Residual migration: two approaches

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Introduction

In SEP-35 Rothman, Levin and Rocca analyzed some practical aspects of residual migration. In that article finite differences were discussed but not implemented. Since then a residual finite difference migration has been written and applied to the same input data of the previous residual Kirchhoff examples. Additional examples of residual Kirchhoff migration have also been generated. The results of the two approaches are, as expected, quite similar with finite difference exhibiting slightly better amplitude control.

Algorithm design for residual finite differencing

Recall from SEP-35 that residual processing involved additional migration where the initial velocity was too low and modeling or "unmigration" where it was too high. Furthermore the residual dip of an event is decreased by the initial migration, allowing the use of a low angle migration algorithm for residual processing. In these examples we chose to use the 15° implicit time domain algorithm as the basis. This choice also simplified switching between migration and modeling - the migration equation, used to step in τ,

$$P_{t\tau} = -\frac{v^2}{8} P_{xx} \tag{1}$$

is simply the time reverse of the corresponding modeling equation

$$P_{t\tau} = +\frac{v^2}{8} P_{xx} \quad . \tag{2}$$

This permitted use of the same implicit tridiagonal marching scheme to handle both cases - only the direction of time recursion changes. Higher accuracy approximations require separate fitting to the asymmetric dispersion relations

$$k_\tau = \sqrt{\omega^2 - v^2(\tau)\, k_x^2} - \omega \tag{3}$$

and

$$k_\tau = \sqrt{\omega^2 + v^2(\tau)\, k_x^2} - \omega \tag{4}$$

respectively. We note that all velocities here are the *interval* velocities implied by appendix A of the above SEP-35 report.

The step size, $\Delta\tau$, was chosen, via formulas (14a,b) of the SEP-35 article, to insure that the phase variation across any τ step did not exceed one tenth of a cycle at the maximum residual velocity, dip, and frequency of interest. This allowed linear interpolation, at each τ step, between panels processed at the minimum and maximum residual interval velocities associated with that layer. Practically speaking, if we were to implement this within the framework of a preexisting finite difference program we would let the program choose its own step size but we would select the number of continuation velocities so as to allow the above linear interpolation criterion to hold.

One notable difference between this algorithm and ordinary finite difference migration is that computation does not stop when the diagonal image plane is reached but instead continued to the end of the grid, either t_o or t_{max} depending on the direction of recursion. If the input data were completely undermigrated this could be avoided at the cost of having to use a smaller extrapolation step. (Completely overmigrated would not work! Think about it.)

Examples

Figure 1a reproduces the unmigrated, stacked section of data from the Gulf of Mexico used in SEP-35. Trace spacing is 33m and sampling interval is 8 msec. True migration velocities range from 1650 m/sec to 2000 m/sec. The steepest dips are between 30 and 40 degrees. Figure 1b reproduces the SEP-35 result of full Kirchhoff migration of the unmigrated data of Figure 1a.

Figure 2b shows a finite difference residual migration of the undermigrated data of Figure 2a where Stolt migration with v_m = 1600 m/sec was performed. We assumed a maximum dip of interest of 30° and a maximum frequency of interest of three quarters Nyquist for choosing the step size. Our $\Delta\tau$ worked out to be approximately 5 times larger than would have been used if no initial migration were performed. Figure 2c reproduces the SEP-35 result for the analogous Kirchhoff residual migration. The results of Figures 1b, 2b, and 2c are virtually identical except for the different behavior at the side boundaries.

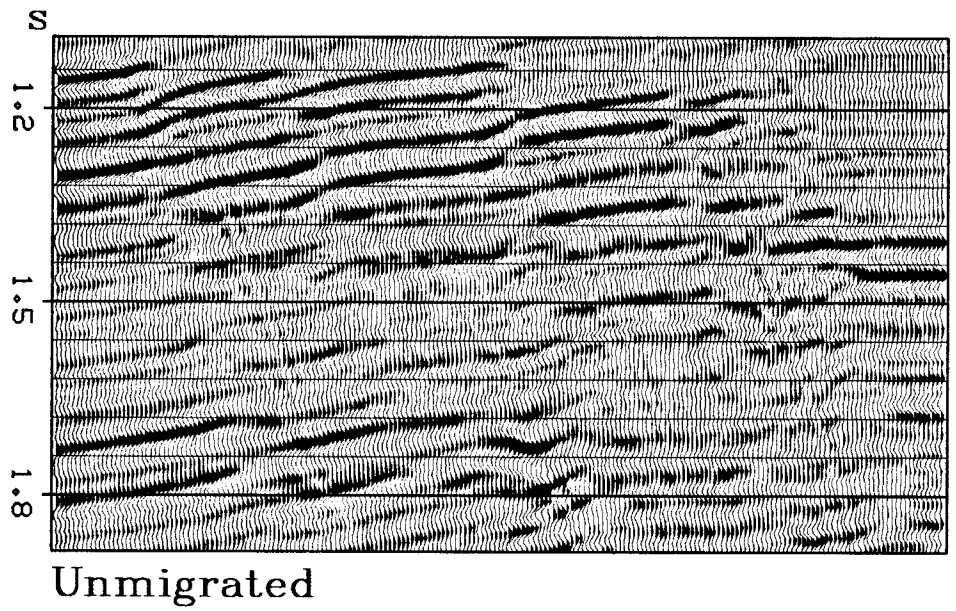
Figures 3a-c compare the results of finite difference and Kirchhoff residual migrations after an initial Stolt migration at an intermediate velocity of 1750 m/sec. The initial migration velocity was chosen so that the outer fault plane reflections are slightly overmigrated and the central fault reflection is somewhat undermigrated (probably because the reflections from this central growth fault are coming from outside the two-dimensional plane of the survey). There is no discernible difference between the finite difference results in Figures 3b and 2b. The Kirchhoff result in Figure 3c exhibits some slight amplitude inconsistencies in the zones where the residual migration velocity was very close to zero (this boundary between under- and overmigration falls approximately on the diagonal lines connecting the top center of the section with the side boundaries at 1.5 sec). The Kirchhoff algorithm was programmed to perform no operation on the data if the half-width of the residual migration aperture was less than $\Delta x/2$. Savings after migration with this intermediate velocity were substantial: the finite difference step size was larger by a factor of 8 and the Kirchhoff migration aperture was smaller, on average, by approximately the same factor.

Lastly, Figures 4a-c show finite difference and residual migrations of data initially overmigrated at 2050 m/sec. Here we see clearly how the residual migration has "unmigrated" overmigrated fault plane reflections. Computational savings for these residual migrations were about 65%, somewhat less than the corresponding 1600 m/sec (undermigrated) savings.

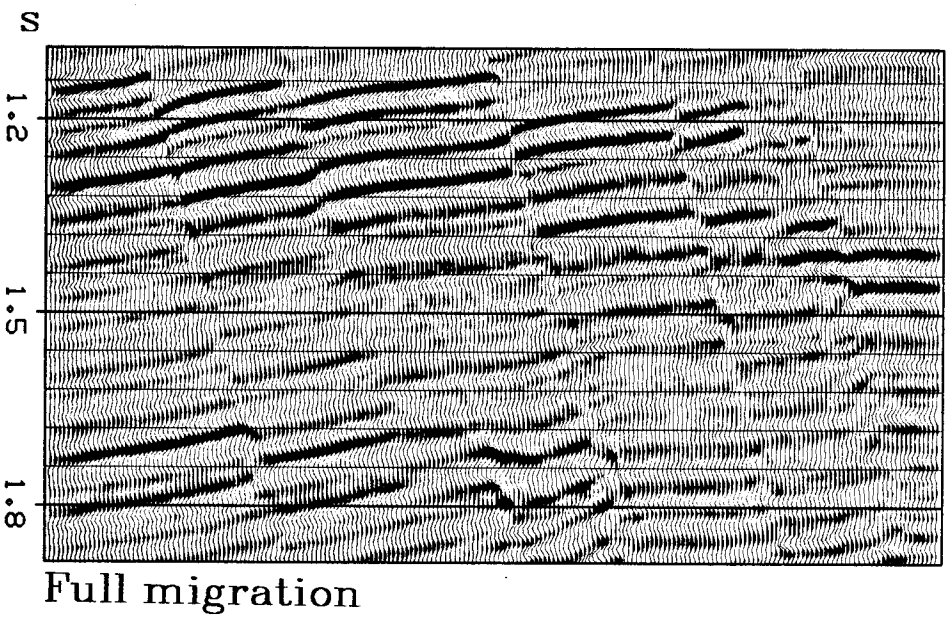
Comparison of the residual migrations in Figures 2-4 with the fully migrated data in Figure 1b shows virtually no difference between ordinary migration and the residual, two-step approach. Moreover, individual comparisons of the finite difference and Kirchhoff results show few disparities. The finite difference results generally provided somewhat better amplitude preservation, though the migrated fault plane reflections are slightly less sharp using this approach.

REFERENCES

Rothman, Levin, and Rocca, 1983, Residual migration: SEP-35 p.153-173.

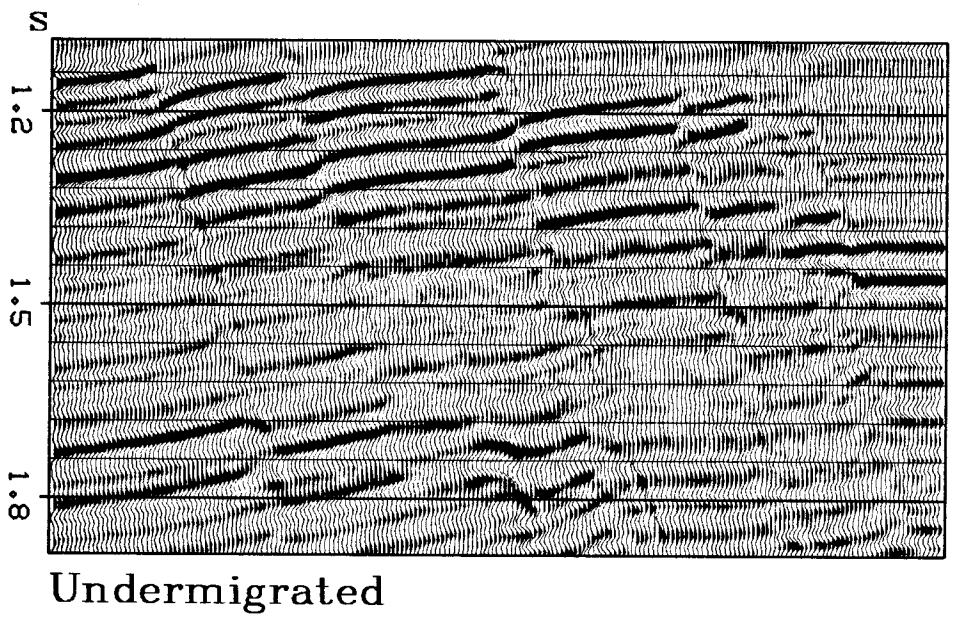


(1a - Unmigrated)

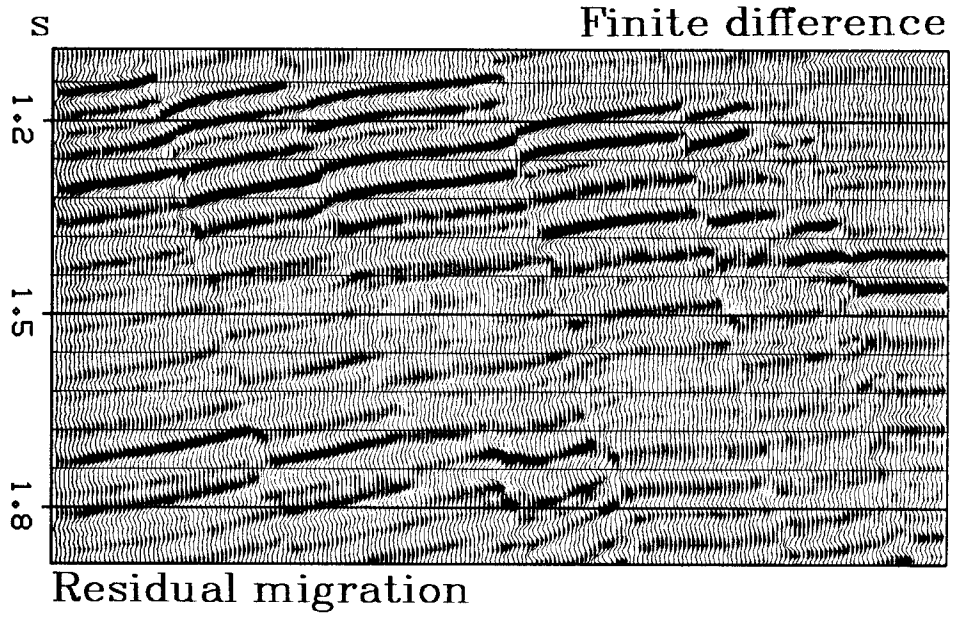


(1b - Fully migrated)

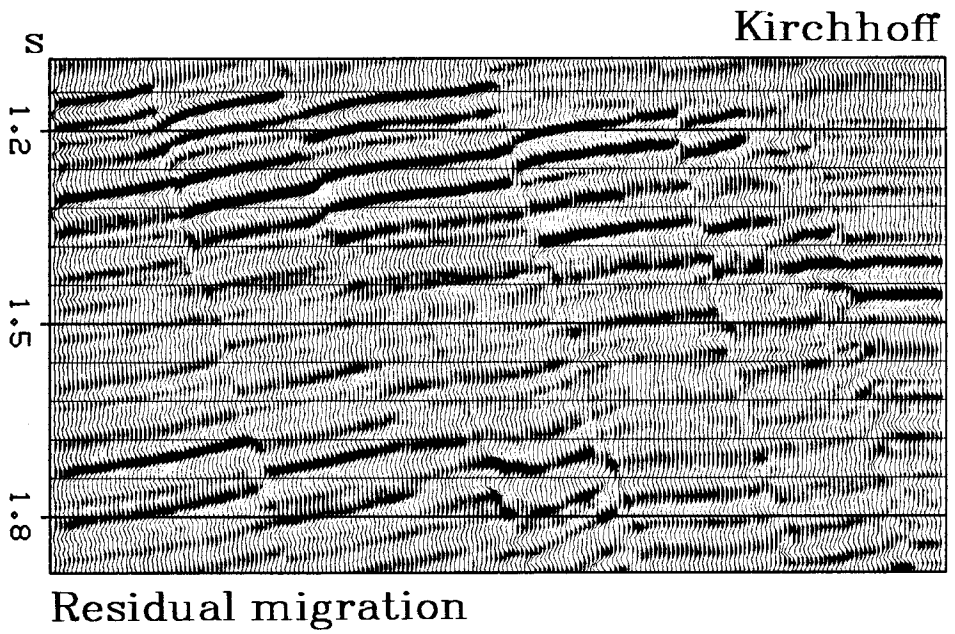
FIG. 1. Unmigrated (1a) and fully migrated (1b), stacked section of data from the Gulf of Mexico used in SEP-35. Trace spacing is 33m and sampling interval is 8 msec. Reflection time is labeled in seconds. True migration velocities range from 1650 m/sec to 2000 m/sec. The steepest dips are between 30 and 40 degrees. The migration used the Kirchhoff algorithm.



(2a - Initial Stolt)

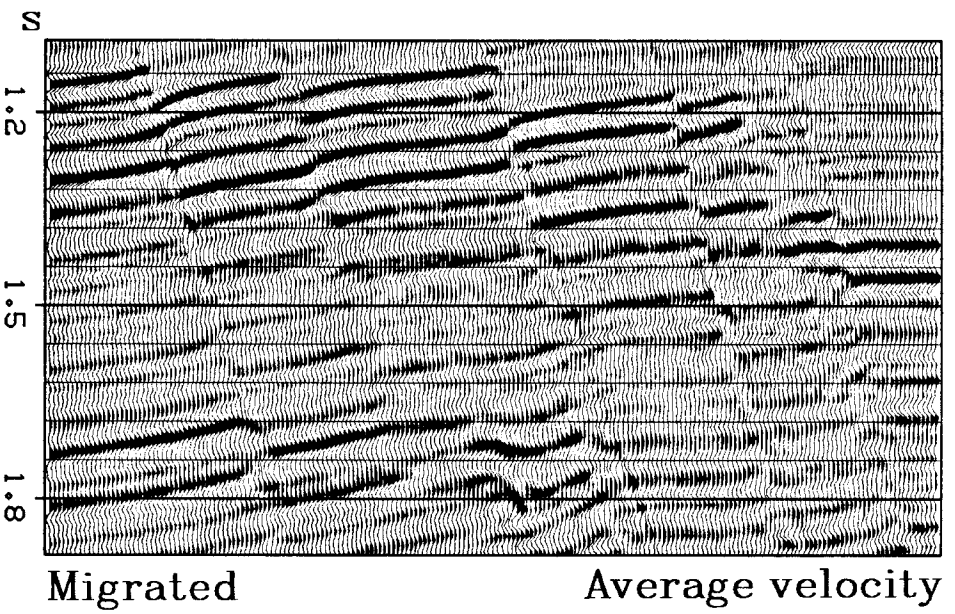


(2b - Finite difference residual)

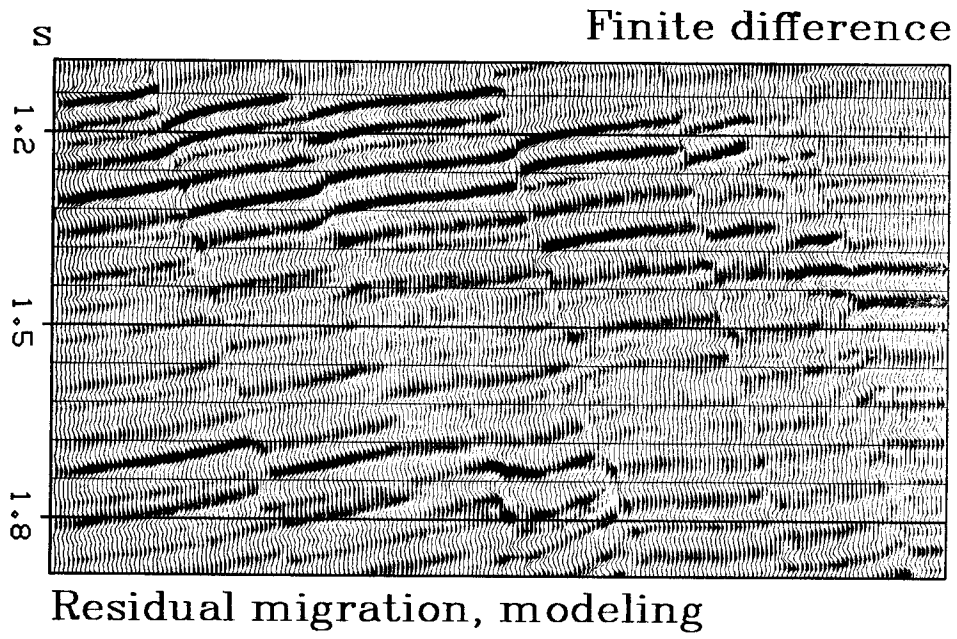


(2c - Kirchhoff residual)

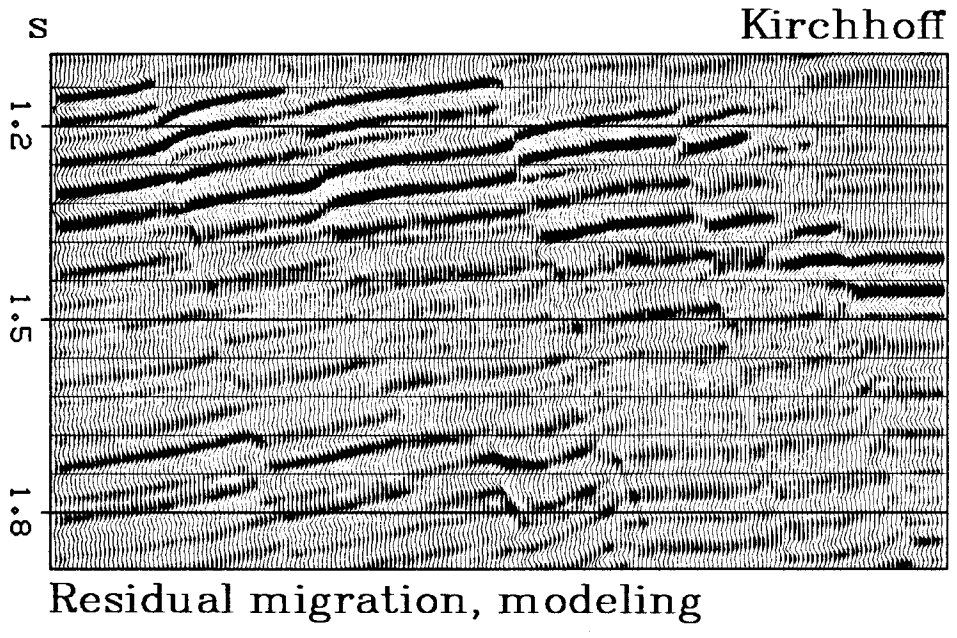
FIG. 2. (2b) is a finite difference residual migration of the data of (2a) where Stolt migration with v_m = 1600m/sec was performed. (2c) is a Kirchhoff residual migration of the same input data.



(3a - Initial Stolt)

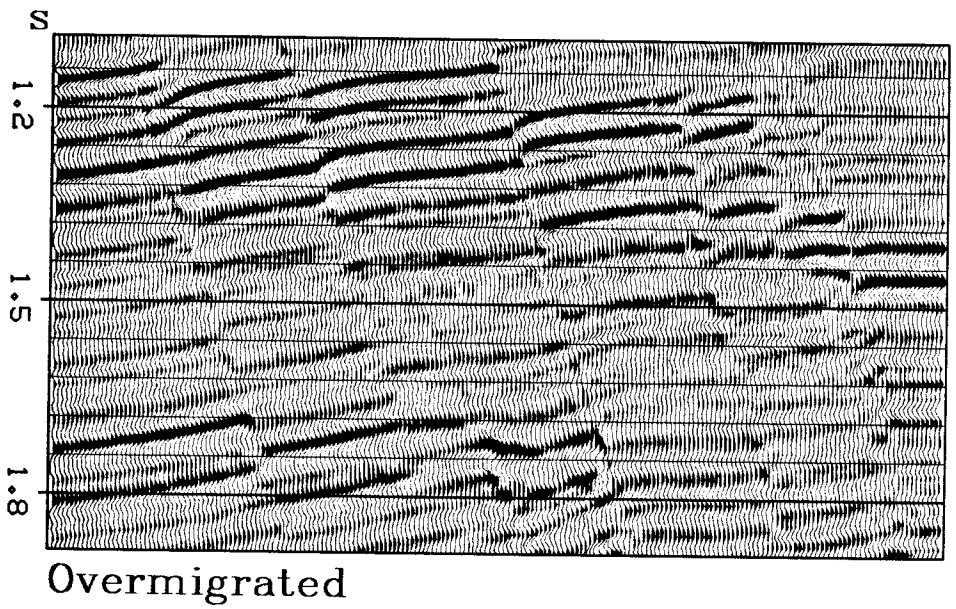


(3b - Finite difference residual)

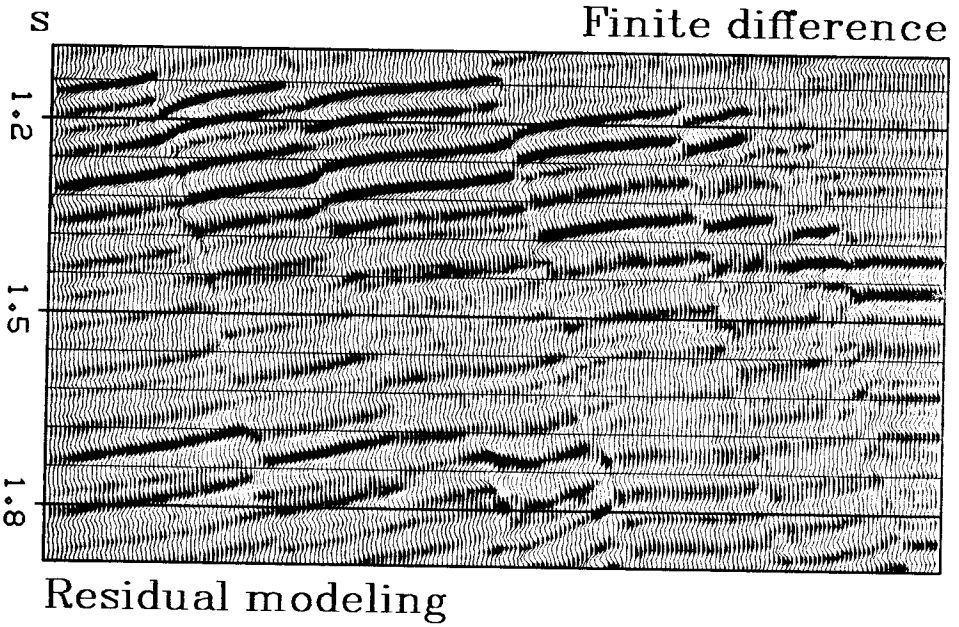


(3c - Kirchhoff)

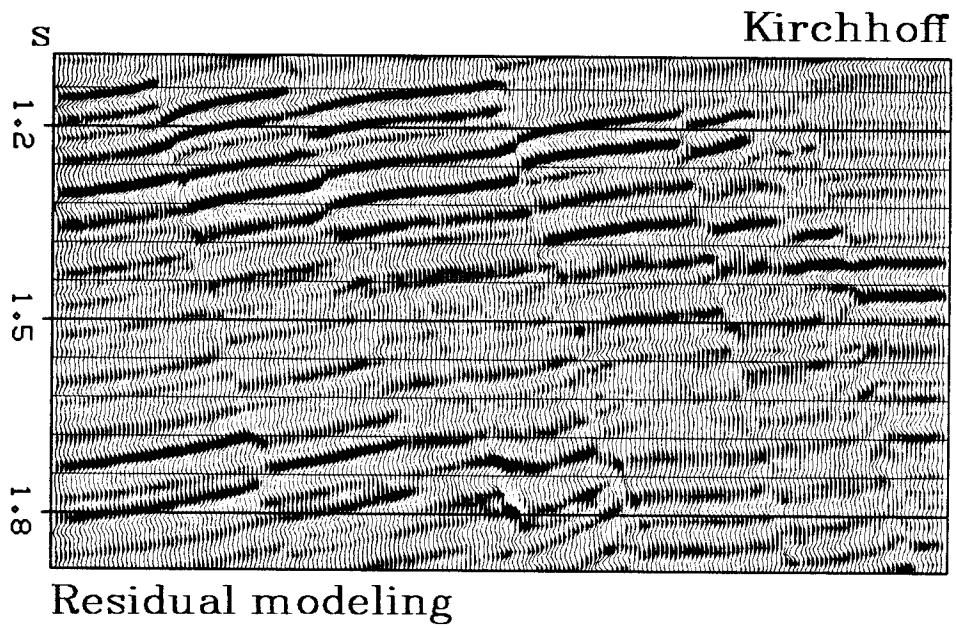
FIG. 3. The result of finite difference (3b) and Kirchhoff (3c) residual migrations after an initial Stolt migration (3a) at an intermediate velocity of 1750 m/sec.



(4a - Initial Stolt)



(4b - Finite difference residual)



(4c - Kirchhoff residual)

FIG. 4. Here the data were initially overmigrated (a) with a Stolt migration at a velocity of 2050 m/sec. Finite difference (4b) and Kirchhoff (4c) residual migrations have successfully "unmigrated" the fault plane reflections to their desired positions.

Revised Itinerary for the Royal Visit to Stanford

Because of the recent flooding and storm damage, changes have been made in the the itinerary for the royal party's visit to Stanford University. Below is the revised schedule for Thursday, March 3.

11:40 The royal motorcade leaves San Francisco.

12:20 The royal motorcade arrives at Stanford University. Queen Elizabeth II and Prince Philip are dropped off at Terman Engineering Building, while the motorcade leaves in search of parking spaces. The Queen and the Prince are greeted by a stirring rendition of *God Save the Queen*, sung by Australian graduate student Peter Mora. Then, in accordance with a quaint Australian custom, Mr. Mora will throw rotten grapes at the two royal visitors.

12:25 The Queen and the Prince are escorted to the basement of Terman, where they join the line for food at Nuts & Mud, a small eating establishment.

12:35 Having purchased their lunch, Queen Elizabeth II and Prince Philip join the line for the microwave oven. Although the menu is being kept secret, it has been rumored that the Queen will have a chicken pot pie, while it has been suggested that the Prince is partial to potato knishes.

12:45 The royal party walks to Mitchell Earth Sciences Building, where they eat lunch while waiting for the elevator.

1:00 Queen Elizabeth II and Prince Philip arrive on the fourth floor for their scheduled tour of the Stanford Exploration Project research facilities. Immediately upon arrival, they participate in a ribbon-cutting ceremony to mark the completion of the new terminal room.

1:05 Graduate students Jeff Thorson and Ron Ullmann give Queen Elizabeth II and Prince Philip a detailed tour of the computer room, concentrating especially on the SEP's unique temperature-monitoring system.

1:20 Graduate students Rick Ottolini and Chuck Sword show the AED graphics device to the Queen, and demonstrate its versatility by showing the Queen all the movies generated by the Project over the past year. (Contrary to rumor, graduate student Paul Fowler will *not* be showing her the *other* SEP film library.) Prince Philip, meanwhile, gives job interviews to all MSE students who are interested in working for British Petroleum for the summer (sign-up sheets are on the third-floor bulletin board).

2:00 When Queen Elizabeth II and Prince Philip are re-united, Professor Jon Claerbout explains to them his theories on why headers on data files are far superior to command-line parameters. He shows them numerous examples on the GIGI graphics terminal.

2:20 Having left via the Mitchell Building stairway in the interest of speed, the royal visitors walk to the Tresidder pay-parking lot, where they are re-united with their motorcade.

2:30 The royal motorcade leaves Stanford University for Cupertino, where Queen Elizabeth II and Prince Philip are scheduled to tour the Hewlett-Packard manufacturing plant.