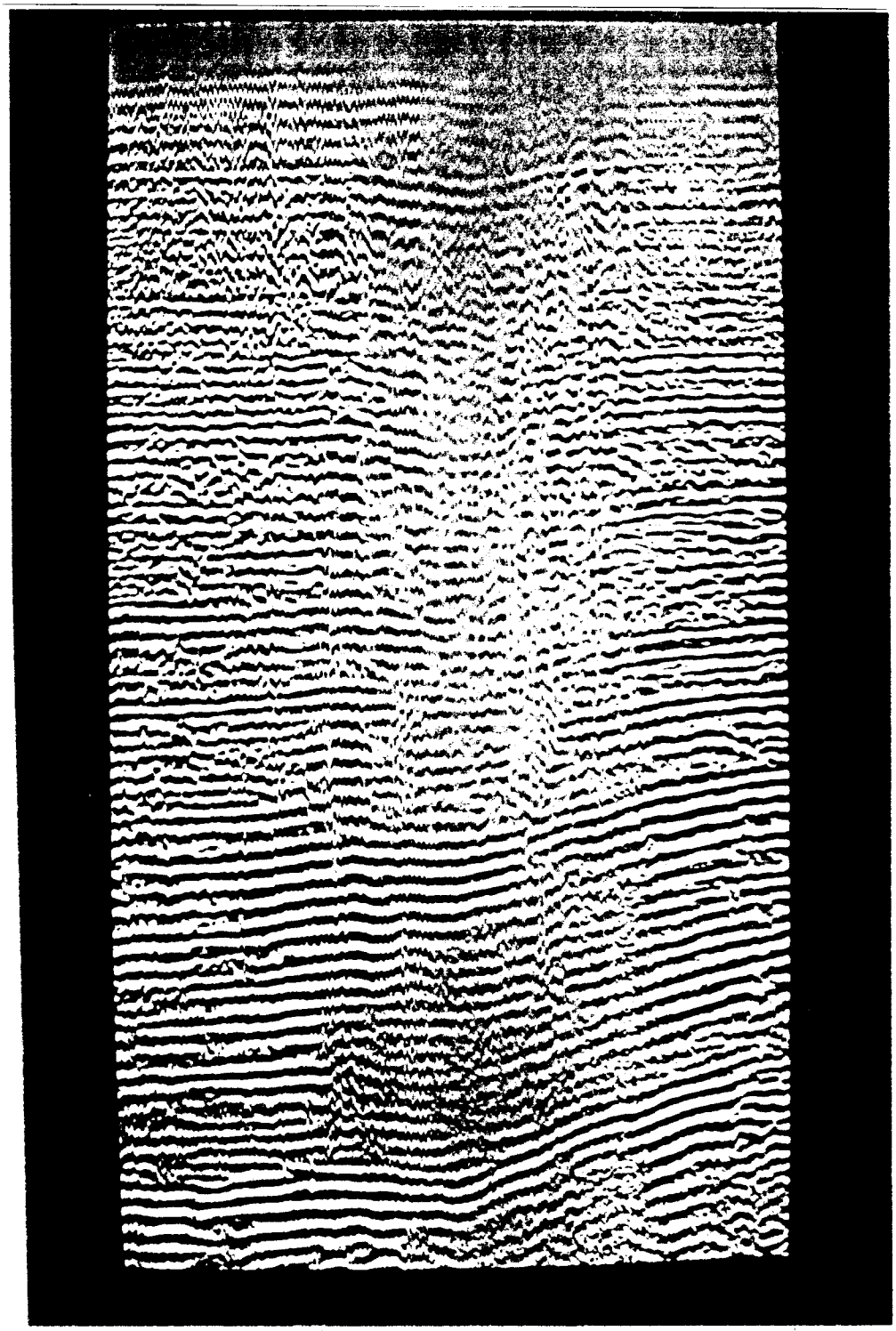
(Raw)

FIG. 2. Stack of undeconvolved input to surface-consistent deconvolution. There were 275 28-fold CMP gathers in the dataset. Trace length is 5 seconds sampled at 4 msec. These data are from the San Joaquin valley. The sag and decreased amplitudes in the center of the section are due to a low-velocity near-surface anomaly.



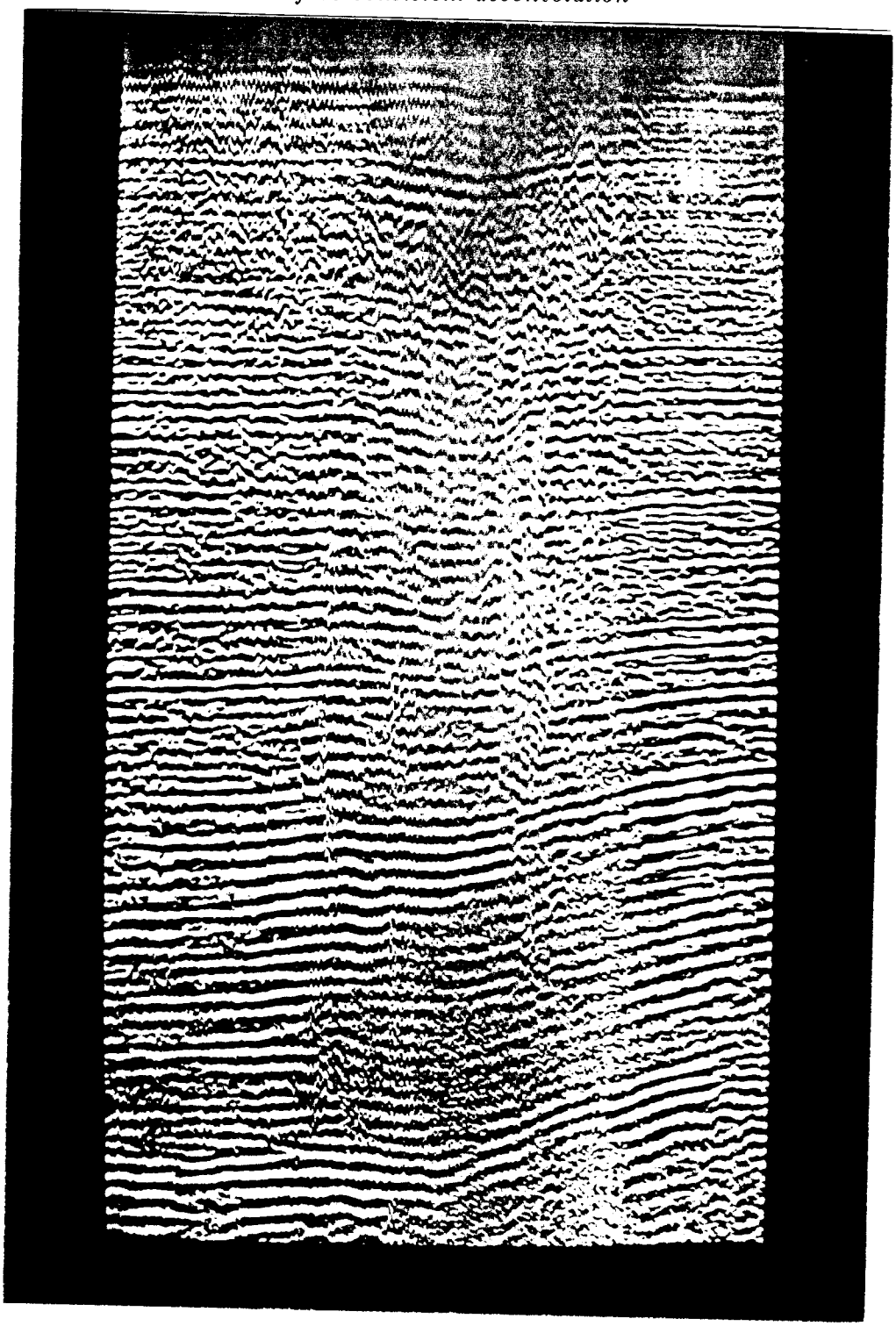
(S)

FIG. 3. Stack after one pass of Wiener prediction-error filtering over shot gathers. A single 128 msec spiking filter was generated for each gather from the average autocorrelation across the gather.



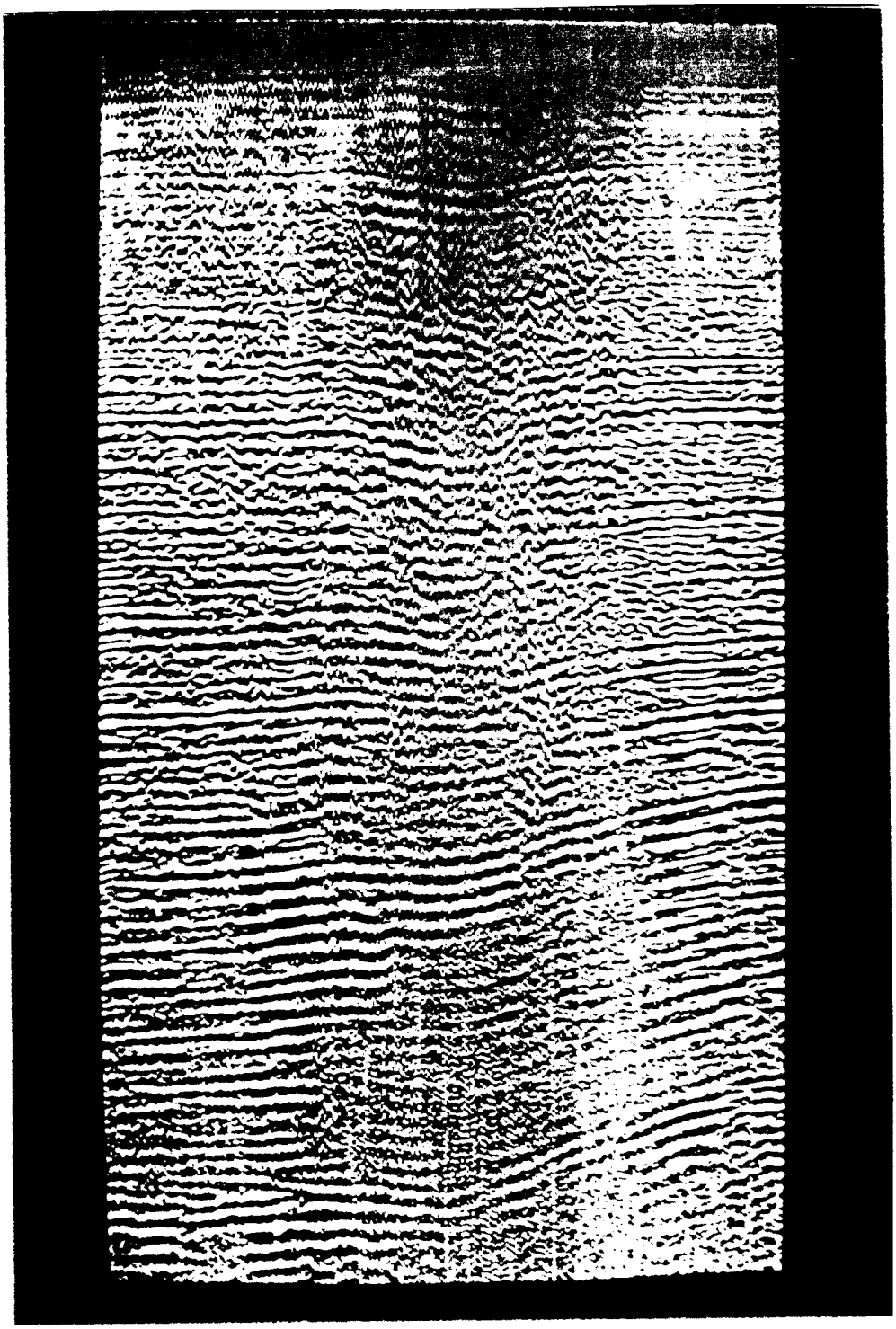
(SG)

FIG. 4. Stack after a iteration through common-geophone gathers. Input was the common-shot iteration shown in the previous figure.



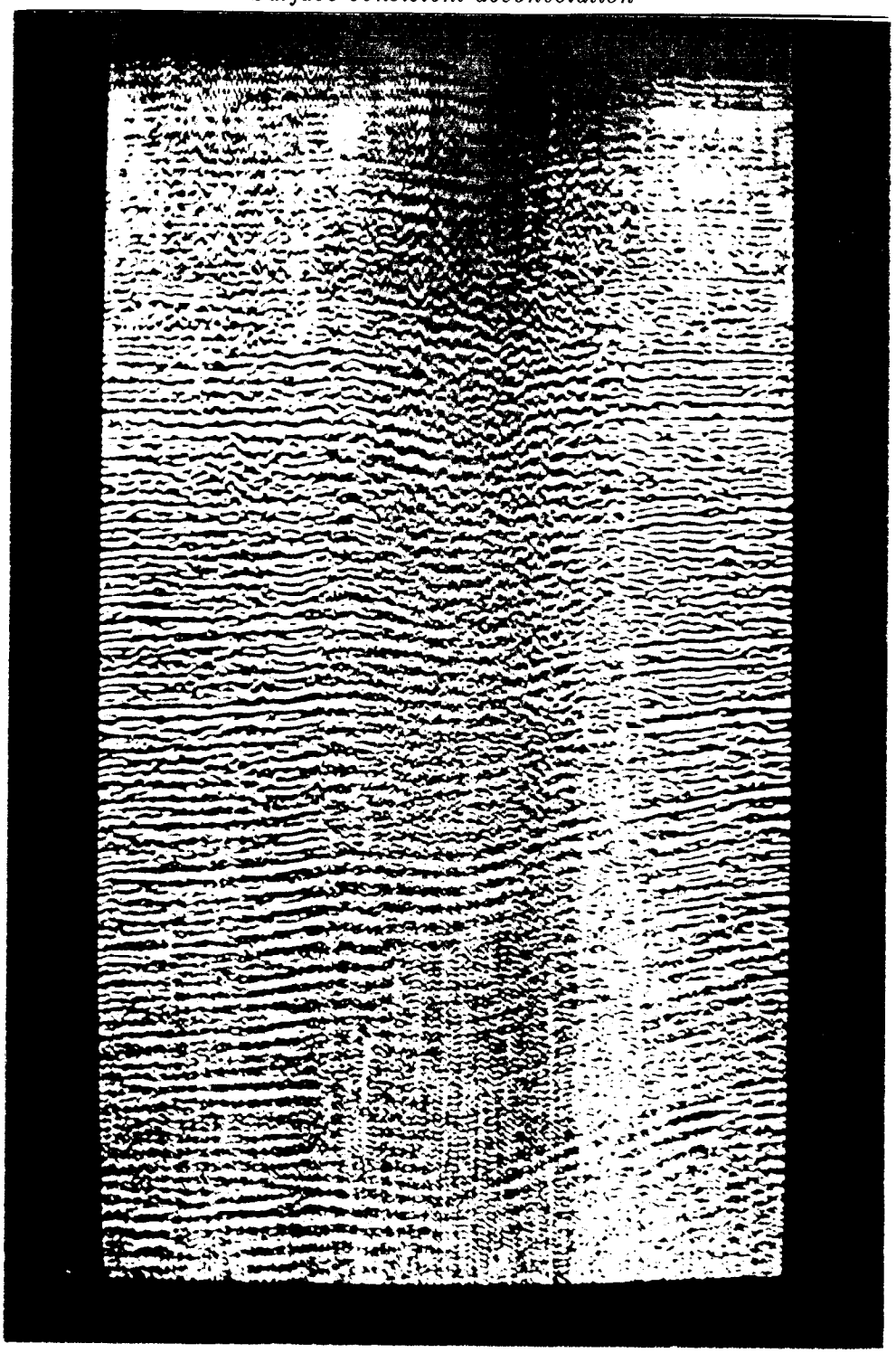
(SGM)

FIG. 5. Stack after another pass, this time over common-midpoint gathers. Note this is not surface-consistent. I include it in this example for more direct comparison with single-trace spiking decon shown below in Figure 10.



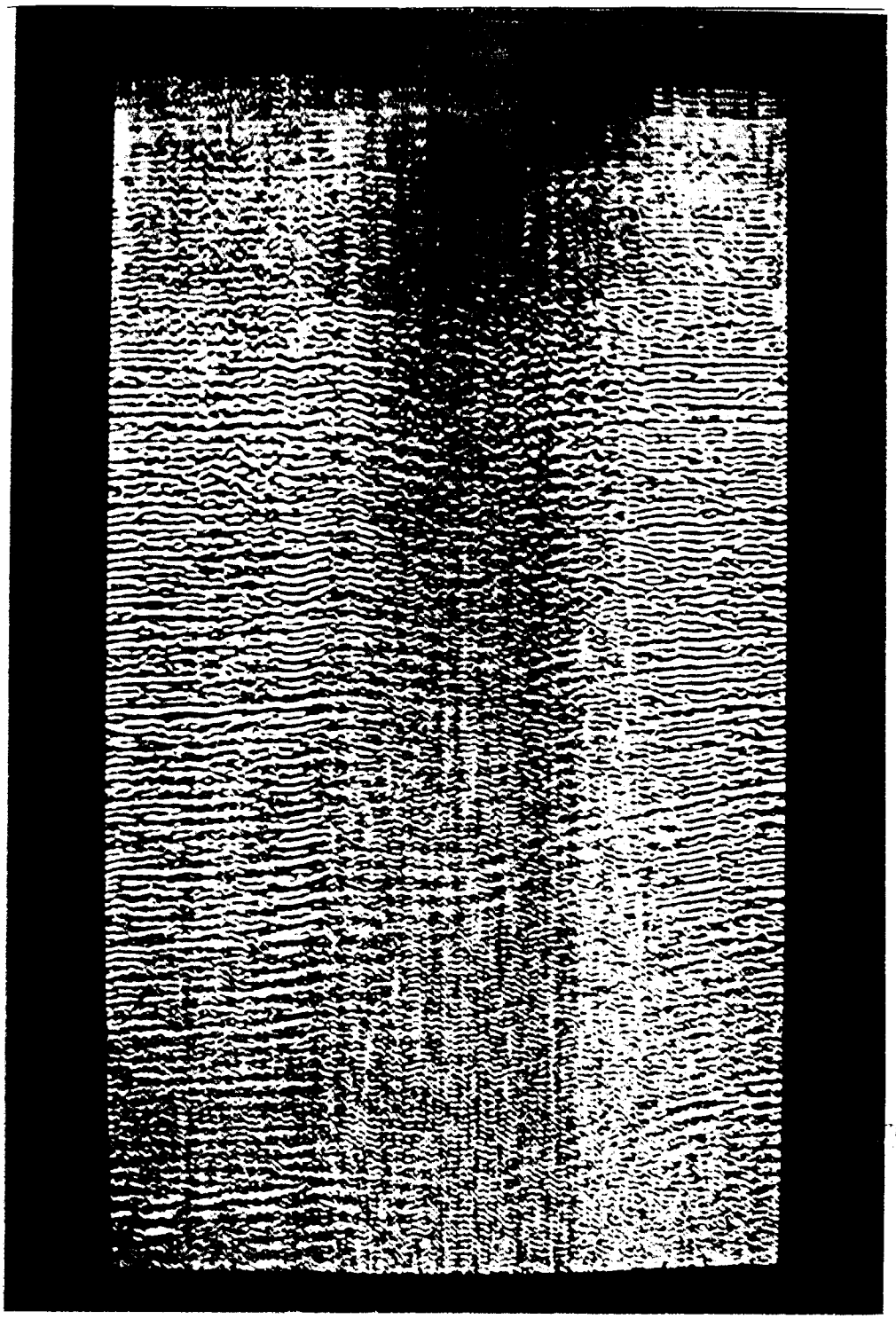
(SGMS)

FIG. 6. Stack after another iteration over common-shot gathers. High frequency noise is quite visible.



(SGMSG)

FIG. 7. Stack following a second iteration over common-geophone gathers. Even more high frequencies but not much compression of the reverberatory reflections.



(SGMSGM)

FIG. 8. Stack after a second iteration over over CMP's.

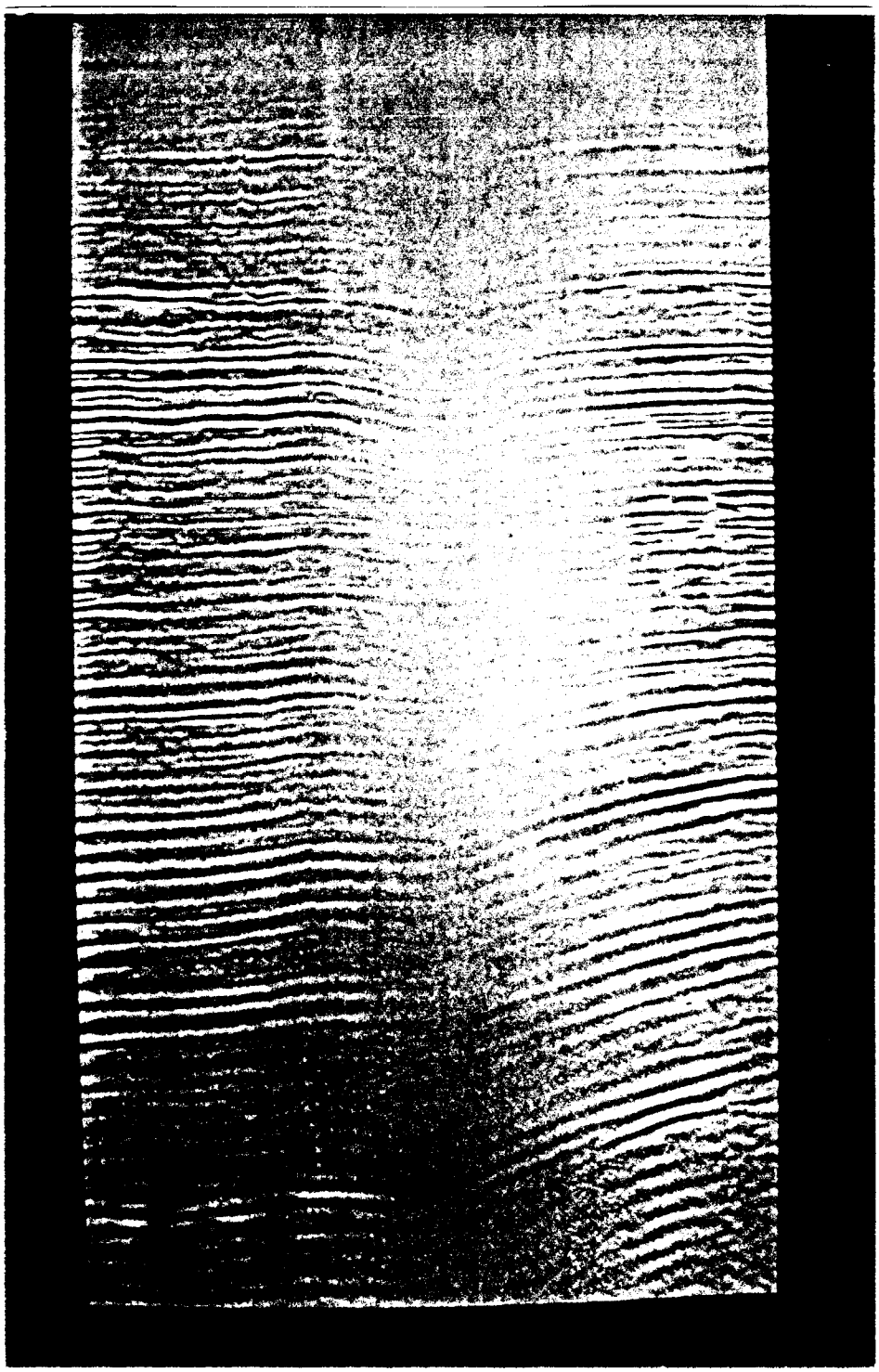


FIG. 9. Stack of original gathers after conventional single-trace spiking deconvolution. Deconvolution has done a good job, despite the fact that the source was Vibroseis.

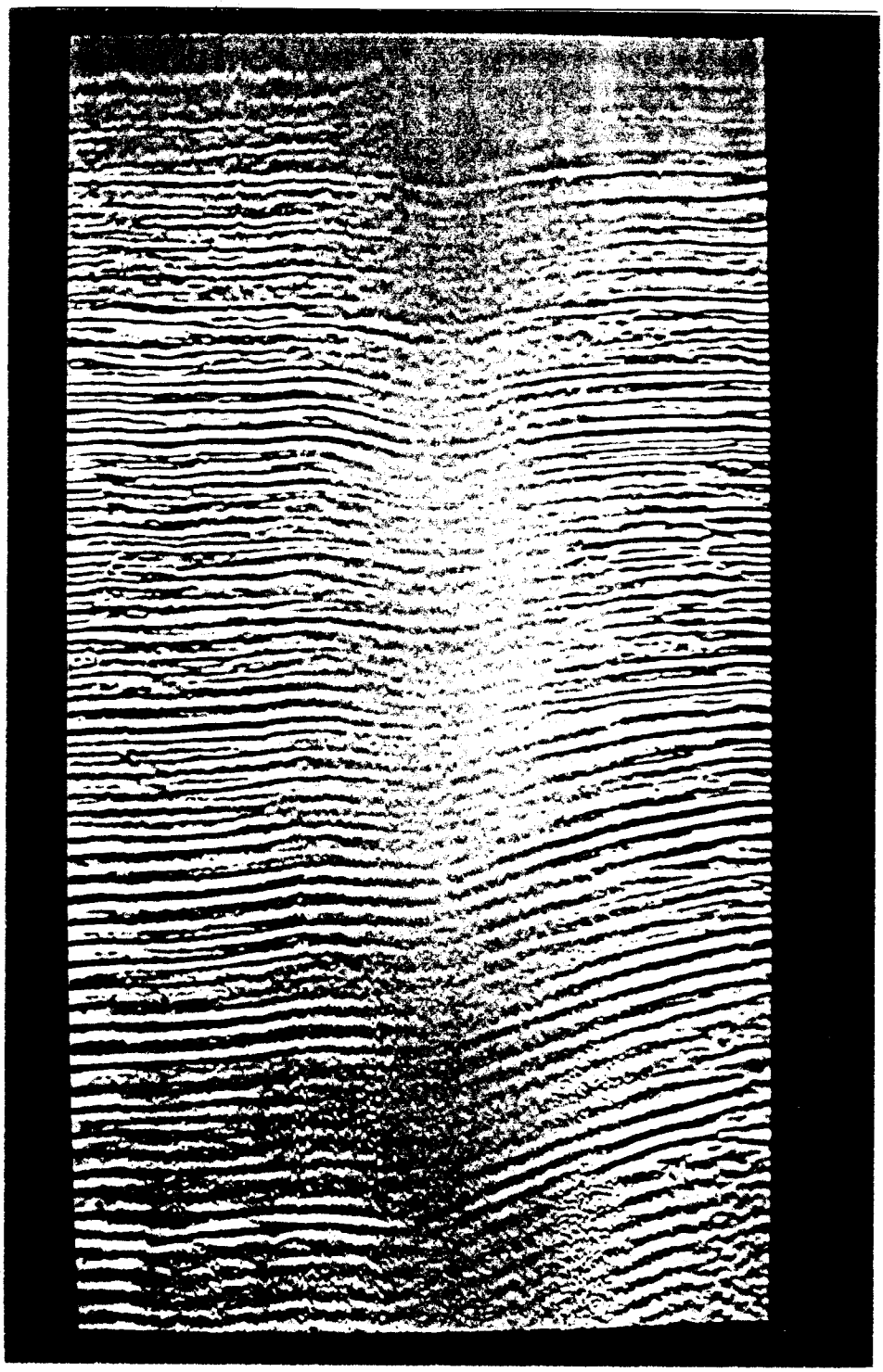


FIG. 10. Single trace spiking decon applied to SG stack in Figure 4. This result is nearly identical to single trace decon before stack (Fig. 9).

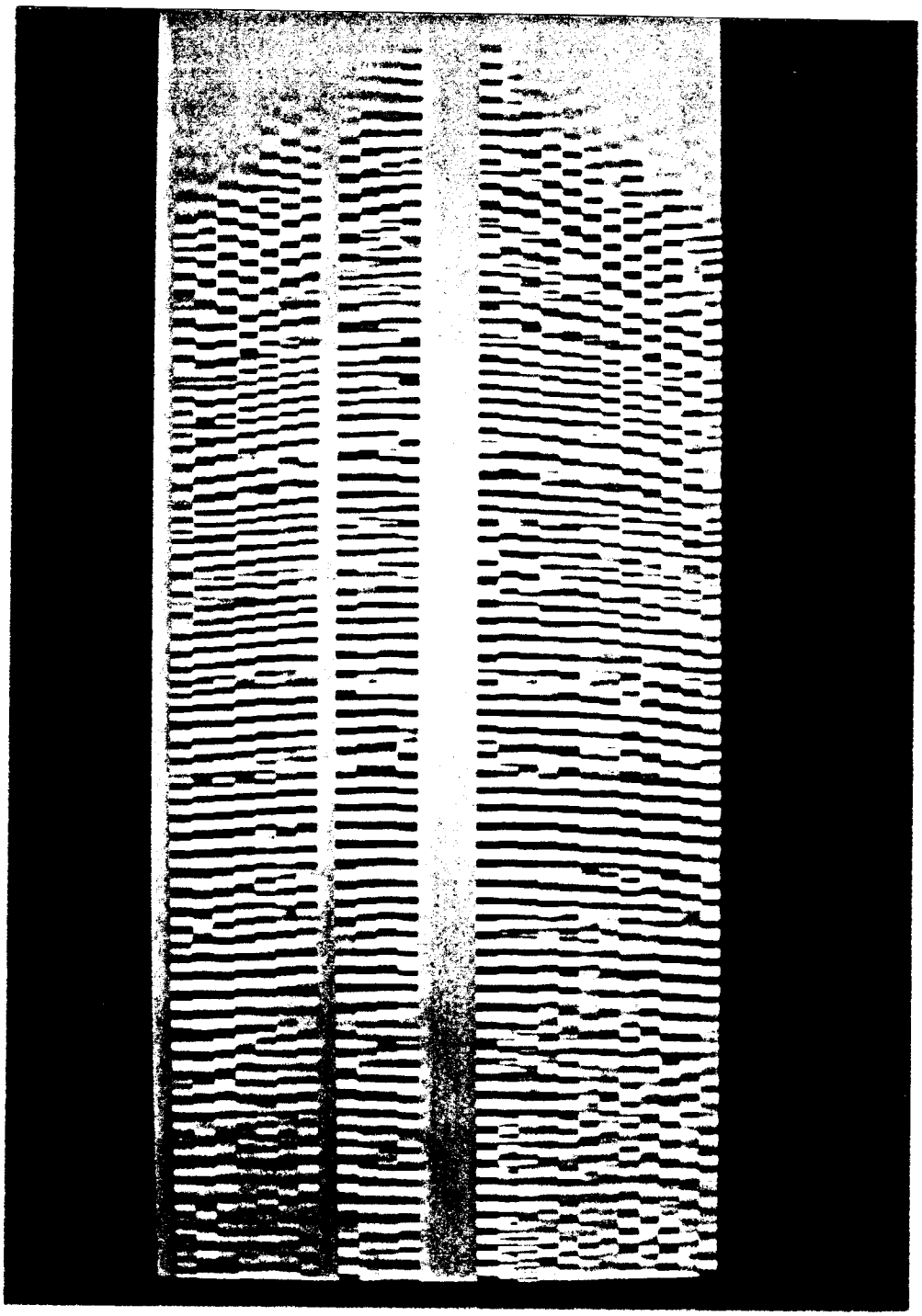


FIG. 11. Input common-midpoint gather used in trace-by-trace iterative deconvolution tests. This CMP is from the section in Figure 2.

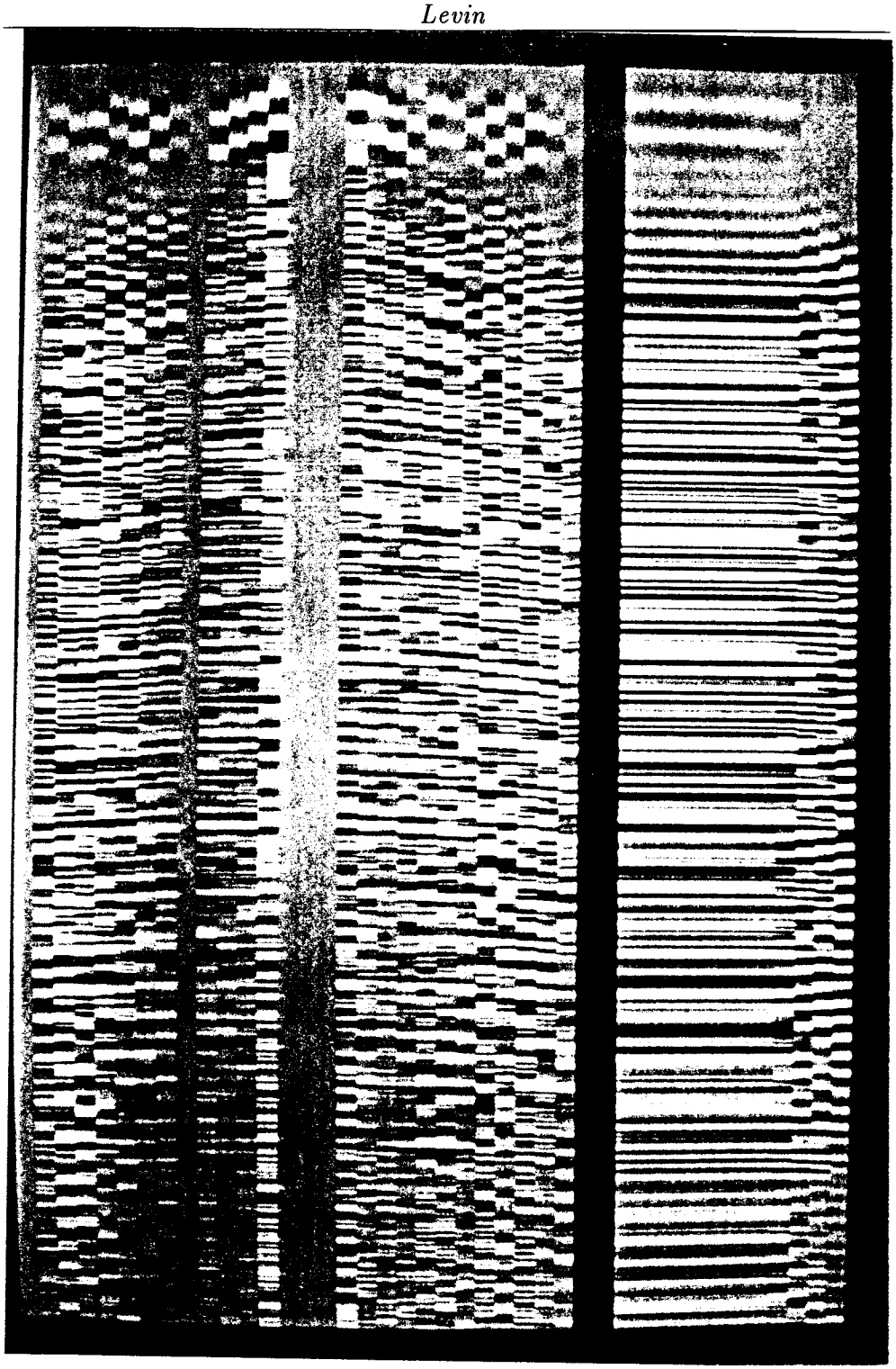


FIG. 12. CMP gather after 11 iterations of spiking deconvolution. $t^2$ gain was applied prior to deconvolution. On the right I show a single trace from that gather as a function of iteration number. We see that after the fourth iteration the changes to the trace are imperceptible.

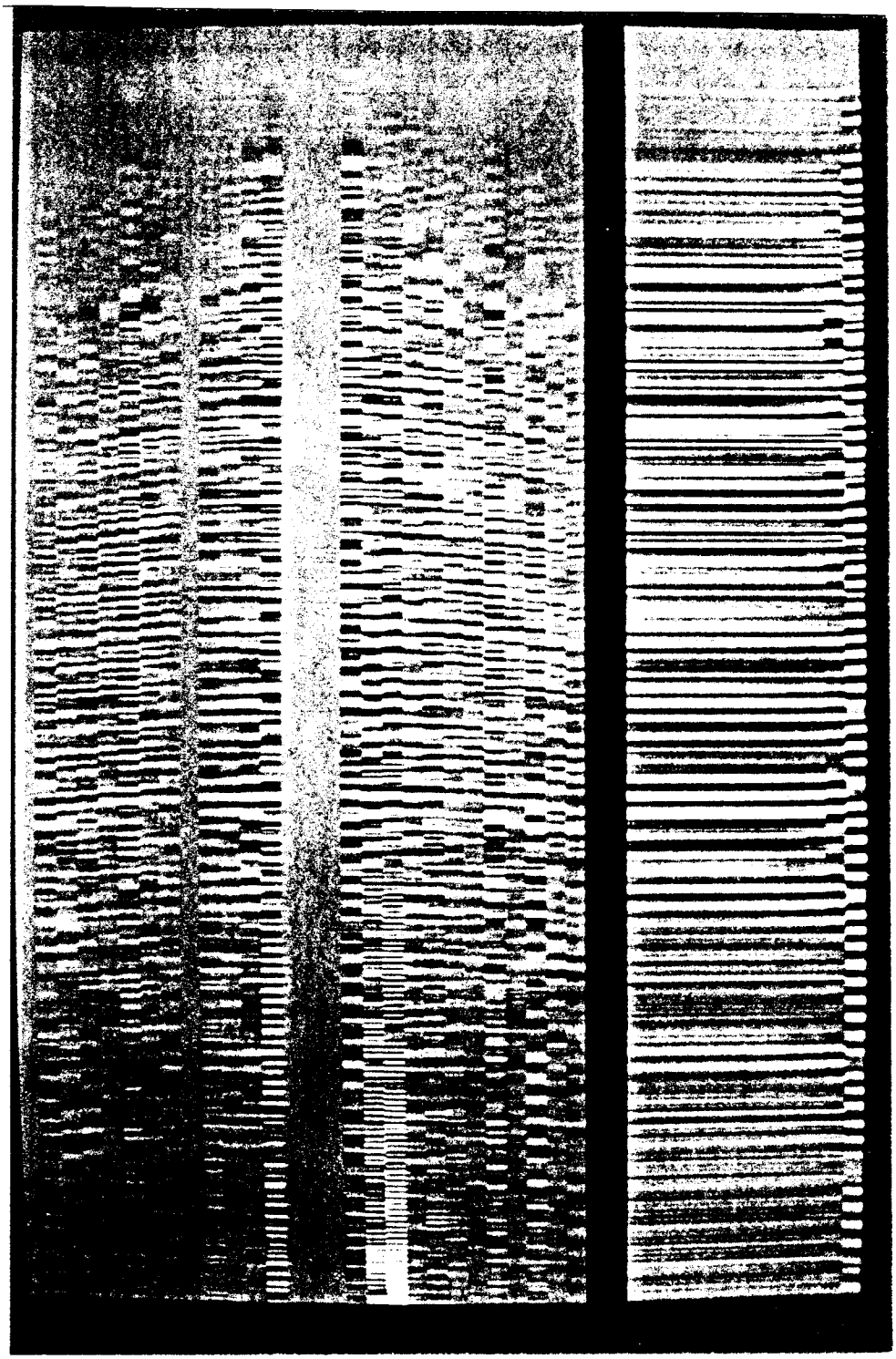


FIG. 13. CMP gather after 11 iterations of Burg deconvolution. $t^2$ gain was applied prior to deconvolution.

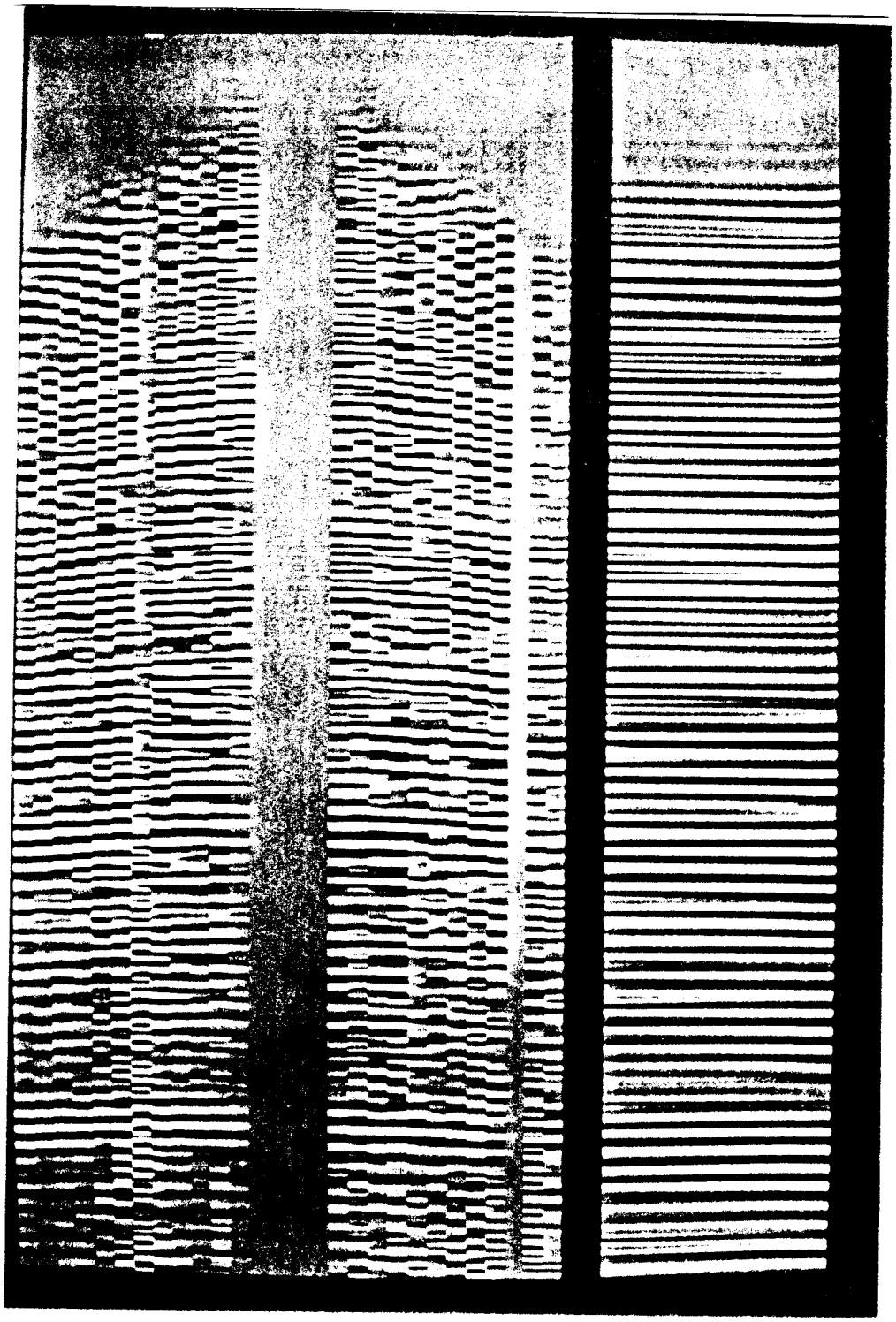


FIG. 14. CMP gather after 11 iterations of minimum entropy deconvolution. The changes are much less apparent than either of the previous two figures bu close examination of the iterated trace on the right shows a very gradual zero-phase compression of some of the stronger reflectors. In a later test not shown here I iterated a few more times and got a marked increase in high frequencies only two iterations later.

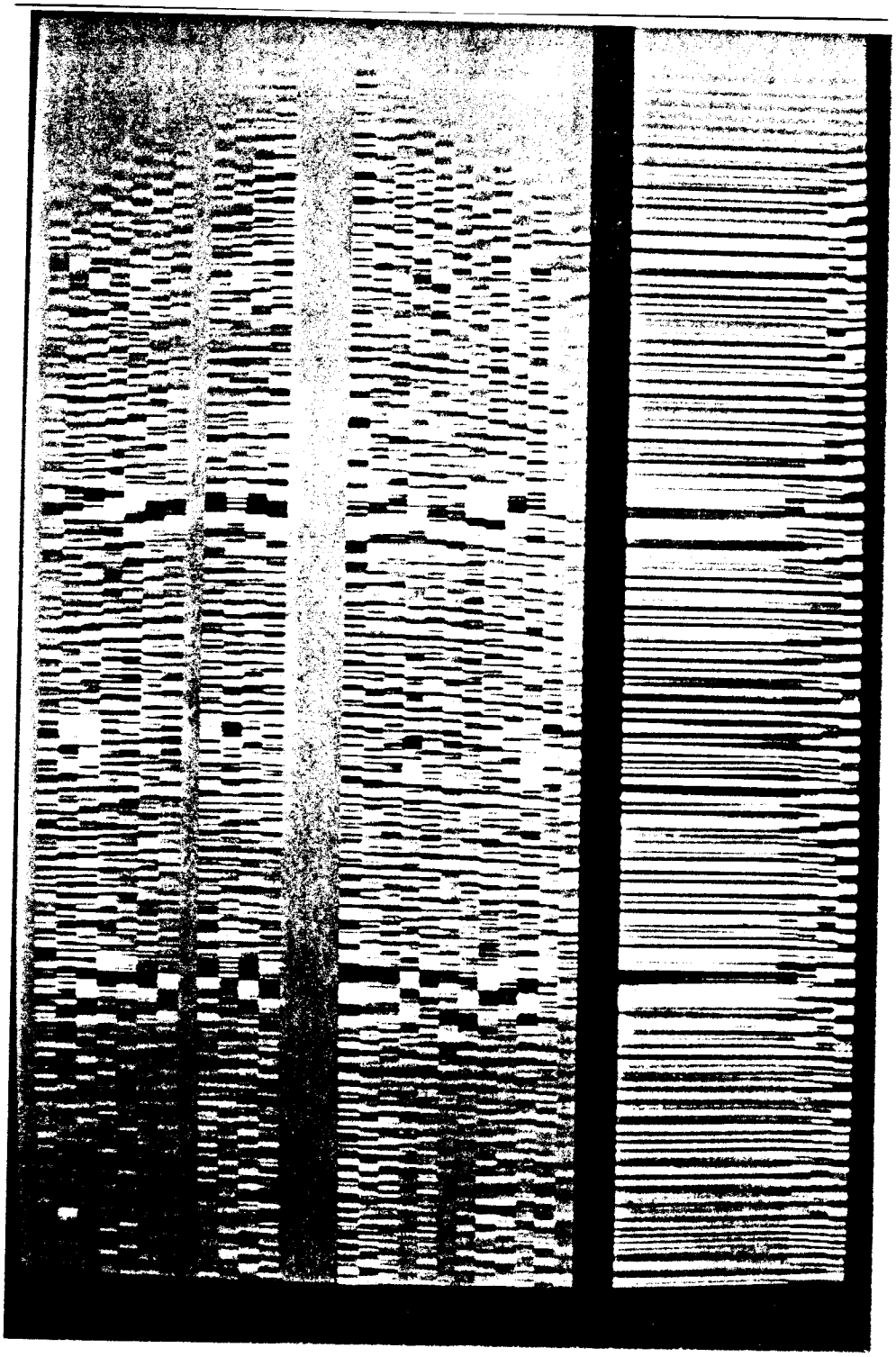


FIG. 15. Spiking deconvolution using overlapping 375 msec windows instead of $t^2$ gain. The banding occurs at the window boundaries. As can be seen from the iteration panel on the right, it becomes noticeable after three iterations.

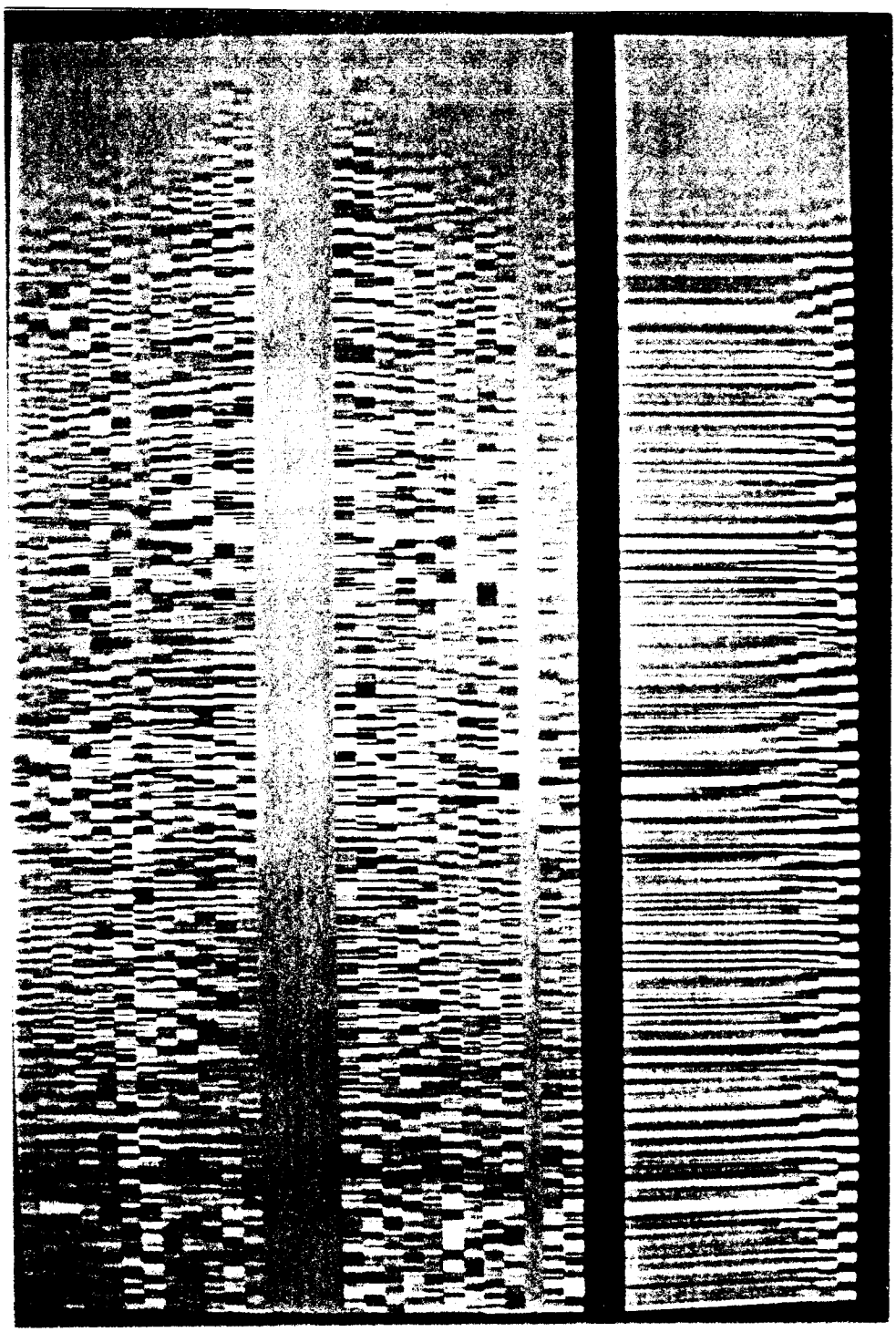


FIG. 16. Spiking deconvolution with overlapping 375 msec windows when a number of samples equal to the inverse filter length is dropped from the beginning of each deconvolved window before blending.

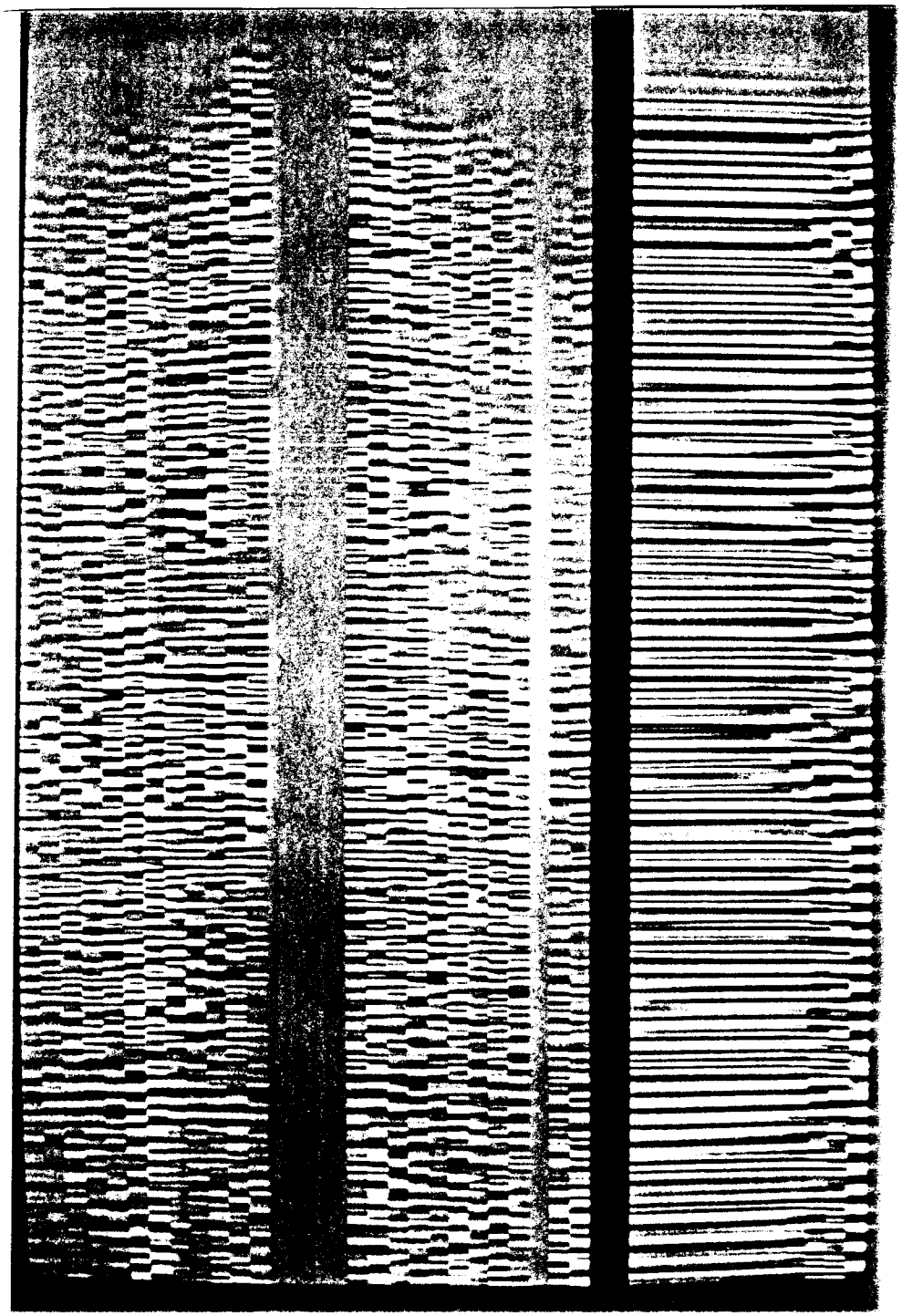


FIG. 17. Bandpass filtered version of Figure 12. The purpose of the filtering is of course to suppress high-frequency noise that was boosted by the deconvolution.

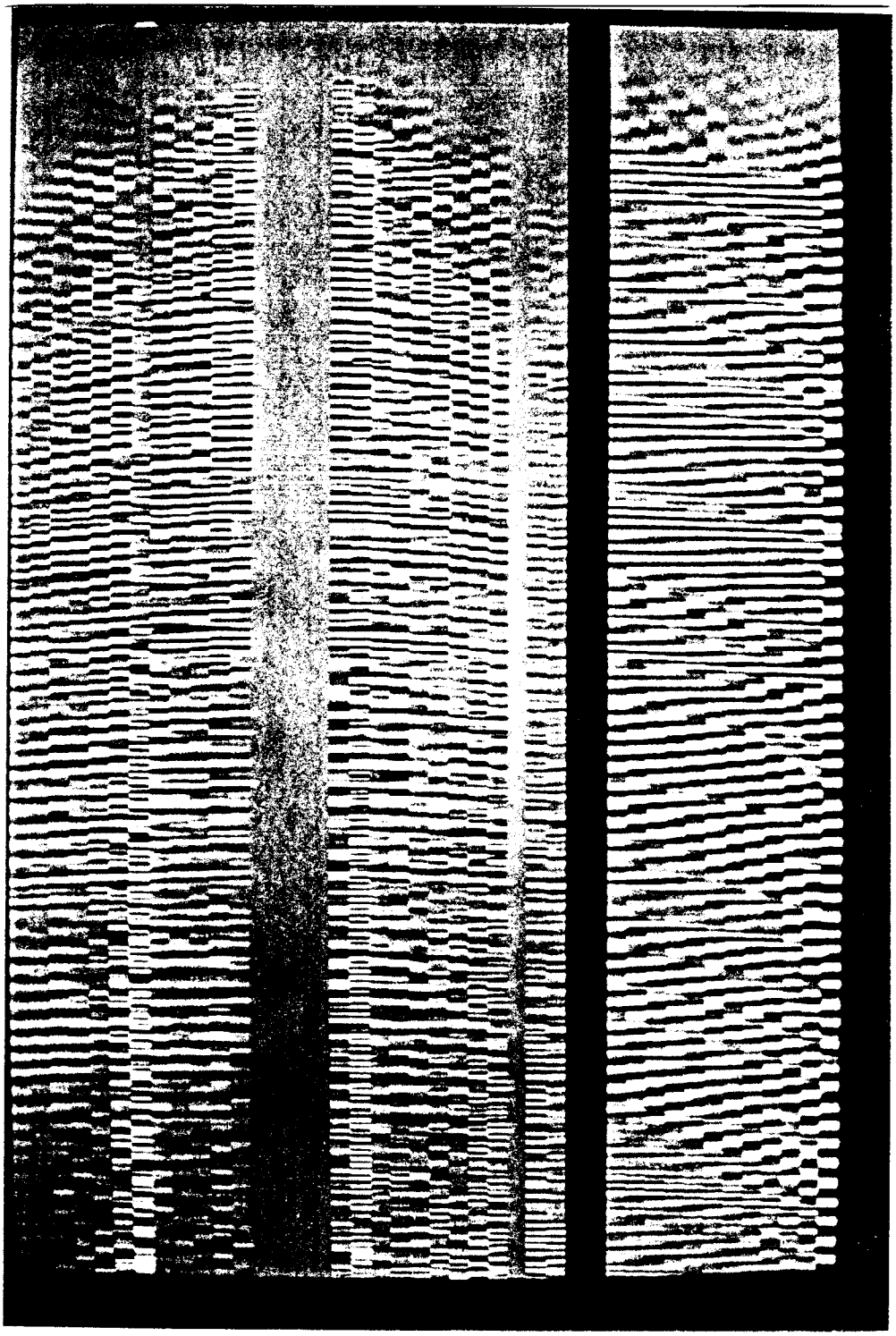


FIG. 18. 11th iteration of zero-phase bandpass filtered spiking deconvolution. The single trace panel clearly shows the problem with this scheme.