Linear properties of Stolt migration and diffraction

William Harlan

The properties

Constant-velocity migration offers to be useful in various descent and signal estimation problems. One may simplify the algebra involved with attention to the linear properties of migration.

Stolt migration exactly images the wavefield received from an exploding reflector in a constant velocity medium. Diffraction solves the corresponding forward problem. We shall write these linear transformations as

$$D = W \cdot G \; ; \qquad G = M \cdot D \; . \tag{1}$$

where W is diffraction, M migration, G the earth model, and D the corresponding data. The Stolt transformations destroy certain information: 1) D may contain noise at high dips, requiring imaginary frequencies in G, frequencies which we do not save; 2) Stolt diffraction does not sample the depth-spatial-frequency axis, k_τ, sufficiently when discrete transforms are used, with the result that late diffraction hyperbolas wrap around to the top--migration treats these wrap-arounds as shallow events. Both D and G may contain information not expressed in the other domain. The second limitation may be ignored by padding zeros onto geologic models. We shall examine integral transforms, so only the first limitation will appear explicitly. Thus diffraction may be exactly inverted, but not migration.

Let us begin by assuming knowledge of the forward modeling operator, W (diffraction). Migration, M, is the left inverse of W, where

$$M \cdot W = I, \tag{2}$$

$$but \;\; W \cdot M = A \neq I. \tag{3}$$

I is the identity operator; A is an operator with zeros and ones on the diagonal in the frequency domain. Thus migration inverts diffraction but not vice versa. We shall demonstrate both (2) and (3) for Stolt's definitions of the migration and diffraction integral transforms. The solution of $D = W \cdot G$ for G is an over-determined problem. The least-squares solution for G given W and D defines migration as

$$G = [(W^* \cdot W)^{-1} \cdot W^*] \cdot D \equiv M \cdot D \ , \tag{4}$$

where the asterisk indicates the adjoint. We shall shortly show that Stolt's migration and diffraction satisfy

$$M = B_1 \cdot W^*, \ \textit{where } B_1 \textit{ is diagonal over } k_x \textit{ and } k_\tau \textit{ and is invertible}, \tag{5}$$

$$\textit{and } W = B_2 \cdot M^*, \ \textit{where } B_2 \textit{ is diagonal over } k_x \textit{ and } \omega \textit{ and is not invertible.} \tag{6}$$

In the frequency domain the diagonal elements of B_1 and B_2 are real and contain the so-called cosine corrections. Except for these corrections, migration is the adjoint of forward modeling. Equations (2) and (5) being satisfied by Stolt's migration and diffraction directly shows (4) to be satisfied--migration is a least-squares solution of (1).

Demonstration of the properties

Let $\overline{G}$ and $\overline{D}$ be the Fourier transforms of G and D. Let $\overline{W}$ and $\overline{M}$ be the Stolt transforms in this new domain.

$$\overline{M} = F \cdot M \cdot F^* \ ; \qquad \overline{W} = F \cdot W \cdot F^* \ . \tag{7}$$

F is the forward Fourier transform. Stolt migration may be written as

$$\overline{G}(k_\tau, k_x) = \overline{M} \cdot \overline{D} \tag{8}$$

$$= \frac{|k_\tau|}{\sqrt{v^2 k_x^2 + k_\tau^2}} \int_{-\infty}^{\infty} \overline{D}(\omega, k_x) \cdot \delta(\omega - sign(k_\tau) \sqrt{v^2 k_x^2 + k_\tau^2}\,) \, d\omega \ .$$

The cosine factor before the integral may be regarded as a diagonal operator over k_x and k_τ. Recall that

$$\delta(f(x)) = \sum_n \frac{1}{|f'(x_n)|} \delta(x - x_n) \ ,$$

where x_n are the zeros of $f(x)$. Thus the adjoint of M is

$$\overline{M}^* \cdot \overline{G} = H(\omega^2 - v^2 k_x^2) \int_{-\infty}^{\infty} \overline{G}(k_\tau, k_x) \delta(k_\tau - sign(\omega) \sqrt{\omega^2 - v^2 k_x^2}) \, dk_\tau \ , \tag{9}$$

$$\textit{where } H(x) \equiv \begin{cases} 1 & x \geq 1 \\ 0 & x < 1 \end{cases} .$$

The derivative of the argument of the delta function and the cosine factor have neatly canceled. The Heaviside function $H(x)$ is necessary because (8) transforms only to real k_τ.

Likewise diffraction may be written

$$\overline{D}(\omega, k_x) = \overline{W}\cdot\overline{G} \tag{10}$$

$$= \frac{|\omega|}{\sqrt{\omega^2 - v^2k_x^2}} H(\omega^2 - v^2k_x^2) \int_{-\infty}^{\infty} \overline{G}(k_\tau, k_x)\delta(k_\tau - sign(\omega)\sqrt{\omega^2 - v^2k_x^2})\, dk_\tau ,$$

and its adjoint

$$\overline{W}^*\cdot\overline{D} = \int_{-\infty}^{\infty} \overline{D}(\omega, k_x)\cdot\delta(\omega - sign(k_\tau)\sqrt{v^2k_x^2 + k_\tau^2}\,)\, d\omega . \tag{11}$$

We observe that

$$M\cdot W\cdot G(k_\tau, k_x) = G(k_\tau, k_x) , \tag{12}$$

but

$$W\cdot M\cdot D(\omega, k_x) = H(\omega^2 - v^2k_x^2)\cdot D(\omega, k_x). \tag{13}$$

We have verified (2) and (3): diffraction is invertible, migration is not. Let

$$\overline{B}_1 = F\cdot B_1\cdot F^* \qquad \overline{B}_2 = F\cdot B_2\cdot F^* \tag{14}$$

Then we observe that

$$\overline{M} = \overline{B}_1\cdot\overline{W}^* \qquad \overline{W} = \overline{B}_2\cdot\overline{M}^* \tag{5,6a}$$

where

$$\overline{B}_1 = \frac{|k_\tau|}{\sqrt{v^2k_x^2 + k_\tau^2}}; \qquad \overline{B}_2 = \frac{|\omega|}{\sqrt{\omega^2 - v^2k_x^2}} H(\omega^2 - v^2k_x^2) . \tag{15}$$

The above transform easily into (5) and (6). Now we may prove that migration is the least-squares inverse of diffraction. By (2) and (5)

$$B_1\cdot W^*\cdot W = M\cdot W = I \quad\rightarrow\quad W^*\cdot W = B_1^{-1}$$

$$\rightarrow\quad (W^*\cdot W)^{-1}\cdot W^* = B_1\cdot W^* = M.$$

16 Mar 1983 1224-PST F.FRANCIS at SU-LOTS-A about REAL programmers -- humor (1943 chars; more?)

brought to you from IBM by t.thekida -- I'm just passing it along. My appologies if such a thing has appeared on system recently

REAL PROGRAMMERS DON'T WRITE SPECS

Real Programmers don't write specs -- users should consider themselves lucky to get any programs at all and take what they get

Real Programmers don't comment their code. If it was hard to write, it should be hard to understand.

Real Programmers don't write application programs; they program right down on the bare metal. Application programming is for feebs who can't do systems programming.

Real Programmers don't eat quiche. They eat Twinkies, and Szechwan food.

Real Programmers don't write in COBOL. COBOL is for wimpy applications programmers.

Real Programmers' programs never work right the first time. But if you throw them on the machine they can be patched into working in "only a few" 30-hour debugging sessions.

Real Programmers don't write in FORTRAN. FORTRAN is for pipe stress freaks and crystallography weenies.

Real Programmers never work 9 to 5. If any real programmers are around at 9 AM, it's because they were up all night.

Real Programmers don't write in BASIC. Actually, no programmers write in BASIC, after the age of 12.

Real Programmers don't write in PL/I. PL/I is for programmers who can't decide whether to write in COBOL or FORTRAN.

Real Programmers don't play tennis, or any other sport that requires you to change clothes. Mountain climbing is OK, and real programmers wear their climbing boots to work in case a mountain should suddenly spring up in the middle of the machine room.

Real Programmers don't document. Documentation is for simps who can't read the listings or the object deck.

Real Programmers don't write in PASCAL, or BLISS, or ADA, or any of those pinko computer science languages. Strong typing is for people with weak memories.
