VELOCITY STACK AND SLANT STACK INVERSION METHODS

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Abstract

An important step central to the analysis of reflection seismic data is NMO (normal moveout) and stacking. This process involves the summation of seismic data over paths represented by a family of hyperbolas. It is a linear transformation, essentially singular, and maps the data into what might be called a *velocity space*: a two-dimensional plane indexed by time and velocity.

The examination of the data in velocity space is useful for the analysis of subsurface velocities and the filtering out of undesired coherent events (multiples). But the filtering step can only be applied if an approximate inverse to the NMO and stack operation is available. Current standard processing makes the implicit assumption that the adjoint $\mathbf{L}^T$ (in which $\mathbf{L}^T$ is defined to be NMO and stacking) and its adjoint **L** are an inverse pair. Replacing $\mathbf{L}^T$ with a closer estimate of the inverse to **L** makes possible more accurate velocity filtering and a better discrimination between signal and noise in velocity space.

The task of designing an estimated inverse to **L** is a generalization of the inversion problem in computerized tomography: deconvolving out the point-spread function after back projection. The problem is complicated by missing data, for data can only be recorded within a limited spatial aperture on the earth's surface.

Two approaches are taken here to estimate the inverse to **L**. The first approach is to design the generalized inverse, in which the components of the data residing in the null space of **L** are constrained to be zero. We derive the

generalized inverse for the case of an arbitrary spatial aperture. As it happens, the generalized inverse for a finite-aperture problem is equivalent to the generalized inverse derived from the corresponding unrestricted-aperture case. No advantage is thus gained in using the finite-aperture filter in lieu of the less expensive unrestricted-aperture filter.

In the second approach, components of the data in the null space of L are allowed to be nonzero: they are to be constrained by a priori information on the solution in velocity space. Starting from a MAP (maximum a posteriori) estimator, a system of equations can be set up in which the a priori information is incorporated into a sparseness, or *parsimony*, measure: the output of the inverse operator should be locally focused to obtain the best possible resolution in velocity space. The size of the resulting nonlinear system of equations is immense, but a few iterations with a gradient descent algorithm has proven adequate to obtain a "good" solution. This so-called *stochastic inverse* is superior to the generalized inverse, with respect to the resolution of the resulting image in velocity space.

The theory developed here may be equally well applied to other large, sparse linear operators. The generalized inverse and stochastic inverse of the *slant stack* operator (a special case of the Radon transform), developed in a parallel manner, have very similar behavior to the corresponding estimated inverses of the NMO and stack operator.

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