

VELOCITY STACK AND SLANT STACK INVERSION METHODS

A DISSERTATION

SUBMITTED TO THE DEPARTMENT OF GEOPHYSICS

AND THE COMMITTEE ON GRADUATE STUDIES

OF STANFORD UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

By

Jeffrey R. Thorson

May 1984

© Copyright 1984

by Jeffrey R. Thorson

printed as Stanford Exploration Project Report No. 39

by permission of the author

Copying for all internal purposes of sponsors
of the Stanford Exploration Project is permitted.

Velocity Stack and Slant Stack Inversion Methods

Jeffrey R. Thorson
Stanford University, 1984

Abstract

An important step central to the analysis of reflection seismic data is NMO (normal moveout) and stacking. This process involves the summation of seismic data over paths represented by a family of hyperbolas. It is a linear transformation, essentially singular, and maps the data into what might be called a *velocity space*: a two-dimensional plane indexed by time and velocity.

The examination of the data in velocity space is useful for the analysis of subsurface velocities and the filtering out of undesired coherent events (multiples). But the filtering step can only be applied if an approximate inverse to the NMO and stack operation is available. Current standard processing makes the implicit assumption that the adjoint L^T (in which L^T is defined to be NMO and stacking) and its adjoint L are an inverse pair. Replacing L^T with a closer estimate of the inverse to L makes possible more accurate velocity filtering and a better discrimination between signal and noise in velocity space.

The task of designing an estimated inverse to L is a generalization of the inversion problem in computerized tomography: deconvolving out the point-spread function after back projection. The problem is complicated by missing data, for data can only be recorded within a limited spatial aperture on the earth's surface.

Two approaches are taken here to estimate the inverse to L . The first approach is to design the generalized inverse, in which the components of the data residing in the null space of L are constrained to be zero. We derive the

generalized inverse for the case of an arbitrary spatial aperture. As it happens, the generalized inverse for a finite-aperture problem is equivalent to the generalized inverse derived from the corresponding unrestricted-aperture case. No advantage is thus gained in using the finite-aperture filter in lieu of the less expensive unrestricted-aperture filter.

In the second approach, components of the data in the null space of L are allowed to be nonzero: they are to be constrained by a priori information on the solution in velocity space. Starting from a MAP (maximum a posteriori) estimator, a system of equations can be set up in which the a priori information is incorporated into a sparseness, or *parsimony*, measure: the output of the inverse operator should be locally focused to obtain the best possible resolution in velocity space. The size of the resulting nonlinear system of equations is immense, but a few iterations with a gradient descent algorithm has proven adequate to obtain a "good" solution. This so-called *stochastic inverse* is superior to the generalized inverse, with respect to the resolution of the resulting image in velocity space.

The theory developed here may be equally well applied to other large, sparse linear operators. The generalized inverse and stochastic inverse of the *slant stack* operator (a special case of the Radon transform), developed in a parallel manner, have very similar behavior to the corresponding estimated inverses of the NMO and stack operator.

Approved for publication:

By _____
For Major Department

By _____
Dean of Graduate Studies & Research

Acknowledgments

My thanks go to the sponsors of the Stanford Exploration Project for their support during my stay at Stanford, and to the sponsoring companies' representatives who, at the annual Tahoe meetings, were a source of ideas. Among the sponsors, Western Geophysical, Amoco and Arco generously contributed data that was used in this thesis.

Talks with Dave Hale on the subject of velocity stacking were of great value. Bert Jacobs provided some key ideas on MAP estimation and variance as a random variable. Some of the others who have had an influence, through discussion, were Rob Clayton, Mat Yedlin, Larry Morley, Shuki Ronen, Bill Harlan and Paul Fowler.

Rick Ottolini and Dave Hale wrote most of the software I used to plot the illustrations. My thanks to Fanny Toldi for her editing work, and to Pat Bartz for typing.

Finally I wish to extend my thanks to my advisor, Jon Claerbout, as a main source of inspiration at the early stages of my work, and a source of patience at the end.

Table of Contents

Abstract	iii
Acknowledgments	v
List of Figures	viii
List of Tables	ix
Chapter I: Invertibility of Velocity Stacks	
1.1 Preface	1
1.2 Velocity stacks	1
1.3 Velocity space	3
1.4 Designing a velocity transform	5
1.5 Limitations	11
1.6 Overview of the remaining chapters	14
1.7 Motivation	15
Chapter II: The Influence of Missing Data on Inverse Filtering	
2.1 Missing data	17
2.2 Examples of missing data restoration	18
2.3 Stacking: velocity stacks and slant stacks	20
2.4 Characterizing missing data by projections	25
2.5 Adjoint operators	28
2.6 Two inverse problems	31
2.7 Pseudoinverses	35
2.8 Summary	41
Chapter III: Slant Stack Generalized Inverses	
3.1 Introduction	43
3.2 Slant stacking	43
3.3 Calculating the slant stack pseudoinverse	45
3.4 Equivalence of the finite offset pseudoinverse and the rho filter	52
3.5 Illustrations of pseudoinverse filtering	56
3.6 Summary	58
Chapter IV: Velocity Stack Generalized Inverses	
4.1 The problem	59
4.2 The impulse response of $L^T P_d L$	61
4.3 The generalized inverse in the p^2, τ^2 domain	63
4.4 Infinite-aperture versus finite-aperture filters	66
4.5 Synthetic and real data examples	69
4.6 Summary: use of the generalized inverse	73
4.A Appendix: derivation of $(L^T P_d L)^+$	74
Chapter V: Velocity Stack Stochastic Inversion	
5.1 Stochastic inverses	77
5.2 Parsimonious inversion and MAP estimation	79
5.3 Parsimony criteria	82

5.4	The multidimensional parsimony functional	91
5.5	Scale invariance and entropy	94
5.6	Gradient descent algorithms	98
5.7	Synthetic data inversion	105
5.8	Real data inversion: Amoco Grand Banks	108
5.9	Real data inversion: Western peg-legs	115
5.10	Real data inversion: Western surface waves	119
5.11	Summary: conditions favorable to inverse velocity stacking	128
5.A	Appendix: equivalence of spectral entropy to statistical entropy	128

Chapter VI: Slant Stack Stochastic Inversion

6.1	Introduction	131
6.2	Stochastic inversion on a vertical seismic profile	132

References	144
------------------	-----

List of Figures

1.1	Velocity stacking	4
1.2	Comparison of the pseudoinverse and stochastic inverse	7
1.3	Comparison of the stochastic inverse and the velocity stack	9
1.4	Reconstructing the original data	12
1.5	Velocity analyses	13
1.6	A comparison of various data reconstructions	14
2.1	Two data sets	21
2.2	Sparseness	34
3.1	Slant stack impulse response $L^T P_d L$	48
3.2	Responses of $L^T P_d L$ and $(L^T P_d L)^+$ to a sinc function	51
3.3	The resolving kernels of the pseudoinverse	57
4.1	Velocity stacking impulse responses	64
4.2	Velocity stack of a synthetic gather	70
4.3	Generalized inverses of synthetic data	71
4.4	The generalized inverse applied to a common-midpoint gather	72
5.1	Parsimony gradient for the continuous-variance case	87
5.2	Parsimony gradient for the discrete-variance case	89
5.3	Maximum entropy versus orthodox statistics	97
5.4	Hypothetical behavior of the norm of $Q_k A Q_k$	105
5.5	Inversion on a synthetic noise-free model	107
5.6	Inversion on a synthetic noisy model	108
5.7	Amoco Grand Banks: data d and Lu	109
5.8	Amoco Grand Banks: u and $L^T d$	110
5.9	Amoco Grand Banks: comparison of data with residual	111
5.10	Amoco Grand Banks: ray tracing model	112
5.11	Amoco Grand Banks: the two possible peg-leg paths	113
5.12	Amoco Grand Banks: synthetic gather	114
5.13	Amoco Grand Banks: velocity stacking function	115
5.14	Amoco Grand Banks: stack of the velocity panels u	116
5.15	Amoco Grand Banks: uniformly weighted stack	117
5.16	Western peg-legs: interpolated shot gather	120
5.17	Western peg-legs: separation of peg-leg paths by velocity	121
5.18	Western peg-legs: high velocity peg-leg paths	122
5.19	Western peg-legs: filtered shot gather	123
5.20	Western surface waves: shot gather d	124
5.21	Western surface waves: comparison of $L^T d$ and u	125
5.22	Western surface waves: velocity stack Lu	126
5.23	Western surface waves: residual $d - Lu$	127
6.1	An Arco vertical seismic profile	133
6.2	Slant stack of a window of the VSP	135
6.3	Aliasing artifacts on a synthetic VSP	136
6.4	Stochastic inverse of the synthetic slant stack	137
6.5	Stochastic inverse of the Arco slant stacked VSP	138
6.6	Rho-filtered slant stack	139
6.7	Restoration of one window of the VSP	140

6.8	Stochastic inverses of five VSP windows	141
6.9	An extrapolated VSP	142

List of Tables

3.1	Slant stack filters	53
4.1	Velocity stack filters	67
5.1	Projected gradient descent algorithm	102