Tpow: an estimator of seismic amplitude decay

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Introduction

Seismic data are generally non-stationary; the clearest evidence is the decay of envelope amplitudes with time. Spectral downshift due to greater attenuation of high frequencies may also be observed concomitant with the amplitude decay. For many purposes this nonstationarity simply is compensated by application of a time varying gain. However, the nonstationarity might contain meaningful information if we knew how to extract useful parameters describing it. Simple spherical divergence in a homogeneous medium predicts an amplitude decay factor inversely proportional to the distance traveled. Other important factors which may affect the observed amplitude decay include intrinsic attenuation and the distribution and strength of reflectors. It is a common practice to gain data by te^{-kt}, where k is an empirically derived constant appropriate to the particular data. The first factor is supposed to allow for divergence and the second for attenuation. An exponential gain function, however, is often really suitable only over a limited time window, and must be tapered or truncated. At SEP we often use a gain function which is just a power of time. We have often found that t^2 provides a good rule of thumb for a gain function to make many data appear much more uniform in time. In this paper we will present a simple argument for the reasonableness of this particular gain function, examine some of the predictions of the theory, and discuss some empirical tests designed to determine how well the theory fits field data.

Simple theories of attenuation predict an exponential decay in amplitude with time for a monochromatic source signal. In practice, the decay of a realistic source signal is more difficult to predict. O'Doherty and Anstey (1971) correctly point out that knowledge of the source spectrum is necessary to predicting the decay law, and suggest that $t^{-3/2}$ is a realistic estimate. Several papers (O'Doherty and Anstey, 1971; Schoenberger and Levin, 1974,1978; Spencer, et al., 1982) discuss the relative importance of intrinsic attenuation

as opposed to scattering or multiple path effects, which can also cause apparent attenuation of the recorded signal. One might like to separate these effects, but in practice it may be necessary to lump them together into an effective attenuation. We will assume for the moment that the observed attenuation and dispersion may be described reasonably well by assigning an effective value to the quality factor Q, whatever the physical mechanism behind it may be.

A constant Q theory for envelope amplitude decay

We base our model of dissipative attenuation on the constant Q theory of Kjartansson (1979). From his constant Q model he derives an equation describing one dimensional wave propagation for a given frequency ω:

$$U(t,x) = e^{-\alpha x} e^{i\omega\left(t-\frac{x}{c}\right)}$$

Here α depends on both Q and $|\omega|$. The phase velocity parameter c also varies with ω, but only very weakly, and we shall treat it as if it were a constant, ignoring dispersion. Thus when we consider envelopes, we can expect for a given frequency component a decay law of

$$U(t,x) = e^{-\alpha(\omega)x}$$

The specific form Kjartansson gives for α is

$$\alpha = \frac{|\omega|}{c}\tan\left(\frac{\pi\gamma}{2}\right)$$

where

$$\gamma = \frac{1}{\pi}\tan^{-1}\left(\frac{1}{Q}\right) \approx \frac{1}{\pi Q}$$

If we assume a constant velocity medium, so that $x = vt$, and we isolate the factor of $|\omega|$ from the expression for α we get the simple relation

$$U(t,\omega) = e^{-\beta|\omega|t} \tag{1}$$

where β is a positive constant approximately equal to $1/2Q$. The amplitude decay of the signal envelope will thus be exponential in both time t and frequency ω. Curves of constant amplitude will be hyperbolas in a $t-\omega$ plot. (See figure 1).

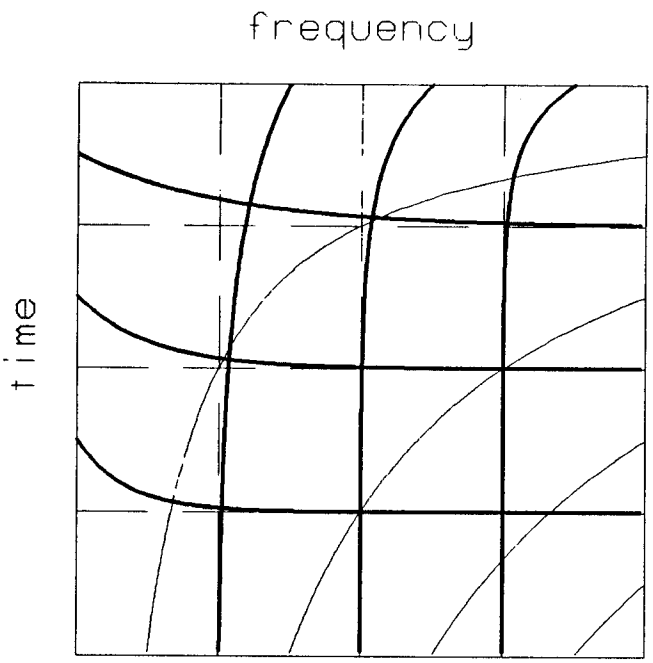

FIG. 1. Schematic plot depicting exponential decay of amplitude in both frequency and time. Curves of constant amplitude are indicated by hyperbolas.

Suppose we now assume an impulsive (white) source function. Since we are ignoring dispersion, we can assume that all phases remain aligned. Even if this assumption is not strictly true, we will be basing our decay envelope essentially on the largest peaks of the time signal, which will come when most frequency components are roughly phase aligned. Assuming then that all frequencies add in phase, we can find the peak amplitude at time t by integrating over all frequencies:

$$U(t) = \int_0^\infty d\omega e^{-\beta\omega t}$$

$$= \frac{1}{\beta t} \tag{2}$$

This, then provides us with a simple justification for our t^2 rule of thumb for gaining data: one power of t for spherical divergence, and one more for attenuation. Note that this simple rule predicts that attenuation will be of the same form independent of the value of Q; Q enters only as a scaling factor.

So far this model has been for one dimensional wave propagation in a homogeneous medium. We have assumed that both velocity and Q are constants independent of location. We also ignore variations with offset, treating all data as one dimensional. The extension to

data from an earth with reflectors requires further assumptions. If reflection losses at interfaces are small enough, the homogeneous model should still work reasonably well for the primary down going field measured in a vertical seismic profile. To extend our results to data recorded at the surface, we consider the upcoming wave field as if it were uniformly sampling the downgoing field without significantly weakening or perturbing it. We treat the variations in reflection coefficients as introducing only a modulation on the general decay trend in which we are interested. We thus assume that primary reflections dominate the data and that the reflection coefficients are both weak and fairly uniform, and that their distribution is stationary with depth. These assumptions are strong ones; for this reason when we fit data to a decay law we will want to be careful that we are really estimating the overall decay of the wave field with time and not primarily the behavior of the reflection sequence.

The Tpow algorithm

To estimate the actual decay which is observed in reflection profiles an algorithm is needed to fit a decay law to seismic data. Our goal is to find that power law of time t which best fits a grossly smoothed envelope of the data. To estimate this envelope, the data is divided into equal length bins down the time axis, and a specified quantile level of the absolute value of the data within each of these bins is extracted. A rapid algorithm for finding such a quantile is given by Canales (1976). This procedure gives a suitably small set of points through which to fit a curve.

A variety of methods are available to fit a power of t curve through the selected envelope points. Least squares is probably the most commonly used data fitting tool, but we choose to use a method which we believe is more robust in the presence of noise contamination, and which pays less attention to missing data and isolated high data values (strong reflectors). We select the appropriate power of t, which we call $tpow$, by minimizing the objective function

$$f(tpow) = \frac{\frac{1}{n}\sum_{i=1}^{n}\left[\,|\,data(t_i)\,|\,t_i^{tpow}\right]^{\gamma}}{\left[\frac{1}{n}\sum_{i=1}^{n}\,|\,data(t_i)\,|\,t_i^{tpow}\right]^{\gamma}}$$

The reader may recognize this as the ratio of the L_γ norm over the L_1 norm. Minimizing it is equivalent to finding the constrained weights which scale the data points so as to approximate a uniform distribution. Note that the objective function is independent of scale in both time and data amplitude. For a discussion of how such objective functions may be derived from Jensen inequalities see Claerbout (1983). Gray also used a similar approach to

estimating decay coefficients, although he assumed an exponentional decay law (Gray, 1978).

Several parameters in this algorithm remain to be specified. The choice of number of bins is pragmatic: we need to balance the desire to have more points for better curve fitting against the need to keep a sufficient number of points in each bin to ensure reasonable statistics. In practice, we found that 20 time bins is usually a good compromise. Choosing the quantile level to use is a bit more sensitive to data variability than might be desired, but for consistency a fixed level needs to be used. If the data were stationary except for a scaling with time, and if our tpow law correctly described this decay, it would make no difference what quantile level we use, since the form of the cumulative distribution function would not change with time. Unfortunately, this is not really true for many data. One major problem with applying a binning and quantization method to seismic data is the presence of many zeroes, due to missing traces, mutes, and the lack of data at far offsets at early times. We want the method to be as insensitive as possible to these zeroes, and as nearly representative of the envelope peaks as possible. As a compromise among these factors we used the 95th quantile level throughout; the effects of this choice might be worthy of further analysis. The last parameter to be chosen is the particular norm ratio to use for defining the objective function. We find the result to be relatively insensitive to the specific choice of γ; we use a value of γ=1.3. Tests of other values for γ showed a variation in the calculated tpow value of less than 5% using any γ between 1.1 and 3.0. Using γ=2 and squaring the objective function is probably the most computationally efficient, if speed is a primary concern.

The application of our algorithm is illustrated on the field profile in figure 2a. These data were collected in Alberta using a dynamite source. The algorithm calculated a best fitting tpow value of 2.332. The representative time bin points used in the calculation and the same bin points after scaling are shown in figures 3a and 3b. These figures illustrate how the scaling drives the points toward a uniform distribution. It also shows up one potential problem mentioned above: the first time bin point is very small, due to the large number of zeroes at outer offsets at early times. Note, however, that the scaling is relatively insensitive to a single outlier such as this. For comparison, the same calculation repeated using only the data after 0.8 seconds gives a tpow value of 2.314, a difference of less than 1%; excluding the early times with their excess of zeroes thus makes little difference. Inserting a zero trace for the missing zero offset record hardly affects the result either, yielding a value of 2.327.

The data of figure 2a may be seen to contain ground roll as well as the reflected energy we wish to study. To check whether this ground roll was significantly biasing our

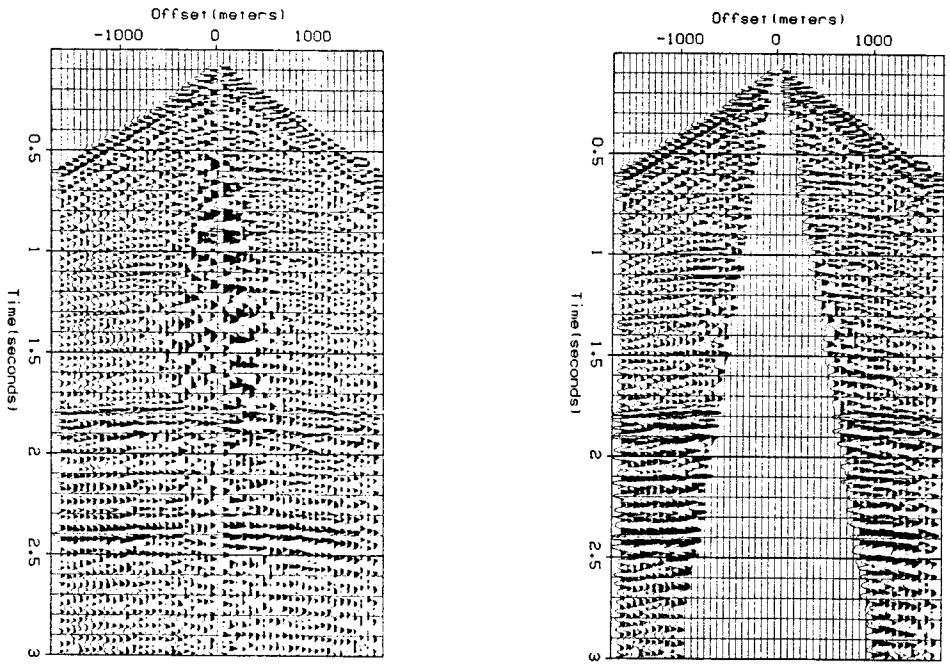

FIG. 2. a) Field profile from Canada (courtesy of Western Geophysics). The energy source is dynamite. A display gain of t^2 has been applied to the data. b) Same profile as 2a with ground roll mute

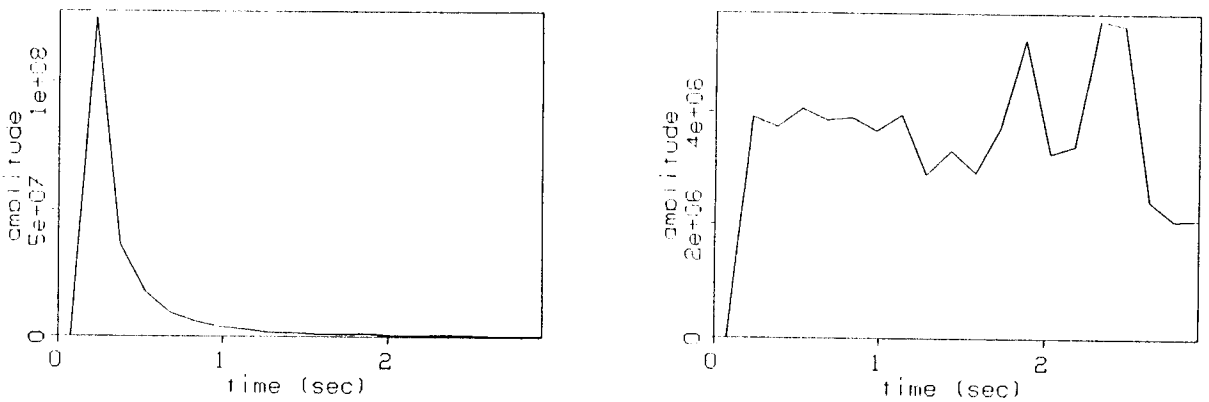

FIG. 3. a) 95th quantile points from 20 time bins for the data of figure 2a b) Time bin points after correction by $t^{2.306}$ gain factor

decay estimate, it was muted out, first with the radial mute shown in figure 23, and second by calculating tpow only for the outermost 12 traces on each side. The radial muting changed the calculated tpow value to 2.375, and the outer trace calculation (using only data after 0.4 seconds, since it is all zeros before then) gives a value of 2.289. We thus conclude that the ground roll is having little effect on the tpow calculation for this data set.

Tpow calculated for field profiles

We calculated tpows for a selection of 36 field profiles from around the world, 20 recorded on land and 16 at sea. The results are summarized in the table below.

Field Profile Tpows				
	entire record		1 to 3 seconds	
record group	mean	σ	mean	σ
all records(36)	2.09	0.73	2.23	0.87
marine(16)	2.50	0.47	2.57	0.83
land(20)	1.78	0.75	1.95	0.81
explosive(8)	2.04	0.71	2.22	0.69
Vibroseis(12)	1.61	0.76	1.77	0.86

These results generally support our t^2 expectation, although there is a large spread in the values obtained. The first two columns of numbers given are for tpow values calculated from the entire length of each record. The last two columns represent the same calculations run using only the data between one and three seconds. This window was chosen to encompass the data of highest visual quality and to minimize bias from excessive zeroes at early times or from high background noise levels at late times. Using the late times in the calculation includes more data with decreased signal to noise ratio. If one assumes a decaying signal and a constant background noise level, using data from late times can be expected to introduce a bias toward slower decay (lower tpow value). This effect probably accounts for the general increase in calculated tpow values for the windowed data relative to the unwindowed.

The effect of a water layer

One consistent observation from this data is that the marine records yield tpow values that are usually greater than two, and are generally higher than the land values. This is not just an artifact of consistent biasing by strong seafloor reflections at early times; the time windowed values exclude the early events, but show the same discrepancy. We might expect that water bottom multiples in marine records would cause calculated tpow values close to one, since the paths these multiples traveled were almost entirely through water, not rock. Such an effect can indeed be found if one looks for cases of extreme multiple contamination. The profile in figure 4 (not one of those included in the statistics above) is a Barents Sea record in which essentially nothing besides sea floor multiples can be seen, and indeed the tpow value calculated is 1.09. This case is an exception, however, which had to be sought out deliberately. The other records considered do not appear to be strongly biased by multiples; indeed the tpow values come out higher than 2, rather than lower.

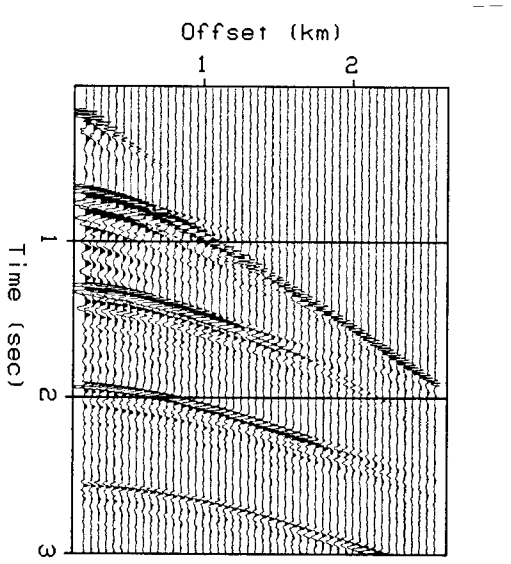

FIG. 4. Field profile from the Barents sea (courtesy of GECO). The computed tpow is nearly one, since water bottom multiples completely dominate the data. Hence there is little or no attenuation and only spherical divergence is seen. Data is plotted with no gain.

We can extend our model quite simply to show how the presence of an overlying water layer will generally raise the computed tpow value. Our assumption of a single homogeneous medium is obviously violated by the presence of two media as dissimilar as water and rock. Let us consider a model comprising two layers, with the upper layer having thickness d_1,

velocity v_1 and quality factor Q_1, and the lower layer being a halfspace with velocity v_2 and quality factor Q_2. Newman (1973) derives the appropriate divergence factor to use in a stratified medium at near zero offset. The divergence correction factor for two way passage through an n layer medium is

$$D_0 = \frac{t\bar{v}^2}{v_1}$$

where $\bar{v}$ is the time weighted rms velocity

$$\bar{v}^2 = \sum_{i=1}^{n} \frac{t_i v_i^2}{t}$$

and t_i is the traveltime through the ith layer. In the case of just two layers this simplifies to

$$D_0 = v_1 t_1 + \frac{v_2^2}{v_1}(t - t_1)$$

From equation (1), we know that at time t_1, each frequency component has decayed by

$$e^{-\frac{\omega t_1}{2Q_1}}$$

If we assume that we started with an impulsive source, then we can treat the attenuated impulse as a new source at time t_1 and find the decay at time $t > t_1$ from evaluating

$$\int_0^\infty d\omega\, e^{-\frac{\omega t_1}{2Q_1}} e^{-\frac{\omega t}{2Q_2}} = \int_0^\infty d\omega\, e^{-\omega\left[\frac{t_1}{2Q_1} + \frac{(t-t_1)}{2Q_2}\right]}$$

$$= \frac{1}{\frac{t_1}{2Q_1} + \frac{(t-t_1)}{2Q_2}}$$

$$= \frac{2Q_1Q_2}{Q_2 t_1 + Q_1(t-t_1)}$$

Hence the correction to be applied to account for attenuation is

$$\frac{Q_1(t-t_1)+Q_2 t_1}{2Q_1Q_2} = \frac{1}{2Q_2}(t-t_1) + \frac{t_1}{2Q_1}$$

In the case of an upper water layer, we can set $Q_1 \approx \infty$ (no attenuation), and the correction for attenuation becomes $\frac{(t-t_1)}{2Q_2}$; that is, a shift of the time origin. This result must be

multiplied by a factor of two if two way travel times are used. When the divergence and attenuation corrections are combined, the total amplitude decay correction factor for times $t > t_1$ is

$$\frac{(t-t_1)}{Q_2}\left[v_1 t_1 + \frac{v_2^2}{v_1}(t-t_1)\right]$$

which can be rearranged as

$$\frac{v_1}{Q_2 v_2^2}(t-t_1)(t-kt_1) \tag{3}$$

where

$$k = 1 - \frac{v_1^2}{v_2^2}$$

For water above rock, k will be between zero and one, so the expected decay correction will be between $t(t-t_1)$ and $(t-t_1)^2$. If the time shift of the origin is not accounted for, and only the single parameter tpow is estimated, the calculated tpow value will be biased upward from what it would be if the water were rock.

The two way travel time depth of the sea floor is between 0.1 and 0.4 seconds for all but one of the marine profiles considered in this study. We can make a rough estimate of the time depth of the sea floor from the marine shot profiles considered in the average above and try to fit such a shifted tpow function. If we set v_1=1500 meters per second and v_2=2500 meters per second, k will be approximately 0.65. Using this value for k, we can modify the tpow algorithm to estimate a value of t_1 that will best fit equation (3) to the data. For a time window of one to three seconds, the mean value of t_1 calculated for the 16 marine profiles is 0.17 seconds. The actual water depths cannot be estimated well enough from the individual profiles to determine whether there is any meaningful correlation between the t_1 values we calculate and the water depth.

Bandlimited sources

Real data does not always have a white source, of course; Vibroseis, for example, uses an explicitly bandlimited signal. For a spectrum which is white between bandlimits A and B, the decay integral becomes

$$U(t) = \int_A^B d\omega e^{-\beta\omega t}$$

$$= \frac{1}{\beta t}\left(e^{-A\beta t} - e^{-B\beta t}\right)$$

$$\approx \frac{1}{\beta t} e^{-A\beta t}$$

So for a source which is deficient in low frequencies, the decay rate should include an exponential factor as well as a power law. If we try to force a power of time to fit such bandlimited data, the exponential factor can be expected to make the time decay appear to be a higher power than 2. Tests on synthetic data with bandlimited white spectra suggest that this effect is inconsequential for Q much greater than 100 or for a low frequency cutoff below 4 Hertz. For 8 to 60 Hertz bandlimited data and Q between 10 and 100, the resulting tpow is increased by up to 25%. Moving the low cutoff up to 16 Hertz with Q=100 results in an increase in the calculated tpow of up to 50%; the exponential term is coming to dominate and the tpow law is no longer applicable. In the field profiles examined, the average tpow calculated for Vibroseis data was lower, not higher, than that for explosive sources. There is no reason for assuming any consistent difference between the groups other than source type. The Vibroseis signal is not really a bandlimited white source, so perhaps the details of the source spectra account for the lower values observed.

Spectral non-stationarity

Shifts in the spectral content of the signal are closely related to the decay of the envelope with time. We expect a broadening of pulse shape during transit through rock layers, and hence a preferential weighting toward low frequencies with increasing time. Gaining up the later times with respect to the early should cause a shift toward lower frequencies, and conversely, lowpass filtering should decrease the apparent decay rate of the signal since low frequencies decay slower than high. Given the simplifying assumptions we have been making, we can make some simple quantitative predictions about the relation between amplitude decay and spectral changes. The differentiation and integration operators give us high and low pass filters whose effects in the frequency domain are particularly easy to understand, as they simply correspond to phase shift and linear weighting of frequency (multiplication by $\pm i\omega$). If we treat differentiation of the data as equivalent to differentiating the source, we modify the calculation from equation (2) to:

$$U(t) = \int_0^\infty d\omega \; \omega e^{-\beta\omega t}$$

More generally, fractional differentiation can be treated as frequency domain multiplication

by fractional powers of $i\omega$, so if we take a source function of $(i\omega)^\gamma$ for some value of gamma, we have

$$U(t) = \int_0^\infty d\omega\ \omega^\gamma e^{-\beta\omega t}$$

For $\gamma > -1$ this integral evaluates to

$$U(t) = \frac{\Gamma(\gamma+1)}{(\beta t)^{\gamma+1}}$$

Thus differentiating corresponds to increasing the decay by an additional power of t. This is a qualitatively reasonable result, since the more the signal energy is concentrated in the higher frequencies, the faster the signal as a whole can be expected to decay. The integral evaluated above must be accepted with some caution, however, since we are indicating a major preferential weighting of high frequencies which may be above our seismic bandwidth.

To test whether this relation between differentiation and tpow decay could be observed in our data, each of the data sets was differentiated zero, one half, one, three halves, and two times. Tpow values were calculated for each resulting data set, and a least squares slope was fitted to the these values. From the calculation above, we expect to get a slope of one; in fact the mean slope is very nearly zero (-0.009 if the whole data set is considered, -0.033 if only the 1 to 3 second time window is considered). No slopes larger than 0.35 in magnitude were found, and most were very close to zero. This suggests that the observed decay of high frequencies relative to lower ones is much less than our simple theory predicts.

To verify the small difference between decay rates at high and low frequencies, we decomposed each record into five bands between 10 and 60 Hertz, each ten Hertz wide, and calculated tpow values for each bandpassed record. We fitted least squares slopes to the five tpow values for the different frequency bands; again, the mean slope is nearly zero (0.0191 for the whole data, 0.123 for 1-3 second time windows). Few records show a clear monotonic trend with frequency. Because the bands considered were narrow, the decay will not be purely a power of time, but will have a substantial exponential contribution as well. The tpow algorithm can be modified readily to fit exponential decay trends instead of powers of time, and the same slope calculation can be made. The resulting exponential decay values shows little more tendency toward a consistent trend than do the tpow values; the mean slope of plots of exponential decay constant versus frequency is 0.018 if the entire record is used, and 0.048 if a 1-3 second window is applied.

Many of the fitted slopes discussed above are actually slightly negative, and for many of the data sets, the plots show much more fluctuation than trend. The simplest explanation is that we are not measuring signal, but noise. A more sophisticated approach might utilize cleaner data (stacks or VSP's) and a closer examination of changes in spectral ratios. Clearly, however, the simple relation between differentiation and tpow suggested above is not readily observed in our data.

A VSP example

Vertical seismic profiles (VSP) can give us a more direct look at wavefields in the earth than reflection profiles do. We might expect that the decay of a given event on a VSP would be much easier to analyze than one on a surface reflection record. In particular, the downgoing first break can be isolated and attenuation estimated from it. Figure 5a shows a portion of a VSP provided by Arco. The direct downgoing first arrival is readily identified, and from its constant slope we can see that our assumption of constant velocity is well satisfied. Moreover, there are no strong upgoing waves visible, so we can assume that the downgoing wavefield is not severely affected by reflection losses within the range of data we consider here. The data has not been deconvolved, so the waveform is extended more than we might like, but the good consistency of the waveform with depth makes it possible to window out the first full wavelength of the downgoing wave. This is shown in figure 5b. The tpow value calculated for this windowed event is not close to 2.0, as we might predict, but is only 1.31. It is worth noting that in this case the tpow value can be calculated from direct measurements of the peak amplitudes on each wavelet; it was reassuring to find that this calculation agreed to within a percent with the result of our binning approach. The data shown go only to a depth of 1075 feet. Data was actually collected down to 2150 feet, although a gradual increase in velocity is visible at greater depths. The tpow for the windowed first break of the whole data set is 1.46, still well below our predicted value.

The source for this data was Vibroseis, so this example is consistent with the lower than average tpow values found for the Vibroseis field profiles. We should be careful, however, of over-reaching the limitations of our very simple model. One indication that our model is too simple is given by more careful analysis of the decay trend of this VSP record; a modification of the tpow algorithm to find the best fitting combination of tpow and exponential decays to fit this data revealed that the decay is dominated by the exponential factor.

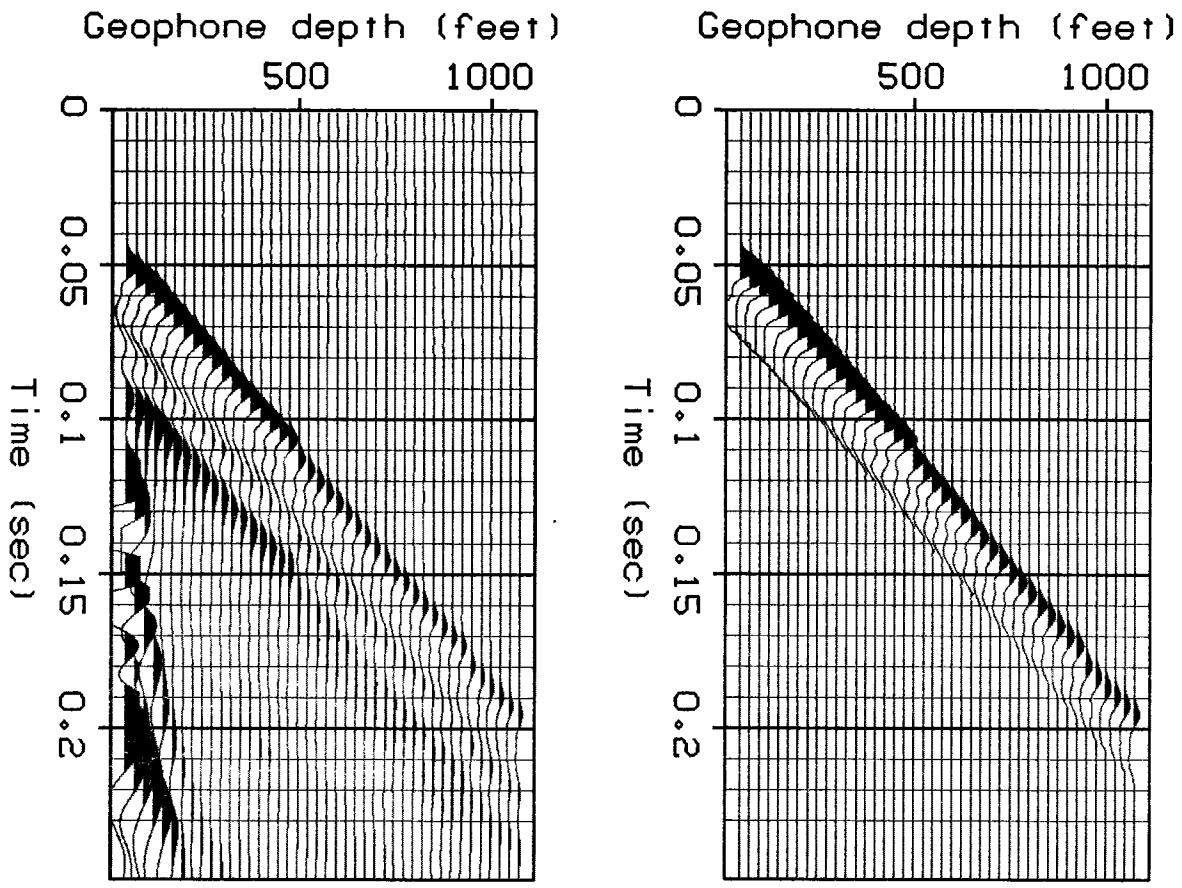

FIG. 5. a) Portion of a VSP record provided by Arco. b) Window of the direct downgoing arrival from figure 5a used in tpow calculation. Data is plotted with no gain.

Discussion and conclusions

Our prediction of a t^2 decay rate for seismic data generally fits field data quite well, considering the extreme simplicity of the model. Some simple extensions of the model, such as including the effects of a water layer, are also possible. However, the predictions about the amount of spectral change expected with time do not match what is observed. Some of the factors we have overlooked could substantially effect the decay rate. Real source spectra are not perfectly white, and Q is not everywhere a constant. Clearly, if velocity increases with depth, the tpow value calculated will be higher, since attenuation really depends on distance traveled, not travel time. Also, the reasonableness of ignoring reflections is open to question. Some previous work would suggest that the effect of short path multiples is at least as important as the intrinsic attenuation (O'Doherty and Anstey, 1971; Schoenberger and Levin, 1974, 1978). More careful consideration of the various factors

which can affect attenuation or otherwise make data non-stationary requires more information than we know how to extract from field profiles; other data such as VSP's, especially in combination with well logs, offer better hope of identifying and separating the major influencing factors.

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