Deformation Transformation

Jon F. Clærbout

Familiar deformation transformations are:

(1) normal moveout correction (NMO)

(2) radial trace transformation (RMO)

(3) frequency domain stretch in Stolt migration

(4) Stolt stretch for depth-variable velocity

In principle, such transforms may be invertible, but in the computer they are often not. In some applications it may be necessary to ensure invertibility in a computer. This generally happens when some process requires iterative transformation and inverse transformation. For example, iteration might be done in studies involving:

(1) missing data

(2) extract wavelet in offset space and radial space

Distributive Property

Consider linear decompositions of a data set. For example, L could be a lowpass filter and $H = 1 - L$ the complementary highpass filter.

$$\mathbf{d} \;=\; H\,\mathbf{d} + L\,\mathbf{d} \tag{1}$$

Alternately, M could be a matrix of ones and zeros where the ones multiply missing data values. The complementary matrix $K = I - M$ selects data values which were recorded, and are known.

$$\mathbf{d} \;=\; M\,\mathbf{d} + K\,\mathbf{d} \tag{2}$$

Let N denote the operation of normal moveout. The distributive property of NMO is:

$$N d \quad = \quad N H d + N L d \quad = \quad N (H + L) d \tag{3}$$

I guess most people's NMO programs are distributive because they are linear operators.

Invertibility

Many of the operators that arise in geophysics are invertible in a subspace, but not in the whole space. This is also true of deformations like NMO and RMO.

A matrix transformation with much the character of NMO is

$$N \quad = \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The transform N throws away the first time point and stretches out the next two. The best we can do at finding an inverse to a matrix like N is

$$N^{-} \quad = \quad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \tag{5}$$

Multiplying shows that the matrix $N^{-}N$ is idempotent, i.e.

$$N^{-}N \quad = \quad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad = \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

It is only around the periphery that information is lost. Idempotence means that repeated transformation and inverse transformation will not accumulate a growing mess, because

$$N^{-}N d \quad = \quad N^{-}N \; N^{-}N d \tag{7}$$

To make a deformation transformation idempotent, it seems to be necessary that the transformation stretches (does not shrink) in the interior region. It is the programmer's job to decide whether invertibility is required in the interior, and if it is, to provide the required space.

In some applications it may be annoying that the inverse of NMO is not its transpose $N^{-} \neq N^{T}$. There are two ways to handle this. First, we could redefine NMO with

$$N \quad = \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{1/2} \\ 0 & 0 & \sqrt{1/2} \end{bmatrix} \tag{8}$$

A problem with the approach of (8) is that the function being stretched will become quite jagged. The spectrum in transform space may be quite different from what you had in mind. The alternative is to define inner products differently, so that something like $N^T N$ never arises, rather, something like $N^T W N$ where

$$N^- N \quad = \quad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad = \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

This means that your NMO program works out the weighting matrix W but does not apply it. You apply the weighting matrix W after filtering but before inverse transformation.

Π SΠ M Γ ο ν π α σίτ ο ξ τ			
υε Cθξ ιρ φε		υhΣ ο σ∂	γ ο ν πφυαυιο ξ
‡ ιμν α φ ∇Bμο ξ δ ι Mε χ ιξ	$P_{t\tau} = \alpha \left(\frac{h}{t}\right)^{\beta} P_{xx}$	"≃→0" ο ζζτ ε υ αξ η με T	χ ε συιγ α μ η σιδ σε τ α ν π μ ξ η ο ξ ε ο ζζτ ε υ
Θ α με	$\omega \left[\simeq + \frac{h^2 k_x^2}{t^2 \omega^{\sim}} \right]^{1/2}$	ψ ιδ e ο ζζτ ε υ αξ η lε τ	σε γ ο ν πφυε πθατε τθιζυ ο ξ ε ο ζζτ ε υ
Ο υυο μξ ι	$\omega \left[\simeq - \frac{r^2 k_{\tilde{x}}}{\sim \omega^2} \right]^{1/2}$	> ≃→0 ο ζζτ ε υ αξ η με τ	ιξ υε σπ ο μαυε ν αξ ∂ ο ζζτ ε υτ ε ατ ∂ ν ιη σαυιο ξ